Lecture #2

Three-phase Induction Machines

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- Winding arrangement for a 2-pole, 3 phase, wye-connected symmetrical induction machine is shown in Fig. 1. Stator windings are identical, sinusoidally distributed windings, displaced by 120°, with N_s equivalent turns and resistance r_s.

- Consider the case when rotor windings are also three identical sinusoidally distributed windings, displaced 120°, with N_r equivalent turns and resistance r_r.

- In abc reference frame, voltage equations can be written as

$$V_{abcs} = r_s i_{abcs} + p\lambda_{abcs}$$
$$V_{abcr} = r_r i_{abcr} + p\lambda_{abcr}$$

$$(f_{abcs})^T = [f_{as} \quad f_{bs} \quad f_{cs}]$$

$$(f_{abcr})^T = [f_{ar} \quad f_{br} \quad f_{cr}]$$

s: denotes variables and parameters associated with the stator circuits

r: denotes variables and parameters associated with the rotor circuits

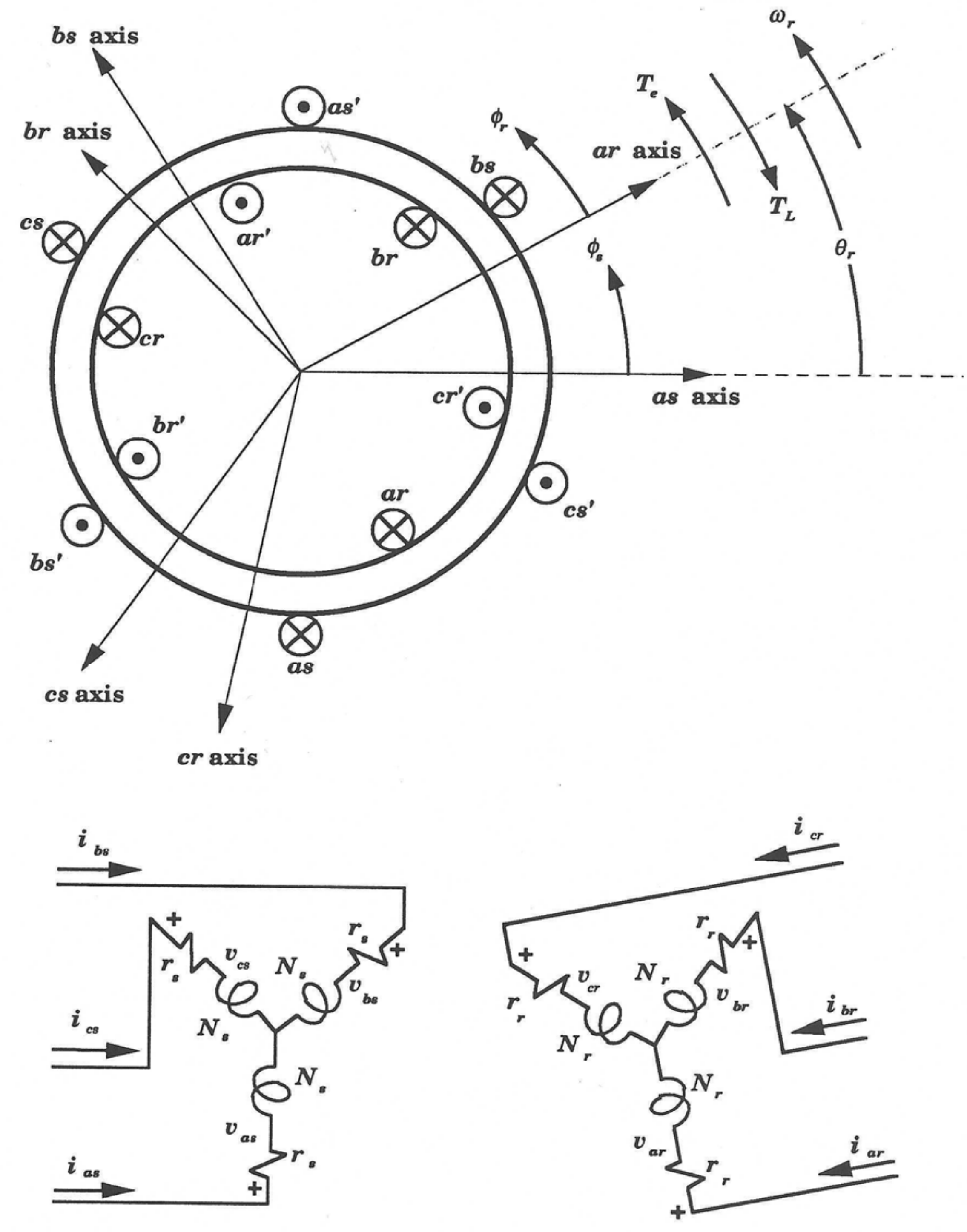

FIGURE 1
A two-pole three-phase symmetrical induction machine.

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ (L_{sr})^T & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix}$$

where

$$L_s = \begin{bmatrix} L_{ls}+L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls}+L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls}+L_{ms} \end{bmatrix}$$

$$L_r = \begin{bmatrix} L_{lr}+L_{mr} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{lr}+L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{lr}+L_{mr} \end{bmatrix}$$

$$L_{sr} = L_{sr} \begin{bmatrix} \cos\theta_r & \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) \\ \cos(\theta_r-\frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r+\frac{2\pi}{3}) \\ \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) & \cos\theta_r \end{bmatrix}$$

L_{ls} and L_{ms} are, respectively, the leakage and magnetizing inductances of the stator windings

L_{lr} and L_{mr} are for the rotor winding (leakage and magnetising inductances)

'L_{sr}': is the amplitude of the mutual inductances between stator and rotor windings

- A majority of induction machines are not equipped with coil-wound rotor windings; instead, the current flows in copper or aluminum bars which are uniformly distributed in a common ring at each end of the rotor. This type of rotor is referred to as a squirrel-cage rotor.

- Rotor variables can be referred to the stator windings by appropriate turns ratio.

$$i'_{abcr} = \frac{N_r}{N_s} i_{abcr}$$

$$V'_{abcr} = \frac{N_s}{N_r} V_{abcr}$$

$$\lambda'_{abcr} = \frac{N_s}{N_r} \lambda_{abcr}$$

$$L_{ms} = \frac{N_s}{N_r} L_{sr}$$

$$[L'_{sr}] = \frac{N_s}{N_r}[L_{sr}]$$

$$= L_{ms} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r \end{bmatrix}$$

Also,

$$L_{mr} = \left(\frac{N_r}{N_s}\right)^2 L_{ms}$$

$$[L'_r] = \left(\frac{N_r}{N_s}\right)^2 [L_r]$$

$$[L'_r] = \begin{bmatrix} L'_{\ell r} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L'_{\ell r} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L'_{\ell r} + L_{ms} \end{bmatrix}$$

where

$$L'_{\ell r} = \left(\frac{N_s}{N_r}\right)^2 L_{\ell r}$$

Flux linkages may be expressed as

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda'_{abcr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ (\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i'_{abcr} \end{bmatrix}$$

Voltage equations expressed in terms of machine variables referred to the stator windings may be written as

$$\begin{bmatrix} V_{abcs} \\ V'_{abcr} \end{bmatrix} = \begin{bmatrix} r_s + p\mathbf{L}_s & p\mathbf{L}'_{sr} \\ p(\mathbf{L}'_{sr})^T & r'_r + p\mathbf{L}'_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i'_{abcr} \end{bmatrix}$$

where

$$r'_r = \left(\frac{N_s}{N_r}\right)^2 r_r$$

Torque Equation in Machine Variables

- Energy stored in the coupling field may be written as

$$W_c = W_f = \frac{1}{2}(\mathbf{i}_{abcs})^T(\mathbf{L}_s - L_{ls}\mathbf{I})\,\mathbf{i}_{abcs} + (\mathbf{i}_{abcs})^T \mathbf{L}'_{sr}\,\mathbf{i}'_{abcr}$$

$$+ \frac{1}{2}(\mathbf{i}'_{abcr})^T(\mathbf{L}'_r - L'_{lr}\mathbf{I})\,\mathbf{i}'_{abcr}$$

$\mathbf{I}$: an identity matrix

$$T_e(i_j, \theta_r) = \frac{P}{2}\frac{\partial W_c(i_j, \theta_r)}{\partial \theta_r}$$

- Since $\mathbf{L}_s$ and $\mathbf{L}'_r$ are functions of θ_r, the above equation for the electromagnetic torque yields.

$$T_e = \left(\frac{P}{2}\right)(\mathbf{i}_{abcs})^T \frac{\partial}{\partial \theta_r}[\mathbf{L}'_{sr}]\,\mathbf{i}'_{abcr}$$

or

$$T_e = -\left(\frac{P}{2}\right)L_{ms}\Big\{\Big[i_{as}\left(i'_{ar} - \frac{1}{2}i'_{br} - \frac{1}{2}i'_{cr}\right) + i_{bs}\left(i'_{br} - \frac{1}{2}i'_{ar} - \frac{1}{2}i'_{cr}\right) + i_{cs}\left(i'_{cr} - \frac{1}{2}i'_{br} - \frac{1}{2}i'_{ar}\right)\Big]\sin\theta_r$$

$$+ \frac{\sqrt{3}}{2}\Big[i_{as}(i'_{br} - i'_{cr}) + i_{bs}(i'_{cr} - i'_{ar}) + i_{cs}(i'_{ar} - i'_{br})\Big]\cos\theta_r$$

The torque and rotor speed are related by

$$T_e = J\left(\frac{2}{P}\right)p\,\omega_r + T_L$$

Equations of Transformation for Rotor Circuits.

- In the analysis of induction machines it is desirable to transform the variables associated with the symmetrical rotor windings to the arbitrary reference frame.

$$f'_{qd0r} = K_r f'_{abcr}$$

$$(f'_{qd0r})^T = [f'_{qr} \; f'_{dr} \; f'_{0r}]$$

$$(f'_{abcr})^T = [f'_{ar} \; f'_{br} \; f'_{cr}]$$

$$K_r = \frac{2}{3}\begin{bmatrix} \cos\beta & \cos(\beta - \frac{2\pi}{3}) & \cos(\beta + \frac{2\pi}{3}) \\ \sin\beta & \sin(\beta - \frac{2\pi}{3}) & \sin(\beta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

where $\beta = \theta - \theta_r$ (see Fig. 2)

$$\theta_r = \int_0^t \omega_r(t)\,dt + \theta_r(0)$$

$$(K_r)^{-1} = \begin{bmatrix} \cos\beta & \sin\beta & 1 \\ \cos(\beta - \frac{2\pi}{3}) & \sin(\beta - \frac{2\pi}{3}) & 1 \\ \cos(\beta + \frac{2\pi}{3}) & \sin(\beta + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

"r" subscript indicates the variables, parameters and transformation associated with rotating circuits.

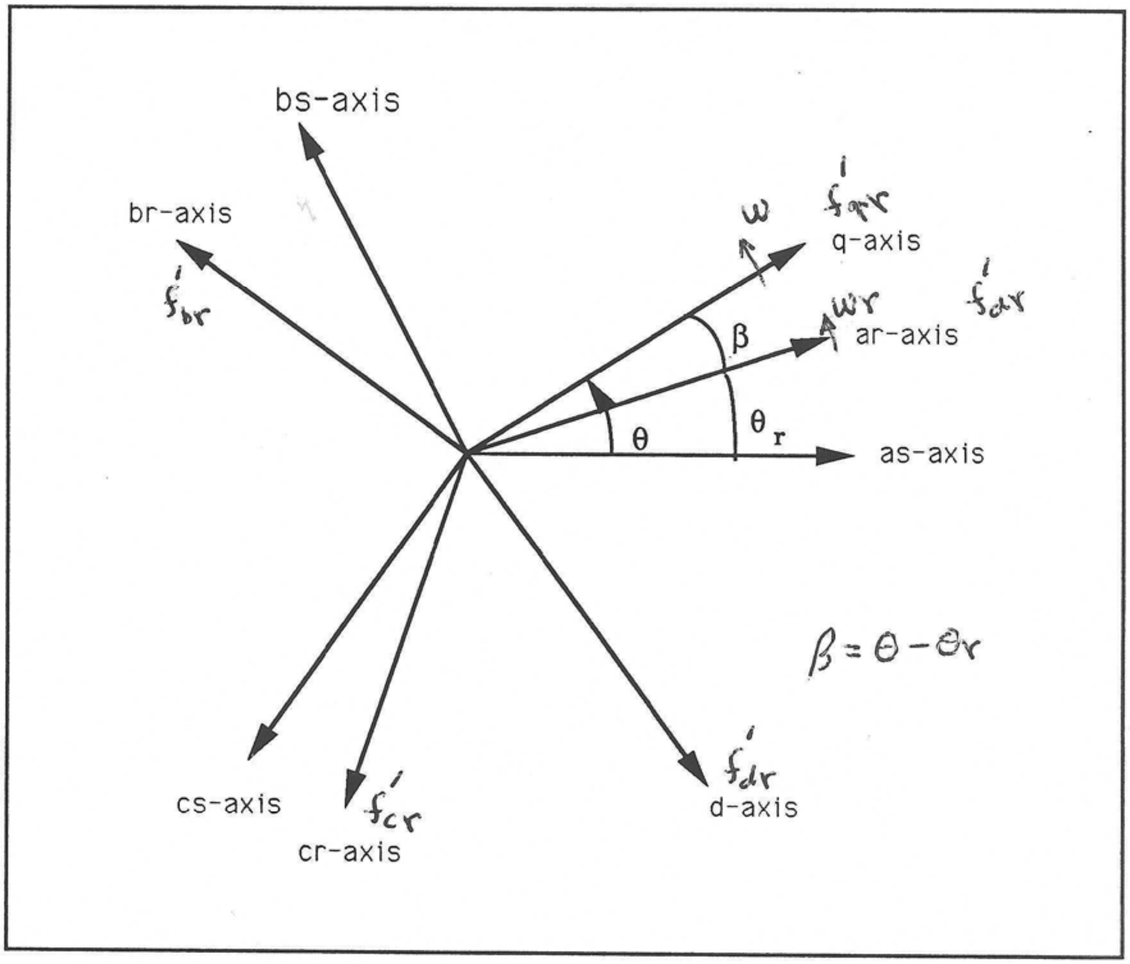

Fig. 2 Axis of 2-pole, 3-phase symmetrical machine