HW #3

EE743

A. Keyhani

Problem 1. An iron-core transformer which has two windings is shown in Fig. 1. $N_1 = 50$ turns, $N_2 = 100$ turns, and $\mu_r = 4000$ Calculate L_{12}, L_{m1} and L_{m2}. and polarity (P.3) of coupled coils with appropriate dots.

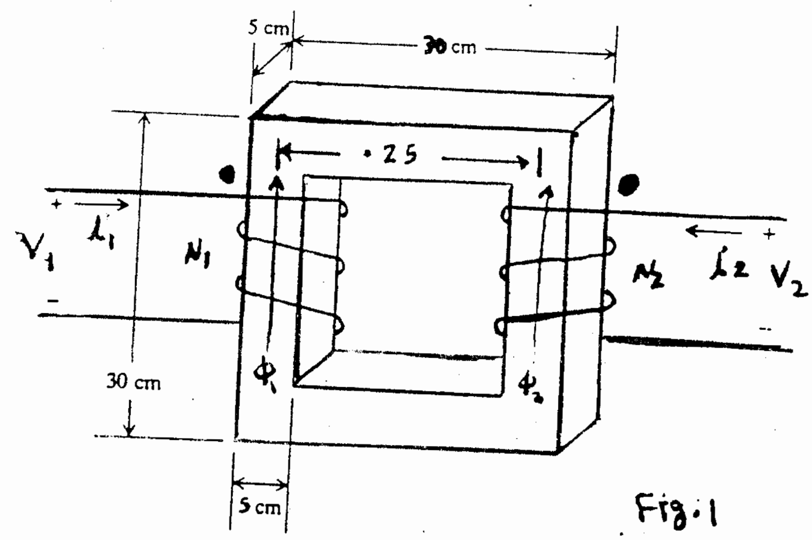


Fig. 1

Solution

$$\mathcal{R}_m = \frac{l}{\mu_i A} \quad H^{-1} \qquad \mu_i = \mu_o \mu_{ri} = (4000)(4\pi \times 10^{-7})$$

$$A = (.05)(.05) = .0025 \quad m^2$$

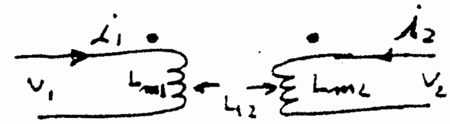
$$l = 4(0.30 - .05) = 4(.25) = 1 \ m$$

$$\mathcal{R}_m = 79{,}577 \ H^{-1}$$

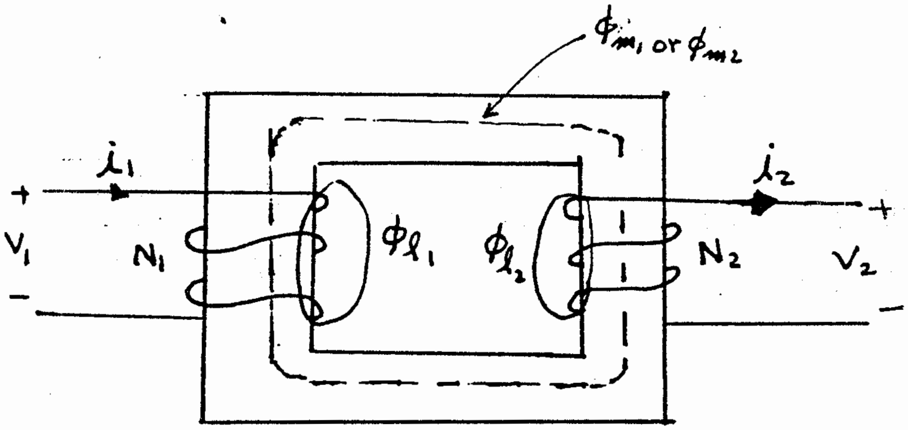
$$L_{12} = \frac{N_1 N_2}{\mathcal{R}_m} = \frac{(50)(100)}{79{,}577} = 0.0628 \ H$$

$$L_{m1} = \frac{N_1^2}{\mathcal{R}_m} = \frac{50^2}{79{,}577} = .0314 \ H \qquad L_{m2} = \frac{N_2^2}{\mathcal{R}_m} = \frac{100^2}{79{,}577} = .1257 \ H$$

Equivalent circuit model:



Problem 2. Two coupled coils have the following parameters:
$r_1 = 10\ (\Omega)$, $L_{\ell 1} = 0.1 L_{11}$, $r_2 = 2.5\ (\Omega)$, $L_{\ell 2} = .1 L_{22}$,
$L_{11} = 100\ (mH)$, $N_1 = 100$ turns, $L_{22} = 25\ mH$, $N_2 = 50$ turns
Develop an equivalent T circuit with (a) Winding 1 as the reference winding and (b) winding (2) as reference winding. (P.6)



$$\Phi_1 = \phi_{\ell_1} + \phi_{m1} + \phi_{m2} = \frac{N_1 i_1}{\mathcal{R}_{\ell 1}} + \frac{N_1 i_1}{\mathcal{R}_m} + \frac{N_2 i_2}{\mathcal{R}_m}$$

$$\lambda_1 = N_1 \Phi_1 = \frac{N_1^2}{\mathcal{R}_{\ell_1}} i_1 + \frac{N_1^2}{\mathcal{R}_m} i_1 + \frac{N_1 N_2}{\mathcal{R}_m} i_2$$

Define

$$L_{11} = \frac{N_1^2}{\mathcal{R}_{\ell 1}} + \frac{N_1^2}{\mathcal{R}_m} = L_{\ell 1} + L_{m1}$$

Similarly.

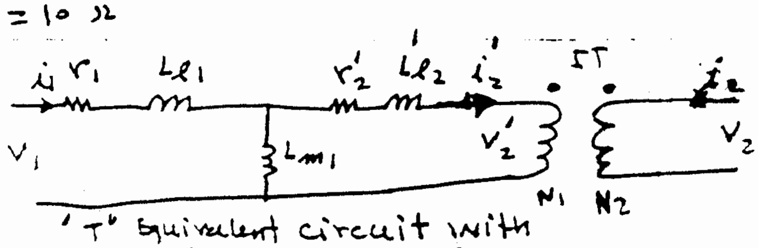
a) $$L_{22} = L_{\ell 2} + L_{m2}$$

$$L_{m1} = L_{11} - L_{\ell 1} = 100 - 0.1(100) = 90\ (mH)$$

$$L_{\ell 2} = 0.1 L_{22} = 0.1(25 mH) = 2.5 (mH)$$

$$L'_{\ell 2} = \left(\frac{N_1}{N_2}\right)^2 L_{\ell 2} = \left(\frac{100}{50}\right)^2 (2.5 \times 10^{-3}) = 10 (mH)$$

$$r'_2 = \left(\frac{N_1}{N_2}\right)^2 r_2 = 10\ \Omega$$



'T' Equivalent circuit with winding 1 as a reference.

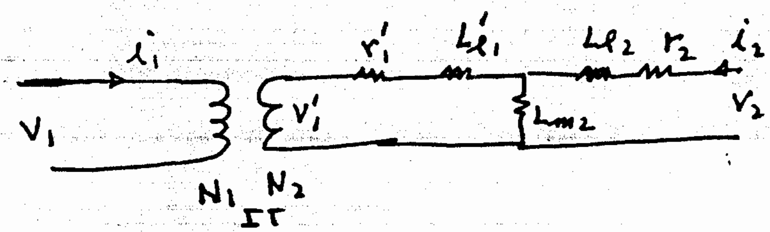
Problem 2 cont. b)

$$L_{22} = L_{\ell 2} + L_{m2} \qquad L_{\ell 2} = 0.1\, L_{22} = 0.1(25) = 2.5 \text{ (mH)}$$

$$L_{m2} = L_{22} - L_{\ell 2} = 25 - 2.5 = 22.5 \text{ (mH)}$$

$$L'_{\ell 1} = \left(\frac{N_2}{N_1}\right)^2 L_{\ell 1} = \left(\frac{50}{100}\right)^2 (10 \times 10^{-3}) = 2.5 \text{(mH)}$$

$$r'_1 = \left(\frac{N_2}{N_1}\right)^2 r_1 = \left(\frac{50}{100}\right)^2 (10) = 2.5 \ (\Omega)$$



Equivalent circuit with Winding 2 as a reference.

Problem 3. Two coupled coils have the following parameters: $r_1 = 10\,\Omega$, $L_{\ell 1} = 0.1 L_{11}$, $r_2 = 2.5\,\Omega$, $L_{\ell 2} = 0.1 L_{22}$, $L_{11} = 100\,(mH)$, $N_1 = 100$ turns, $L_{22} = 25\,(mH)$, $N_2 = 50$ turns. Develop ~~an~~ coupled coil equivalent circuit and compared with its T equivalent circuit.

Solution: Recall

$$\Phi_1 = \phi_{\ell 1} + \phi_{m1} + \phi_{m2} = \frac{N_1 i_1}{\mathcal{R}_{\ell 1}} + \frac{N_1 i_1}{\mathcal{R}_m} + \frac{N_2 i_2}{\mathcal{R}_m}$$

$$\lambda_1 = N_1 \Phi_1 = \frac{N_1^2}{\mathcal{R}_{\ell 1}} i_1 + \frac{N_1^2}{\mathcal{R}_m} i_1 + \frac{N_1 N_2}{\mathcal{R}_m} i_2$$

Define self inductance

$$L_{11} = \frac{N_1^2}{\mathcal{R}_{\ell 1}} + \frac{N_1^2}{\mathcal{R}_m} = L_{\ell 1} + L_{m1}$$

$$L_{12} = \frac{N_1 N_2}{\mathcal{R}_m}$$

$$\lambda_2 = N_2 \Phi_2 = \frac{N_2^2}{\mathcal{R}_{\ell 2}} i_2 + \frac{N_2^2}{\mathcal{R}_m} i_2 + \frac{N_1 N_2}{\mathcal{R}_m} i_1$$

Define $$L_{22} = \frac{N_2^2}{\mathcal{R}_{\ell 2}} + \frac{N_2^2}{\mathcal{R}_m} = L_{\ell 2} + L_{m2}$$

$$L_{21} = \frac{N_1 N_2}{\mathcal{R}_m} \qquad \boxed{L_{21} = L_{12}}$$

and also $$L_{m2} = \frac{N_2^2}{\mathcal{R}_m}$$

$$\frac{L_{21}}{L_{m2}} = \frac{N_1 N_2}{\mathcal{R}_m} \times \frac{\mathcal{R}_m}{N_2^2} = \frac{N_1}{N_2}$$

$$L_{21} = \frac{N_1}{N_2} L_{m2} = \frac{N_2}{N_1} L_{m1}$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$

Problem 3 cont.

$$L_{m1} = L_{11} - L_{\ell 1} = 100 - 0.1(100) = 90\,(mH)$$

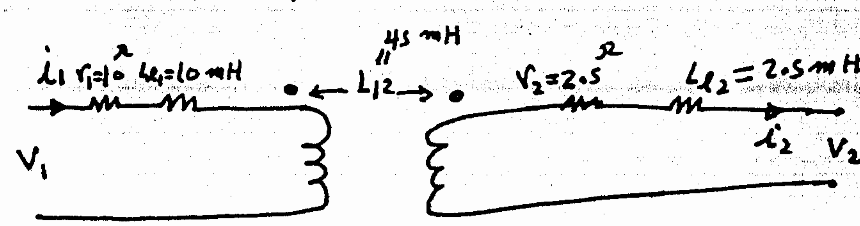
$$L'_{\ell 2} = 0.1\, L_{\ell 2} = 0.1\,(25\, mH) = 2.5\,(mH)$$

$$L_{m2} = L_{22} - L_{\ell 2} = 25 - 0.(25) = 22.5\,(mH)$$

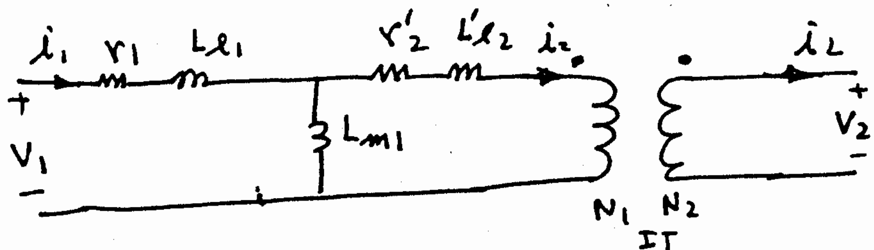
$$L_{\ell 1} = 0.1\, L_{11} = 0.1(100) = 10\,(mH)$$

$$L_{21} = \frac{N_1}{N_2} L_{m2} = \frac{100}{50}(22.5) = 45\,(mH)$$

$$L_{12} = \frac{N_2}{N_1} L_{m1} = \frac{50}{100}(90) = 45\,(mH)$$



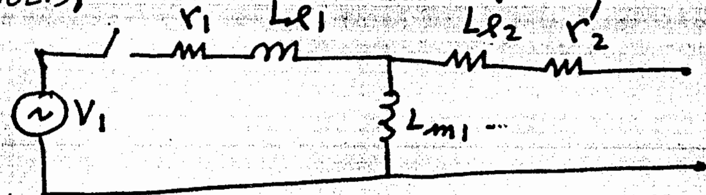
coupled coil equivalent circuit.



"T" Equivalent circuit with winding '1' as a reference.

Problem 4. Consider a two-winding transformer with "T" equivalent circuit model. Assume $r_1 = 6\ \Omega$, $L_{m1} = 263.9\ (mH)$, $L_{\ell 1} = 13.5\ (mH)$, $r'_2 = 5\ \Omega$, $L'_{\ell 2} = 13.5\ mH$. Compute the transient response of the transformer winding '1' current $i_1(t)$, assuming the windings are initially unexcited and winding "2" is open-circuited. Assume a) when the switch is closed, the sinusoidal input voltage is at its maximum value, b) the input voltage is at its zero crossing point. [$\omega_e = 377$ rad/sec and $V_{max} = \sqrt{2}\ 110$ volts) (P.12)

Solution



$$V_1 = r_1 i_1 + \frac{d\lambda_1}{dt}, \quad \lambda_1 = (L_{\ell 1} + L_{m1}) i_1 = L_{11} i_1$$

$$V_1 = r_1 i_1 + L_{11}\frac{di_1}{dt}, \quad i_1 = i_{tr} + i_{ss}, \quad i_{tr} = K e^{-r_1/L_{11} t}$$

a)

$$V_1 = \sqrt{2}\, 110 \cos \omega_e t = \sqrt{2}\, V_s \cos \omega_e t$$

$$i_{ss} = \frac{\sqrt{2}\, V_s}{|Z|}\cos(\omega_e t - \phi), \quad \phi = \tan^{-1}\frac{\omega_e L_{11}}{r_1}$$

$$i_1 = K e^{-r_1/L_{11} t} + \frac{\sqrt{2}\, V_s}{|Z|}\cos(\omega_e t - \phi)$$

where at $t=0$ $i_1(0) = 0$, thus

$$K = -\frac{\sqrt{2}\, V_s}{|Z|}\cos(-\phi)$$

b) $V_1 = \sqrt{2}\, V_s \sin \omega_e t$

$$i_1 = K e^{-r_1/L_{11} t} + \frac{\sqrt{2}\, V_s}{|Z|}\sin(\omega_e t - \phi)$$

where

at $t=0$ $i_1(0) = 0$, thus

$$K = -\frac{\sqrt{2}\, V_s}{|Z|}\sin(-\phi) = \frac{\sqrt{2}\, V_s}{|Z|}\sin\phi$$

Problem 4) For part a)

$$Z = \sqrt{(6)^2 + [(377)(13.5 + 263.9)\times 10^{-3}]^2} = 104.75\ \Omega$$

$$\phi = \tan^{-1} \frac{(377)(13.5 + 263.9)\times 10^{-3}}{6} = -86.7^\circ$$

$$K = -\frac{\sqrt{2}\ 110}{104.75} \cos(-86.7^\circ) = -0.085$$

$$\frac{r_1}{L_{11}} = \frac{6}{.2774} = 21.6$$

$$i_1(t) = -.085\, e^{-21.6t} + 1.48 \cos(377t - 86.7^\circ)\ A$$

For part b)

$$K = +\frac{\sqrt{2}\ 110}{104.75} \sin(86.7^\circ) = 1.48$$

$$i_1(t) = 1.48\, e^{-21.6t} + 1.48 \sin(377t - 86.7^\circ)\ A$$

Problems 5. Consider the elementary electromagnet shown in Fig. 1. Assume that the cross-sectional area of the stationary and movable member is the same and $A_i = A_g = 4\ cm^2$. Assume $l_i = 20\ cm$, $N = 500$ and $\mu_{ri} = 1000$. a) Derive an expression for the magnetizing inductance $L_m(x)$. b) compute the value of the magnetizing inductance when $x = 1\ mm$.

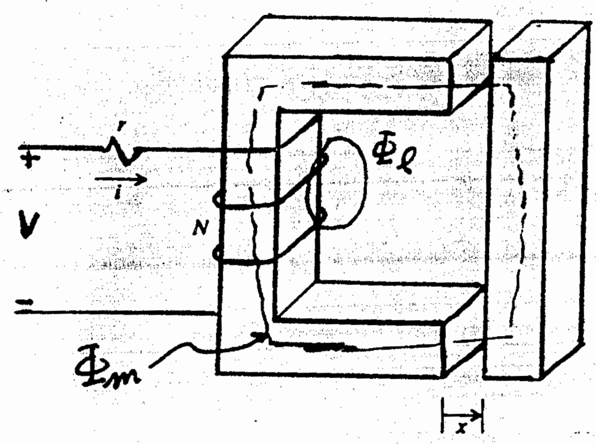


Fig. 1 Elementary electromagnet

Solution.

$$v = ri + \frac{d\lambda}{dt}$$

Flux linkage

$$\lambda = N\Phi \quad , \Phi = \Phi_l + \Phi_m$$

$$\Phi_l = \frac{Ni}{\mathcal{R}_l} \qquad \Phi_m = \frac{Ni}{\mathcal{R}_m}$$

$$\lambda = \left(\frac{N^2}{\mathcal{R}_l} + \frac{N^2}{\mathcal{R}_m}\right) i \qquad L_l\ (\text{leakage}) = \frac{N^2}{\mathcal{R}_l}$$

Magnetizing inductance

$$L_m = \frac{N^2}{\mathcal{R}_m}$$

Reluctance of the magnetizing path

$$\mathcal{R}_m = \mathcal{R}_i + 2\mathcal{R}_g$$

$\Re_i$: total reluctance of the magnetic material of the stationary and movable members.

$\Re_g$: reluctance of the air gaps.

$$\Re_i = \frac{l_i}{\mu_{ri}\mu_0 A_i} \quad , \quad \Re_g = \frac{x}{\mu_0 A_g}$$

Assume $A_g = A_i$

$$\Re_m = \frac{1}{\mu_0 A_i}\left(\frac{l_i}{\mu_{ri}} + 2x\right)$$

$$\lambda_m = N\Phi_m = \frac{N^2}{\Re_m} i \qquad L_m = \frac{N^2}{\Re_m}$$

$$L_m = \frac{N^2}{(1/\mu_0 A_i)(l_i/\mu_{ri} + 2x)} = L_m(x)$$

$$L_m(x) = \frac{(N^2 \mu_0 A_i/2)}{(l_i/2\mu_{ri}) + x} = \frac{k}{k_0 + x}$$

$$k = \frac{N^2 \mu_0 A_i}{2} = \frac{(500)^2 4\pi \times 10^{-7} \times 4 \times 10^{-4}}{2} = 2\pi \times 10^{-5}$$

$$k_0 = \frac{l_i}{2\mu_{ri}} = \frac{20 \times 10^{-2}}{2(1000)} = 10^{-4}$$

$$L_m(x) = \frac{2\pi \times 10^{-5}}{10^{-4} + x} \quad (H)$$

$$L_m(10^{-3}) = \frac{2\pi \times 10^{-5}}{10^{-4} + 10^{-3}} = 57.09 \text{ mH}$$

$$L_m(0) = \frac{2\pi \times 10^{-5}}{10^{-4}} = 62.8 \text{ mH}$$