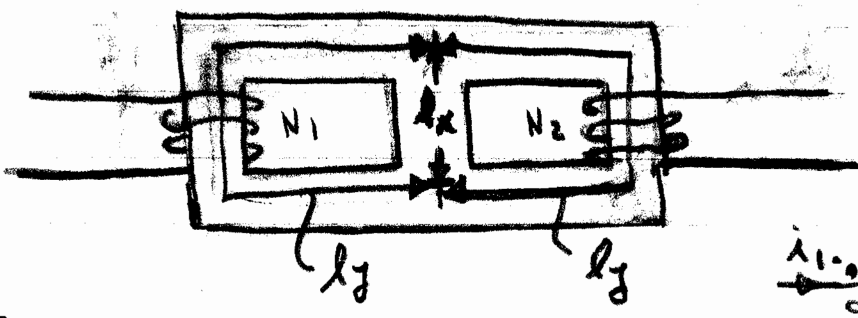
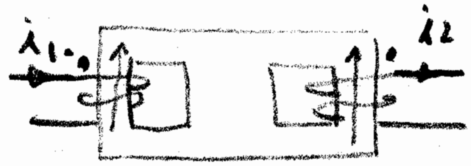
①

Problem: For the magnetic system given below:




$l_y = 12$ cm $\qquad l_x = 4$ cm

$N_1 = 400$ T $\qquad N_2 = 800$ T

$\mu_{ry} = 20000 \quad , \mu_y = 4\pi \times 10^{-7} \times 2 \times 10^{4} = 2.5 \times 10^{-2}$

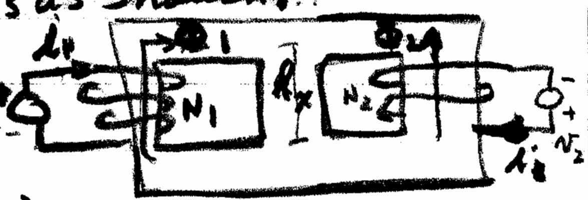
$\mu_{rx} = 2000 \qquad \mu_x = 2.5 \times 10^{-3} \qquad \mathcal{R}_y = \dfrac{12 \times 10^{-2}}{2.5 \times 10^{-2} \times 10^{-4}} = 4.8 \times 10^{4} (H^{-1})$

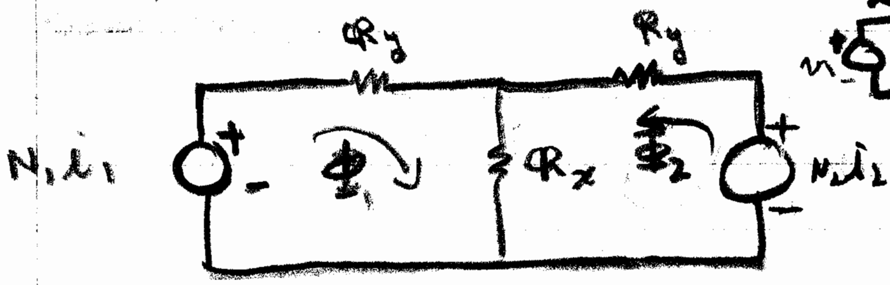
$A_x = 1\ cm^2 \qquad A_y = 1\ cm^2 \qquad \mathcal{R}_x = \dfrac{4 \times 10^{-2}}{2.5 \times 10^{-3} \times 10^{-4}} = 1.6 \times 10^{5} (H^{-1})$

1. Determine the dot notation for both windings

2 Compute $L_{11}$, $L_{12}$ and $L_{22}$

solution: assume that $i_1$ and $i_2$ directions as shown:




$$N_1 i_1 = \mathcal{R}_y \Phi_1 + (\Phi_1 + \Phi_2)\mathcal{R}_x +$$
$$N_2 i_2 = \mathcal{R}_y \Phi_2 + (\Phi_2 + \Phi_1)\mathcal{R}_x$$

$$\begin{bmatrix} N_1 i_1 \\ N_2 i_2 \end{bmatrix} = \begin{bmatrix} \mathcal{R}_y + \mathcal{R}_x & +\mathcal{R}_x \\ +\mathcal{R}_x & \mathcal{R}_y + \mathcal{R}_x \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$$

Solution Cont.

$\mathcal{R}_y = 4.8 \times 10^4 \; (H^{-1})$

$\mathcal{R}_x = 1.6 \times 10^5 \; (H^{-1})$

$N_1 = 400$

$N_2 = 800$

$$\Phi_1 = \frac{\begin{vmatrix} N_1 i_1 & +\mathcal{R}_x \\ N_2 i_2 & \mathcal{R}_y + \mathcal{R}_x \end{vmatrix}}{\begin{vmatrix} \mathcal{R}_y + \mathcal{R}_x & +\mathcal{R}_x \\ +\mathcal{R}_x & \mathcal{R}_y + \mathcal{R}_x \end{vmatrix}}$$

$$\Phi_1 = \frac{(N_1 i_1)(\mathcal{R}_x + \mathcal{R}_y) - \mathcal{R}_x N_2 i_2}{(\mathcal{R}_y + \mathcal{R}_x)^2 - \mathcal{R}_x^2}$$

$$= \frac{\qquad}{\mathcal{R}_x^2 + \mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y - \mathcal{R}_x^2} = \frac{\qquad}{\mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y}$$

$$\lambda_1 = N_1 \Phi_1$$

$$\lambda_1 = \frac{N_1^2 (\mathcal{R}_x + \mathcal{R}_y)}{\mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y} i_1 - \frac{\mathcal{R}_x N_1 N_2}{\mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y} i_2$$

$$\lambda_1 = L_{11} i + L_{12} i_2$$

$$L_{11} = \frac{N_1^2 (\mathcal{R}_x + \mathcal{R}_y)}{\mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y} \qquad L_{12} = \frac{\mathcal{R}_x N_1 N_2}{\mathcal{R}_y^2 + 2\mathcal{R}_x \mathcal{R}_y} > 0$$

$$L_{11} = \frac{(400)^2 (1.6 \times 10^5 + 4.8 \times 10^4)}{(4.8 \times 10^4)^2 + 2(1.6 \times 10^5)(4.8 \times 10^4)} = 1.88\,H$$

$L_{12} = 2.90\,H$ $\qquad$ $L_{22} = 7.54\,H$

Solution

$$\Phi_2 = \frac{\begin{bmatrix} R_y + R_x & N_1 i_1 \\ + R_x & N_2 i_2 \end{bmatrix}}{R_y^2 + 2 R_x R_y} = \frac{(R_y + R_x) N_2 i_2 + R_x N_1 i_1}{R_y^2 + 2 R_x R_y}$$

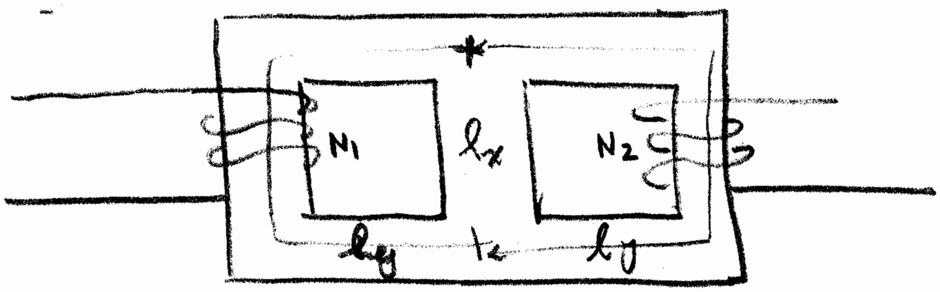
$$\lambda_2 = N_2 \Phi_2$$

$$\lambda_2 = \frac{- R_x N_1 N_2}{R_y^2 + 2 R_x R_y} i_1 + \frac{(R_y + R_x) N_2^2}{R_y^2 + 2 R_x R_y} i_2$$

$$\lambda_2 = L_{21}(-i_1) + L_{22} i_2$$

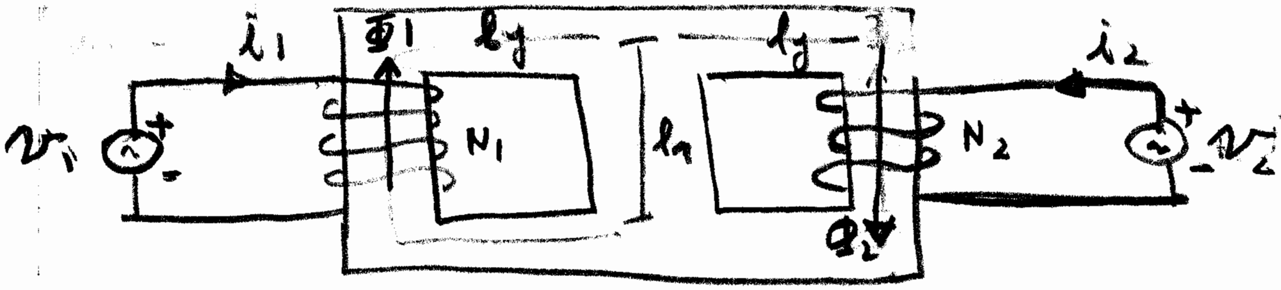
$$L_{21} = \frac{R_x N_1 N_2}{R_y^2 + 2 R_x R_y} > 0 \qquad L_{22} = \frac{(R_y + R_x) N_2^2}{R_y^2 + 2 R_x R_y} > 0$$

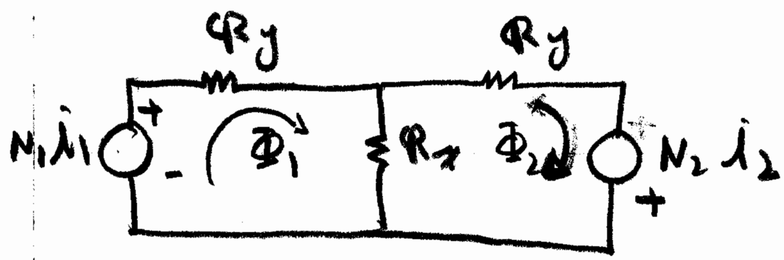
Problem 1: Alternate solution




Compute $L_{11}$ and $L_{12}$

Assume that $i_1$ and $i_2$ directions as shown:



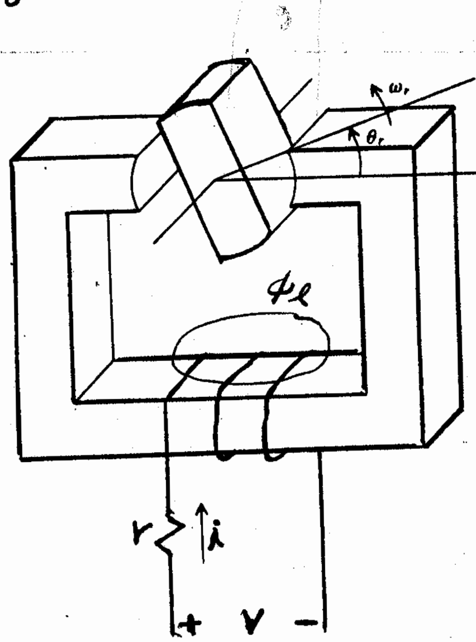

$$\begin{bmatrix} N_1 i_1 \\ N_2 i_2 \end{bmatrix} = \begin{bmatrix} \mathcal{R}_y + \mathcal{R}_x & -\mathcal{R}_x \\ -\mathcal{R}_x & \mathcal{R}_y + \mathcal{R}_x \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$$

$$\lambda_1 = N_1 \Phi_1 = \frac{N_1^2(R_x + R_y)}{2 R_x R_y + R_y^2} i_1 + \frac{N_1 N_2 \mathcal{R}_x}{2 R_x R_y + R_y^2} i_2$$

$$L_{11} = \frac{N_1^2 (R_x + R_y)}{2 R_x R_y + R_y^2} \qquad L_{12} = \frac{N_1 N_2 R_x}{2 R_x R_y + R_y^2} > 0$$

$L_{11}$ and $L_{12}$ are positive

Problem 3. consider the elementary reluctance machine shown in Fig. 1. a) Derive an expression for the magnetizing inductance as a function of $\theta_r$. b) write an expression for the flux linkage equation and the voltage equation of the reluctance machine. Assume the movable member rotates at an angular displacement and angluar velocity of $\theta_r$ and $\omega_r$ respectively.




$\begin{cases}\theta_r = 0 \\ g = g_{max}\end{cases}$ rotor is in vertical position

$\begin{cases}\theta_r = \pi/2 \\ g = g_{min}\end{cases}$ rotor is in horizontal position

Fig. 1

Solution.

$$v = ri + \frac{d\lambda}{dt}$$

$$\lambda = (L_l + L_m)\, i$$

$L_l$ : leakage inductance , $L_m$: magnetizing inductance.

$$L_m(\theta_r) = \frac{N^2}{R_m(\theta_r)}$$

with $\theta_r = 0$ $R_m(\theta_r) =$ Maximum, since the airgap is Largest, that is the rotor is in vertical position. Hence $L_m$ has a minimum value in this position.

$$L_m(0) = \frac{N^2}{\Re_m(0)}$$ and $L_m(0)$ has a minimum value.

with $\theta r = \pi$, again $\Re_m(\pi) = max.$ and

$$L_m(\theta r = \pi) = \frac{N^2}{\Re_m(\pi)}$$ and $L_m(\pi)$ has a minimum value.

Also, when $\theta r = 2\pi, 3\pi, 4\pi,$

Now, with $\theta r = \pi/2$, $\Re_m(\theta r = \pi/2)$ has a minimum value and

$$L_m(\theta_r = \pi/2) = \frac{N^2}{\Re_m(\pi/2)}$$ has a maximum value. The same situation occurs at $\theta r = \frac{3}{2}\pi, \frac{5}{2}\pi$. Hence $L_m(\theta r)$ varies between maximum and minimum positive value twice per revolution of the rotating member (rotor).

Thus.

$$L_m(\theta_r) = L_A - L_B \cos 2\theta_r$$

where $L_m(0) = L_A - L_B$ (minimum)

$L_m(\theta_r = \pi/2) = L_A + L_B$ (maximum)

self inductance of reluctance machine can be expressed as

$$L(\theta_r) = L_\ell + L_m(\theta_r)$$
$$= L_\ell + L_A - L_B \cos 2\theta_r$$

b)

$$V = ri + \frac{d\lambda}{dt}$$

$$\lambda(i, \theta_r) = L(\theta_r)\, i$$

$$\frac{d\lambda}{dt} = \frac{d}{dt}\left(\lambda(\theta_r, i)\right) = \frac{\partial \lambda}{\partial i}\frac{di}{dt} + \frac{\partial \lambda}{\partial \theta_r} \cdot \frac{d\theta_r}{dt}$$

$$= \frac{\partial \lambda(\theta_r, i)}{\partial i}\frac{di}{dt} + \frac{\partial \lambda(\theta_r, i)}{\partial \theta_r} \cdot \frac{d\theta_r}{dt}$$

$$= L(\theta_r)\frac{di}{dt} + i\,\frac{\partial L(\theta_r, i)}{\partial \theta_r} \cdot \frac{d\theta_r}{dt}$$

$$= [L_\ell + L_A - L_B \cos 2\theta_r]\frac{di}{dt} +$$
$$+ i(-L_B)(-\sin 2\theta_r)(2)\frac{d\theta_r}{dt}$$

$$V = ri + [L_\ell + L_A - L_B \cos 2\theta_r]\frac{di}{dt} +$$
$$2\omega_r i L_B \sin 2\theta_r$$

②

```
clear

V_type = menu('Type of input:','trapezoidal function','zero input');
X_type = menu('Position of x','spring at rest','spring at max. stress');

% Parameters:

        K       = 2667;
        D       = 10;
        N       = 500;
        m       = 0.8;
        d       = 1E-4;
        g       = 3E-4;
        R       = 10;
        Ag      = 2E-4;
        uo      = 4*pi*1E-7;

% Matrix Equation:

        x = 0;
        i = 0;

        Lx      = N^2*Ag*uo/(x+d);

        dLx     = -N^2*Ag*uo/(x+d)^2;

        Fem     = 0.5*i^2*dLx;

        LL = [  1       0       0
                -D      -m      0
                -i*dLx  0       -Lx     ];

        RR = [  0       1       0
                K       0    -0.5*i*dLx
                0       0       R       ];

        DD = [  0       0
                0       -K*g
                -1      0       ];

% The given system in terms of its matrix equation with input Vs:
        A = inv(LL)*RR;
        B = inv(LL)*DD;

%  sampling time:
        if V_type == 1
                T = .0001;
        elseif V_type == 2
                T = .001;
        end

% Trapezoidal Integration Method:

        % Damping constant: 0 < alpha < 1
                alpha = 0.15;
```