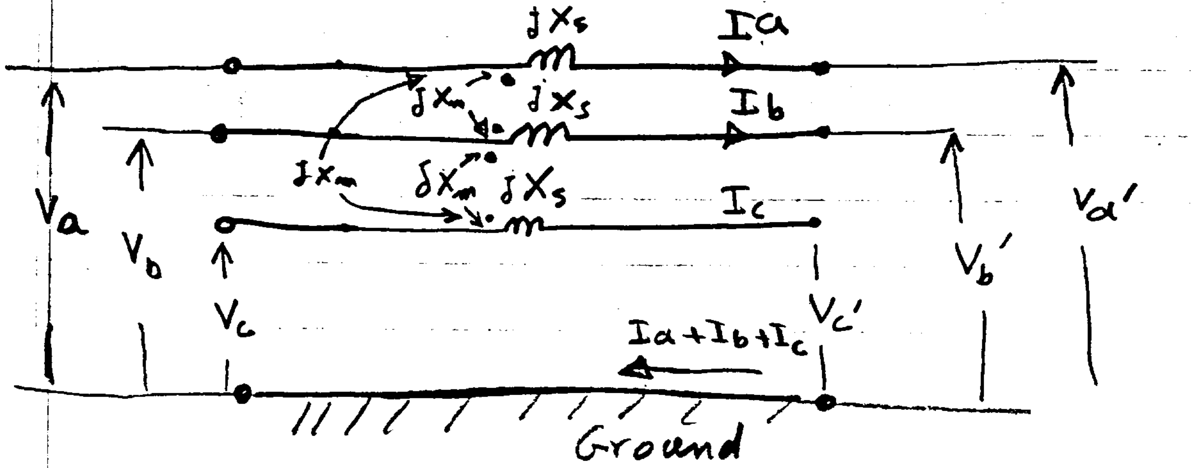
Sequence Impedances of 3-ϕ balanced transmission lines



$$V_a = jX_s I_a + jX_m I_b + jX_m I_c + V_a'$$
$$V_b = jX_s I_b + jX_m I_a + jX_m I_c + V_b'$$
$$V_c = jX_s I_c + jX_m I_a + jX_m I_b + V_c'$$

In matrix notation

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\left[V_{abc} \right] - \left[V_{abc}' \right] = j \left[Z^{abc} \right] \left[I_{abc} \right] \qquad (1)$$

Recall

$$\left[V_{abc} \right] = \left[T_s \right] \left[V_{0,1,2} \right] \qquad (2)$$

$$\left[I_{abc} \right] = \left[T_s \right] \left[I_{012} \right] \qquad (3)$$

Replace V_{abc} by Eq (2)
" I'_{abc} by Eq (3)

$$[T_s][V_{012}] - [T_s][V'_{012}] = [Z_{abc}][T_s][I_{012}] \quad (4)$$

multiply (4) by T_s^{-1}

$$\underbrace{[T_s]^{-1}[T_s]}_{I}[V_{012}] - \underbrace{[T_s]^{-1}[T_s]}_{I}[V'_{012}] = \underbrace{[T_s]^{-1}[Z_{abc}][T_s]}[I_{012}]$$

$$[V_{012}] - [V'_{012}] = [Z_{012}][I_{012}] \quad (4)$$

$$[Z_{012}] = [T_s]^{-1}[Z_{abc}][T_s]$$

$$[Z_{012}] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$[Z_{012}] = \frac{j}{3}\begin{bmatrix} (X_s+2X_m) & (X_s+2X_m) & X_s+2X_m \\ (X_s-X_m) & (aX_s+(1+a^2)X_m & (a^2X_s+(1+a)X_m) \\ (X_s-X_m) & (a^2X_s+(1+a)X_m) & (aX_s+(1+a^2)X_m) \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$Z_{012} = j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix} \quad (5)$$

Z_0 = Zero sequence impedance: $= j(X_s + 2X_m)$

Z_1 = Positive sequence impedance: $= j(X_s - X_m)$

Z_2 = Negtive sequence impedance $= j(X_s - X_m)$

Note some book use the notation "0 12" for (0, + −)

$$V_{abc} = [T_s][V_{012}]$$

$$I_{abc} = [T_s][I_{012}]$$

$$Z^{012} = Z^{0+-}$$

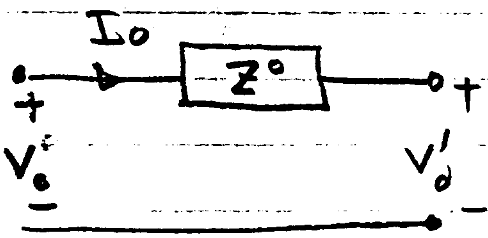
From Eq. (4)

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} - \begin{bmatrix} V_0' \\ V_1' \\ V_2' \end{bmatrix} = j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

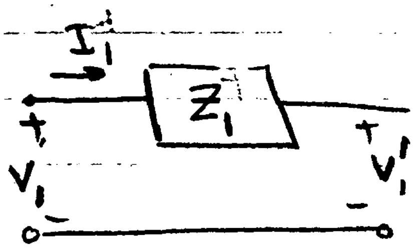
$$V_0^0 - V_0' = j\,(X_s + 2X_m)\,I_0$$

$$V_1^+ - V_1'^+ = j\,(X_s - X_m)\,I_1^+$$

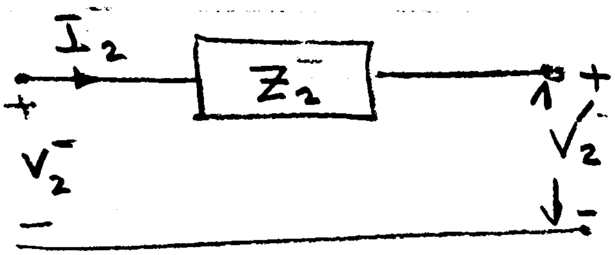
$$V_2^- - V_2'^- = j\,(X_s - X_m)\,I_2^-$$



Zero seq Network



(+) Seq Network



(−) Seq. Network

Power.

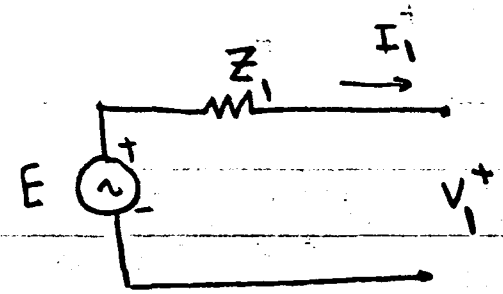
$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

$$S_{3\phi} = \left[V_{abc}\right]^t \left[I_{abc}^*\right]$$

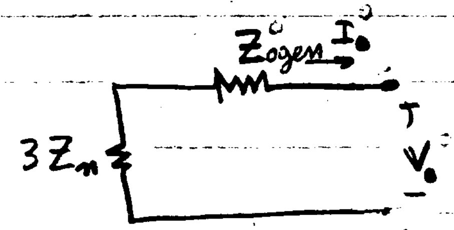
$$= \left\{[T][V_{012}]\right\}^t \left[T\, I_{012}\right]^*$$

$$S_{3\phi} = 3\left[V_0 I_0^* + V_1 I_1^* + V_2 I_2^*\right]$$

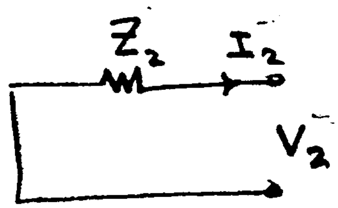
$$= 3\left[S_{012}\right]$$



Positive seq. Network



Zero Seq. Network



Negative sequence Network.