Lecture #1
Reference - Frame Theory.

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- Synchronous and induction machine inductances are functions of the rotor speed, therefore the coefficients of the differential equations (voltage equations) which describe the behavior of these machines are time-varying.

- A change of variables can be used to reduce the complexity of machine differential equations, and represent these equations in another reference frame with constant coefficients

- A change of variables which formulates a transformation of the 3-phase variables of stationary circuit elements to the arbitrary reference frame may be expressed

$$f_{qd0s} = K_s f_{abcs}$$

$$(f_{qd0s})^T = [f_{qs} \;\; f_{ds} \;\; f_{0s}]$$

$$(f_{abcs})^T = [f_{as} \;\; f_{bs} \;\; f_{cs}]$$

$$K_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\theta = \int_0^t \omega(t)\,dt + \theta(0)$$

Reference. Analysis of Electric Machinery, P.C. Krause

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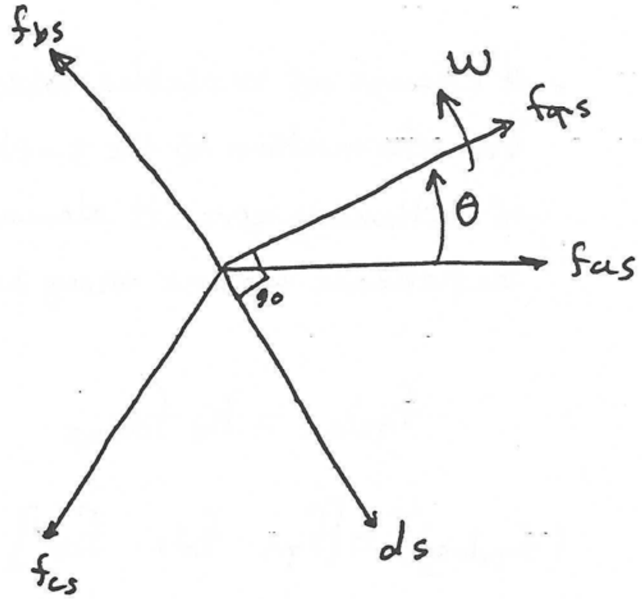
$$(K_s)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix}$$

"f" can represent either voltage, current, or flux linkage.

"s" indicates the variables, parameters and transformation associated with stationary circuits.

"w" represent the speed of reference frame.



$w = 0$ stationary reference frame

$w = w_e$ synchronously rotating reference frame

$w = w_r$ rotor reference frame (i.e the reference frame is fixed on the rotor)

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- f_{as}, f_{bs} and f_{cs} may be thought of as the direction of the magnetic axes of the stator windings.
- f_{qs} and f_{ds} can be considered as the direction of the magnetic axes of the "new" fictious windings located on qs and ds axis which are created by the change of variables.

- $$P_{abcs} = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs}$$

$$P_{qdos} = P_{abcs}$$

$$= \frac{3}{2} (v_{qs} i_{qs} + v_{ds} i_{ds} + 2 v_{os} i_{os})$$

- Stationary circuit variables Transformed to the arbitrary reference frame.

- Resistive elements. For a 3-phase resistive circuit

$$v_{abcs} = \bar{r}_s \, i_{abcs}$$

$$i_{abcs} = (K_s)^{-1} i_{qdos}$$

$$v_{abcs} = (K_s)^{-1} v_{qdos}$$

$$(K_s)^{-1} v_{qdos} = \bar{r}_s (K_s)^{-1} i_{qdos}$$

$\times (K_s)$

$$v_{qdos} = (K_s) \, \bar{r}_s \, (K_s)^{-1} i_{qdos}$$

$$v_{qdos} = \bar{r}_s \, i_{qdos}$$

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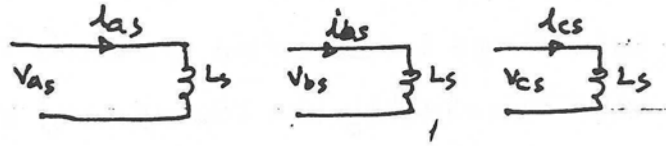
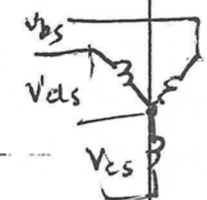
$$(K_s)\bar{r}_s(K_s^{-1}) = r_s \qquad \bar{r}_s = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}$$

- Inductive elements. For a 3-phase inductive circuit

$$v_{abcs} = p\,\lambda_{abcs} \qquad p = \frac{d}{dt}$$





$$\lambda_{abcs} = L_s\, i_{abcs} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

- In terms of the substitute variables, we have

$$\boxed{\begin{aligned} V_{qdos} &= K_s\, p\left[K_s^{-1}\lambda_{qdos}\right] \\ \text{or} \quad V_{qdos} &= K_s\, p\left[(K_s)^{-1}\right]\lambda_{qdos} + K_s(K_s^{-1})\,p\,\lambda_{qdos} \end{aligned}}$$

where,

$$p\left[(K_s)^{-1}\right] = \omega \begin{bmatrix} -\sin\theta & \cos\theta & 0 \\ -\sin(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 0 \\ -\sin(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) & 0 \end{bmatrix}$$

- After some work, we can show that

$$K_s\, p\left[(K_s)^{-1}\right] = \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v_{qdos} = K_s \underbrace{P\left[(K_s)^{-1}\right]}\lambda_{qdos} + \underbrace{K_s(K_s)^{-1}} P\lambda_{qdos}$$

$$v_{qdos} = \omega\lambda_{dqs} + P\lambda_{qdos}$$

where

$$(\lambda_{qds})^T = [\lambda_{ds} \quad -\lambda_{qs} \quad 0]$$

Vector equation v_{qdos} can be expressed as

$$v_{qs} = \omega\lambda_{ds} + P\lambda_{qs}$$

$$v_{ds} = -\omega\lambda_{qs} + P\lambda_{ds}$$

$$v_{os} = P\lambda_{os}$$

where "$\omega\lambda_{ds}$" term and "$\omega\lambda_{qs}$" term are referred to as a "speed voltage" with the speed being the angular velocity of the arbitrary reference frame.

- when the reference frame is fixed in the stator that is, the stationary reference frame ($\omega = 0$), the voltage equations for the three-phase circuit become the familiar time rate of change of flux linkage in abcs reference frame

For the three-phase circuit shown, L_s is a diagonal matrix, and

$$\lambda_{abcs} = L_s i_{abcs}$$

$$\lambda_{qdos} = \underbrace{K_s L_s K_s^{-1}} i_{qdos}$$

$$\lambda_{qdos} = L_s i_{qdos}$$

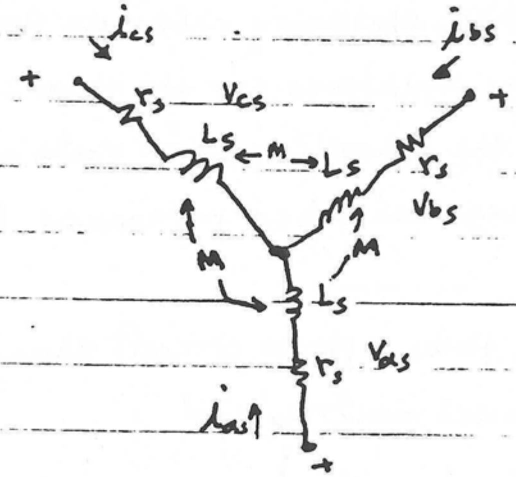
- For three-phase induction or synchronous machines, L_s matrix is expressed as

$$L_s = \begin{bmatrix} L_{ls}+L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls}+L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls}+L_{ms} \end{bmatrix}$$

L_{ls} : leakage inductance , L_{ms} : magnetising inductance.

$$K_s L_s (K_s)^{-1} = \begin{bmatrix} L_{ls}+\frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls}+\frac{3}{2}L_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

- Consider the stator windings of a symmetrical induction or round rotor synchronous machine shown below:



$$r_s = \mathrm{diag}\,[\,r_s \quad r_s \quad r_s\,]$$

$$L_s = \begin{bmatrix} L_s & M & M \\ M & L_s & M \\ M & M & L_s \end{bmatrix}$$

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- $L_s = L_{\ell s} + L_{ms}$, $M = -\frac{1}{2} L_{ms}$

For each phase voltage, we write the following equations

$$V_{as} = r_s i_{as} + p\lambda_{as}$$
$$V_{bs} = r_s i_{bs} + p\lambda_{bs}$$
$$V_{cs} = r_s i_{cs} + p\lambda_{cs}$$

$$V_{qdos} = K_s V_{abcs}$$
$$i_{qdos} = K_s i_{abcs} \qquad \lambda_{abcs} = L_s i_{abcs}$$
$$\lambda_{qdos} = K_s \lambda_{abcs}$$

In vector form,

$$V_{abcs} = r_s i_{abcs} + p\lambda_{abcs}$$

· x by K_s

$$K_s V_{abcs} = K_s r_s (i_{abcs}) + K_s p (\lambda_{abcs})$$

replace i_{abcs} and λ_{abcs} using the transformation equations

$$K_s V_{abcs} = K_s r_s (K_s^{-1} i_{qdos}) + K_s p [K_s^{-1} \lambda_{qdos}]$$

$$V_{qdos} = r_s i_{qdos} + \bar{\omega} \bar{\lambda}_{dqos}$$

or

$$V_{qs} = r_s i_{qs} + \omega \lambda_{ds} + p\lambda_{qs}$$
$$V_{ds} = r_s i_{ds} - \omega \lambda_{qs} + p\lambda_{ds}$$
$$V_{os} = r_s i_{os} + p\lambda_{os}$$

where

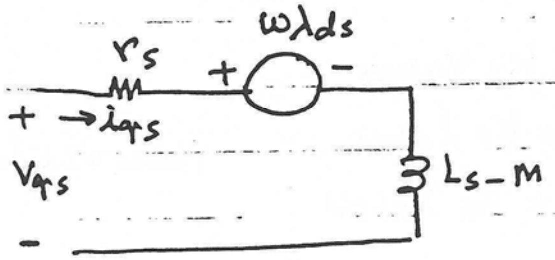
$$\bar{\omega} = \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

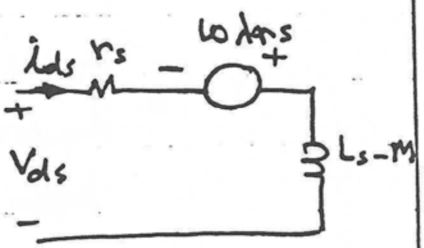
$$\lambda_{qs} = (L_s - M)\, i_{qs}$$

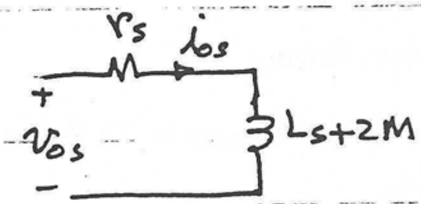
$$\lambda_{ds} = (L_s - M)\, i_{ds}$$

$$\lambda_{os} = (L_s + 2M)\, i_{os}$$

our equivalent circuit in arbitrary reference frame can be represented as







commonly Used Reference Frames

ω = unspecified Stationary circuit variables referred to the arbitrary reference frame. The variables are referred to as f_{qdos} or f_{qs}, f_{ds} and f_{os} and transformation matrix is designated as K_s

$\omega = 0$ stationary circuit variables referred to the stationary reference frame. The variables are referred to as f^s_{qdos} or f^s_{qs}, f^s_{ds} and f^s_{os} and transformation matrix is designated as K^s_s

$w = w_r$ stationary circuit variables referred to a reference frame fixed in the rotor. The variables are referred to as f^r_{qdos} or f^r_{qs}, f^r_{ds} and f_{os} and transformation matrix is designated as K^r_s.

$w = w_e$ Stationary circuit variables referred to the synchronously rotating reference frame. The variables are referred to as f^e_{qdos} or f^e_{qs}, f^e_{ds} and f_{os} and transformation matrix is designated as K^e_s

f^s_{qdos}
s → stationary reference frame
qdos → d-q axes of stator variables

f^r_{qdos}
r → reference frame fixed on the rotor with speed of w_r.
qdos → d-q axes of stator variables

$$\theta_r = \int_0^t \omega_r(t)\,dt$$

f^e_{qdos}
e → synchronously rotating reference frame
qdos → d-q axes of stator variables

$$\theta_e = \int_0^t w_e(t)\,dt$$

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- Transformation of a balance set.
- Consider a three-phase circuit which is excited by a balanced three-phase voltage set. Assume the balanced set is a set of equal-amplitude sinusoidal quantities which are displaced by 120°.

$$f_{as} = \sqrt{2}\, f_s \cos \theta_{ef}$$

$$f_{bs} = \sqrt{2}\, f_s \cos\left(\theta_{ef} - \frac{2\pi}{3}\right)$$

$$f_{cs} = \sqrt{2}\, f_s \cos\left(\theta_{ef} + \frac{2\pi}{3}\right)$$

$$f_{as} + f_{bs} + f_{cs} = 0 \quad \text{(balanced set)}$$

$$\theta_{ef} = \int_0^t \omega_e(t)\,dt + \theta_{ef}(0)$$

θ_{ef}: Angular position of each electrical variable (voltage, current, and flux lenkage) is θ_{ef} with the f subscript used to denote the specific electrical variable.

θ_e: Angular position of the synchronously rotating reference frame is θ_e.

θ_e and θ_{ef} differ only in the zero position $\theta_e(0)$ and $\theta_{ef}(0)$, since each has the same angular velocity of ω_e.

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- f_{as}, f_{bs} and f_{cs} can be transformed to the arbitrary reference frame

$$\bar{f}_{qdos} = \bar{K}_s \bar{f}_{abcs}$$

After transformation, we will have.

$$f_{qs} = \sqrt{2}\, f_s \cos(\theta_{ef} - \theta)$$

$$f_{ds} = -\sqrt{2}\, f_s \sin(\theta_{ef} - \theta)$$

$$f_{os} = 0$$

- qs and ds variables form a balanced two-phase set in all reference frames except when $\omega = \omega_e$

$$f_{qs}^{e} = \sqrt{2}\, f_s \cos[\theta_{ef}(0) - \theta_e(0)]$$

$$f_{ds}^{e} = -\sqrt{2}\, f_s \sin(\theta_{ef}(0) - \theta_e(0)]$$

- In q_s^e and d_s^e reference frame, sinusoidal quantities appear as constants or dc quantities.

Balanced Steady State Phasor Relationships

- For balanced steady state conditions ω_e is constant and sinusoidal quantities can be represented as phasor variables.

$$F_{as} = \sqrt{2} \cdot F_s \cos\left[\omega_e t + \theta_{ef}(0)\right]$$

$$= Re\left[\sqrt{2}\, F_s\, e^{j\theta_{ef}(0)}\, e^{j\omega_e t}\right]$$

$$F_{bs} = \sqrt{2}\, F_s \cos\left[\omega_e t + \theta_{ef}(0) - \frac{2\pi}{3}\right]$$

$$= Re\left[\sqrt{2}\, e^{j(\theta_{ef}(0) - 2\pi/3)}\, e^{j\omega_e t}\right]$$

$$F_{cs} = \sqrt{2}\, F_s \cos\left[\omega_e t + \theta_{ef}(0) + \frac{2\pi}{3}\right]$$

$$= Re\left[\sqrt{2}\, F_s\, e^{j(\theta_{ef}(0) + 2\pi/3)}\, e^{j\omega_e t}\right]$$

Balanced steady-state qs-ds variables are

$$F_{qs} = \sqrt{2}\, F_s \cos\left[(\omega_e - \omega)t + \theta_{ef}(0) - \theta(0)\right]$$

$$= Re\left[\sqrt{2}\, F_s\, e^{j(\theta_{ef}(0) - \theta(0))}\, e^{j(\omega_e - \omega)t}\right]$$

$$F_{ds} = -\sqrt{2}\, F_s \sin\left[(\omega_e - \omega)t + \theta_{ef}(0) - \theta(0)\right]$$

$$= Re\left[j\sqrt{2}\, e^{j(\theta_{ef}(0) - \theta(0))}\, e^{j(\omega_e - \omega)t}\right]$$

f_{as} phasor can be expressed as

$$\tilde{F}_{as} = F_s\, e^{j\theta_{es}(0)}$$

for arbitrary reference frame ($\omega \neq \omega_e$),

$$\hat{F}_{qs} = F_s\, e^{j(\theta_{ef}(0) - \theta(0))}$$

$$\hat{F}_{ds} = j\, \hat{F}_{qs}$$

Selecting $\theta(0) = 0$,

$$\tilde{F}_{as} = \tilde{F}_{qs}$$

- Thus, in all <u>asynchronously</u> rotating reference frames ($\omega \neq \omega_e$) with $\theta(0) = 0$, the phasor representing the as variables is equal to the phasor representing the qs variables.

- In the synchronously rotating reference frame $\omega = \omega_e$, F^e_{qs} and F^e_{ds} can be expressed as

$$F^e_{qs} = Re\left[\sqrt{2}\, F_s\, e^{j(\theta_{ef}(0) - \theta_e(0))}\right]$$

$$F^e_{ds} = Re\left[j\sqrt{2}\, F_s\, e^{j(\theta_{ef}(0) - \theta_e(0))}\right]$$

Let $\theta_e(0) = 0$, then

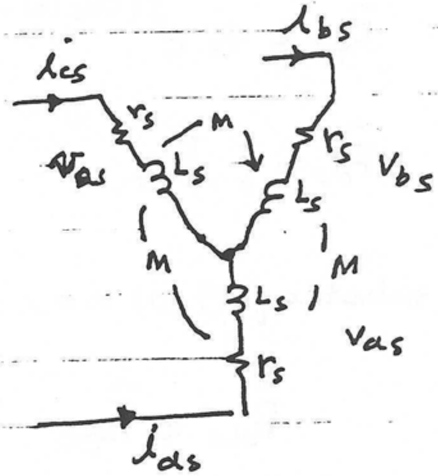
$$F^e_{qs} = \sqrt{2}\, F_s \cos(\theta_{ef}(0))$$

$$F^e_{ds} = -\sqrt{2}\, F_s \sin(\theta_{ef}(0))$$

$$\sqrt{2}\, \hat{F}_{as} = F^e_{qs} - j F^e_{ds}$$

Since $\tilde{F}_{as} = F_s\, e^{j(\theta_{es}(0))} = F_s \cos(\theta_{ef}(0)) + j F_s \sin(\theta_{ef}(0))$

• Consider the stator winding of a symmetrical induction machine.



• Assume the stator winding is excited by a balanced 3-phase sinusoidal voltage set.

For phase as, we will have ($p = \frac{d}{dt}$)

$$V_{as} = r_s i_{as} + L_s p i_{as} + M p i_{bs} + M p i_{cs}$$

For balanced conditions

$$V_{as} + V_{bs} + V_{cs} = 0$$

$$i_{as} + i_{bs} + i_{cs} = 0$$

$$M p i_{as} = - M p (i_{bs} + i_{cs})$$

$$V_{as} = r_s i_{as} + (L_s - M) p i_{as}$$

For steady-state conditions, $p = j\omega_e$

$$\hat{V}_{as} = r_s \tilde{I}_{as} + [(L_s - M) j\omega_e] \tilde{I}_{as}$$

qs and ds voltage equations in the arbitrary reference frame, can be written as

$$v_{qs} = r_s i_{qs} + \omega \lambda_{ds} + p \lambda_{qs}$$
$$v_{ds} = r_s i_{ds} - \omega \lambda_{qs} + p \lambda_{ds}$$

$$\lambda_{qs} = (L_s - M) i_{qs}$$
$$\lambda_{ds} = (L_s - M) i_{ds}$$

Let $\omega = \omega_e$, then

$$V_{qs}^e = r_s i_{qs}^e + \omega_e \lambda_{ds}^e + p \lambda_{qs}^e$$
$$V_{ds}^e = r_s i_{ds}^e - \omega_e \lambda_{qs}^e + p \lambda_{ds}^e$$

$$\lambda_{qs}^e = (L_s - M) i_{qs}^e$$
$$\lambda_{ds}^e = (L_s - M) i_{ds}^e$$

For balanced steady-state conditions the variables in the synchronously rotating reference frame are constants, therefore, $p\lambda_{qs}^e$ and $p\lambda_{ds}^e$ are zero. Therefore, the above can be expressed as

$$V_{qs}^e = r_s I_{qs}^e + \omega_e (L_s - M) I_{ds}^e$$

$$V_{ds}^e = r_s I_{ds}^e - \omega_e (L_s - M) I_{qs}^e$$

Recall $\sqrt{2}\, \tilde{F}_{as} = F_{qs}^e - j F_{ds}^e$

Thus

$$\sqrt{2}\, \tilde{V}_{as} = V_{qs}^e - j F_{ds}^e$$

$$\sqrt{2}\hat{V}_{as} = r_s I_{qs}^e + \omega_e (L_s - M) I_{ds}^e - j\left[r_s I_{ds}^e - \omega_e (L_s - M) I_{qs}^e\right]$$

Now $\sqrt{2}\,\hat{I}_{as} = I_{qs}^e - j I_{ds}^e$

$$j\sqrt{2}\,\hat{I}_{as} = I_{ds}^e + j I_{qs}^e$$

substituting in the above equation, we WILL have

$$\hat{V}_{as} = \left[r_s + j\omega_e (L_s - M)\right]\hat{I}_{as}$$