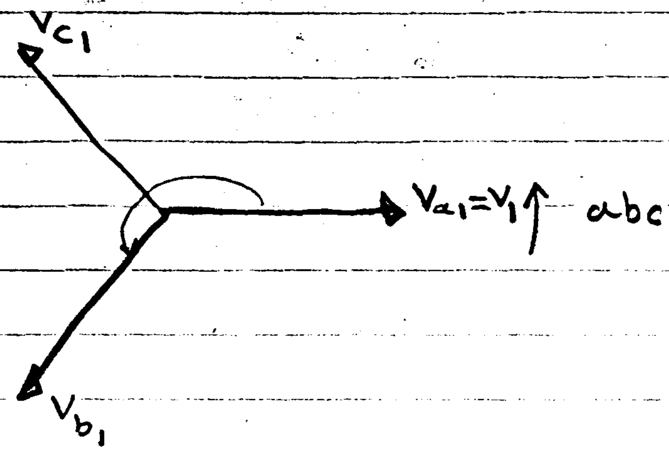
A Keyhani

EE 740

1 Lecture #10

# Symmetrical Components

Operator positive sequence voltage 1 = "+"




$V_a = V_{a_1} = V\angle 0 = V_1\angle 0$

$V_b = V_{b_1} = V_1\angle 240$

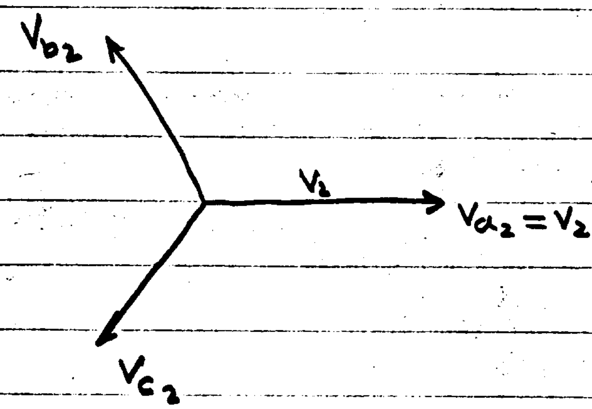
$V_c = V_{c_1} = V_1\angle 120$

Let $a = 1\angle 120$ $\quad a^2 = 1\angle 240$

$V_{a_1} = V_1 \qquad V_{b_1} = a^2 V_1 \qquad V_{c_1} = a V_1$

$$\begin{bmatrix} V_{a_1} \\ V_{b_1} \\ V_{c_1} \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} V_1$$

"−" sequence = 2




$$V_{a2} = V_2$$

$$V_{b2} = a V_2$$

$$V_{c2} = a^2 V_2$$

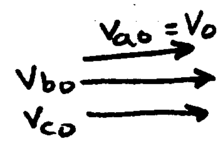
$$\begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix} V_2$$

"0" sequence

$$V_{a0} = V_0$$

$$V_{b0} = V_0$$

$$V_{c0} = V_0$$




We will show that a set of unbalanced voltages $V_a, V_b, V_c$ can be written as

$$V_a = V_{a0} + V_{a1} + V_{a2} \Rightarrow V_a = V_0 + V_1 + V_2$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \Rightarrow V_b = V_0 + a^2 V_1 + a V_2$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \Rightarrow V_c = V_0 + a V_1 + a^2 V_2$$

Compact form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$$[V_{abc}] = [T][V_{012}]$$

$$[T]^{-1}[V_{abc}] = [T]^{-1}[T][V_{012}]$$

$$[V_{012}] = [T]^{-1}[V_{abc}]$$

$$[T]^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$\therefore$

$$V_0 = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_1 = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

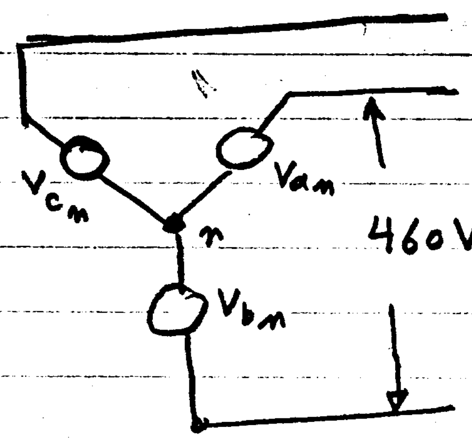
$$V_2 = \frac{1}{3}(V_a + a^2 V_b + aV_c)$$

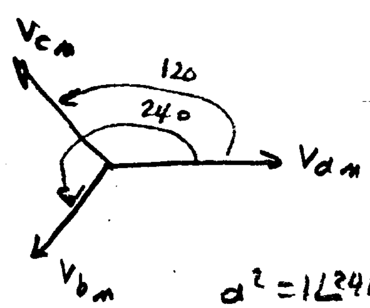
similarly

$$[I_{abc}] = [T][I_{012}]$$

$$[I_{012}] = [T]^{-1}[I_{abc}]$$

Problem. Consider a balanced, Y-connected, 460 volts generator. Compute the positive, negative and zero sequence voltages




$$V_{an} = \frac{460\angle 0}{\sqrt{3}} = 265.9$$

$$V_{bn} = 265.9\angle 240 = 265.9\,a^2$$

$$V_{cn} = 265.9\angle 120^\circ = 265.9\,a$$

$a^2 = 1\angle 240$

$a = 1\angle 120$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \overset{=V_{an}}{265.9} \\ \overset{=V_{bn}}{265.9\,a^2} \\ \overset{=V_{cn}}{265.9\,a} \end{bmatrix}$$

$$V_0 = \frac{1}{3}(265.9 + 265.9\,a^2 + 265.9\,a)$$

$$= \frac{265.9}{3}(\underbrace{1 + a + a^2}_{\text{show this}}) = 0$$

$$V_0 = 0$$

$$V_1 = \frac{1}{3}(265.9 + 265.9a^3 + 265.9a^3)$$

Recall $a^3 = 1\angle 0$

$$V_1 = 265.9\angle 0$$

$$V_2 = \frac{1}{3}[265.9 + 265.9a^4 + 265.9a^2]$$

$a^4 = a = 1\angle 120$

$$V_2 = \frac{1}{3}[265.9)(\underbrace{1 + a + a^2}_{0}) = 0$$

Conclusions.

- For balanced three-phase sources, we have:
  - Only positive sequence voltages exist.
  - Zero sequence voltages do not exist.
  - negative sequence voltages " .

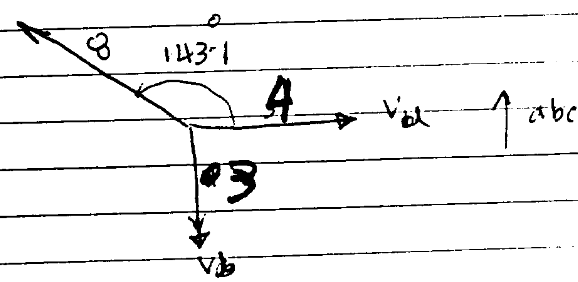
## Problem

Consider for a 3-φ unbalanced set of voltages:

$V_a = 4\angle 0$

$V_b = 3\angle -90$

$V_c = 8\angle 143.1^\circ$




determine $V_0$, $V_1$ and $V_2$.

sol.

$$V_0 = \frac{1}{3}(V_a + V_b + V_c)$$

$$= \frac{1}{3}[4.0 - j3 + 8(-0.8 + j0.6)] = -0.8 + j0.6$$

$$= 1\angle 143.1^\circ$$

$$V_{a0} = V_{b0} = V_{c0} = V_0 = 1\angle 143.1^\circ$$

$$V_1 = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

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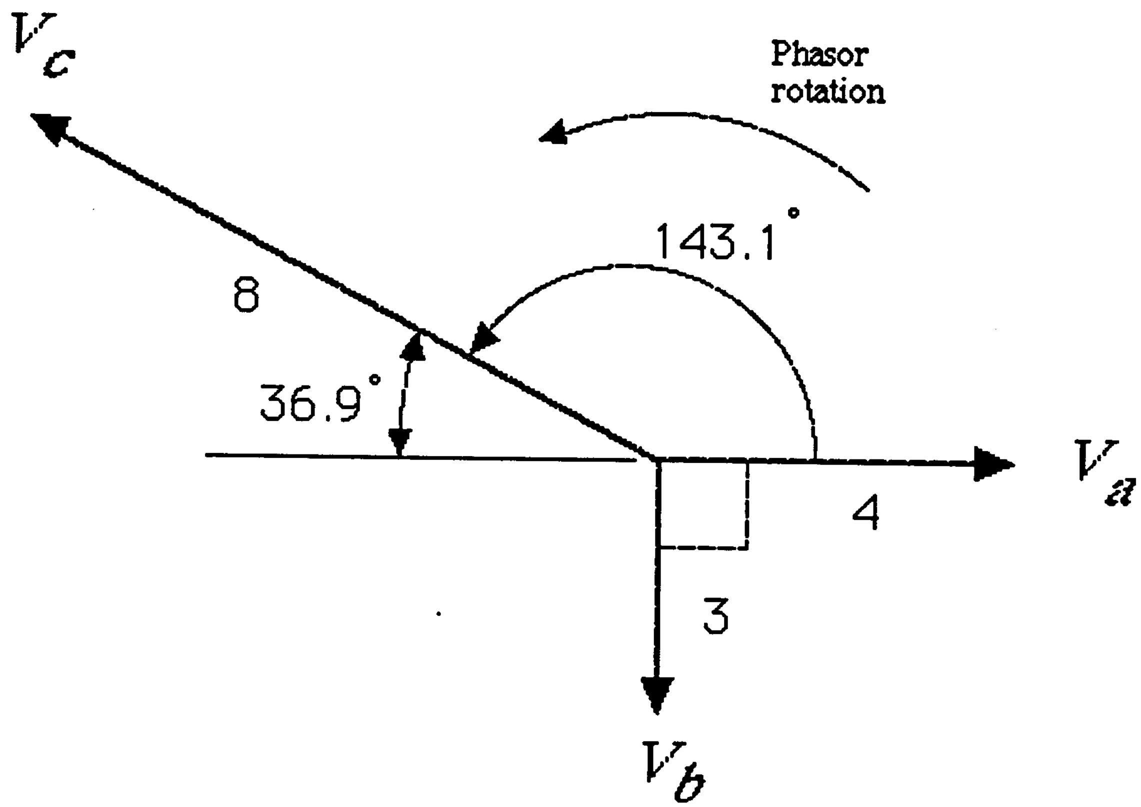



Unbalanced Three-Phase System

#7B

$$V_1 = \frac{1}{3}\left[4\angle 0 + (1\angle 120)(3\angle -90) + (1\angle -120)(8\angle 143.1)\right]$$

$$V_1 = \frac{1}{3}\left[13.96 + j4.64)\right] = 4.9\angle 18.4^\circ$$

$$V_{a_1} = V_1 = 4.9\angle 18.4^\circ$$

$$V_{b_1} = V_1\angle -120 = 4.9\angle -101.6$$

$$V_{c_1} = V_1\angle 120 = 4.9\angle 138.4$$

$$V_2 = \frac{1}{3}\left[V_a + a^2 V_b + a V_c\right]$$

$$V_2 = 2.15\angle -86.2$$

$$V_{a_2} = V_2 = 2.15\angle -86.2$$

$$V_{b_2} = V_2\angle 120 = 2.15\angle 33.8$$

$$V_{c_2} = V_2\angle -120 = 2.15\angle -206.2$$

8A

~~Note~~ . check

$V_a = V_{a_0} + V_{a_1} + V_{a_2}$

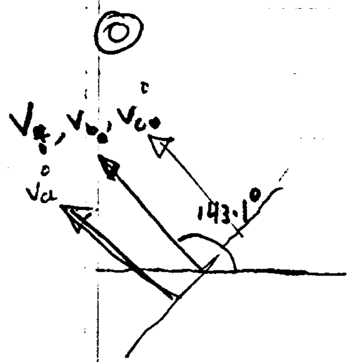
$= 1\angle 143.1^\circ + 4.9\angle 18.4^\circ + 2.15\angle -86.2 = 4\angle 0$

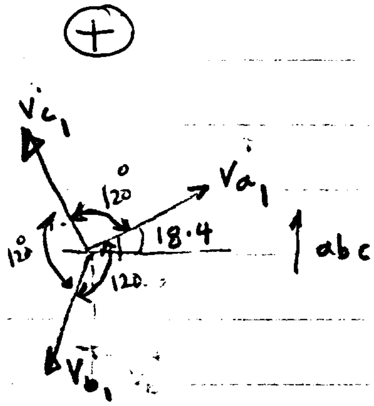
$V_b = V_{b_0} + V_{b_2} + V_{b_1} = 1\angle 143.1^\circ + 4.9\angle -106.6 + 2.15\angle 33.8$

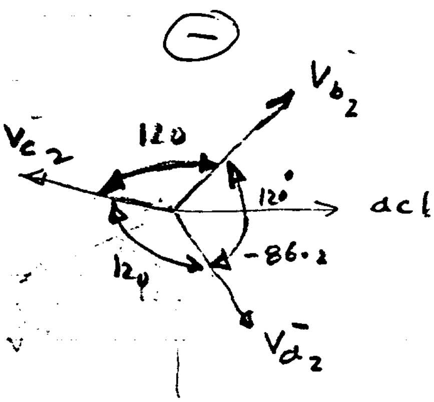
$= 3\angle -90^\circ$

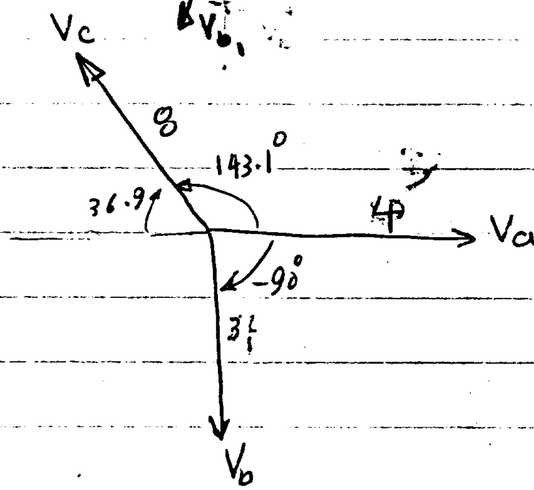
$V_c = V_{c_0} + V_{c_1} + V_{c_2} = 1\angle 143.1 + 4.9\angle 138.4 + 2.15\angle -206.2$

$= 8\angle 143.1^\circ$



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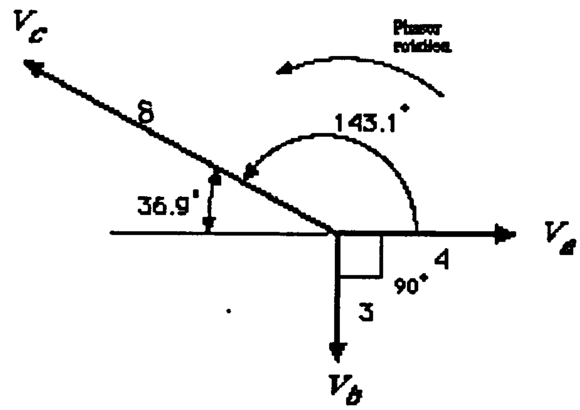
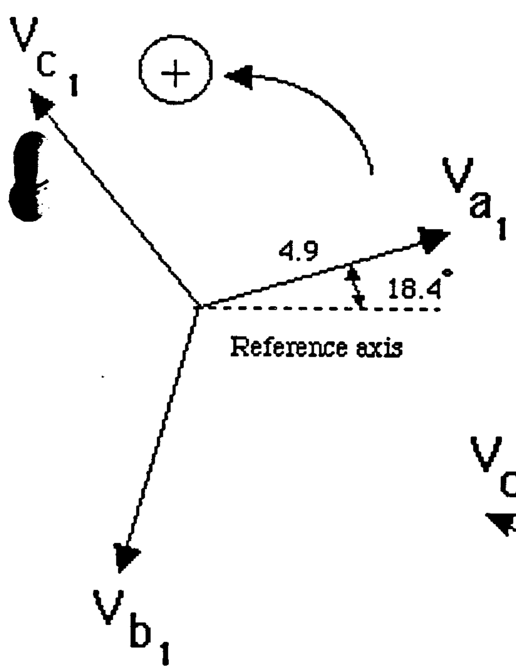
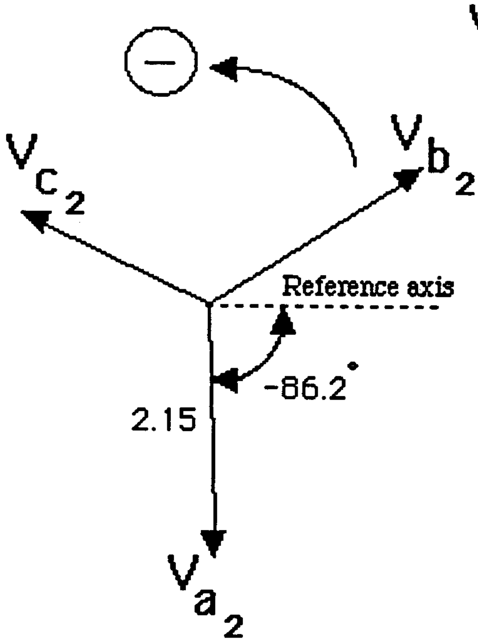


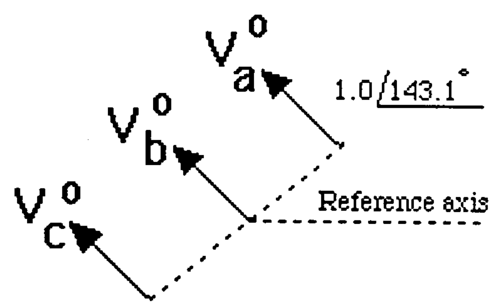

Original Unbalanced Three-Phase System




Positive Sequence



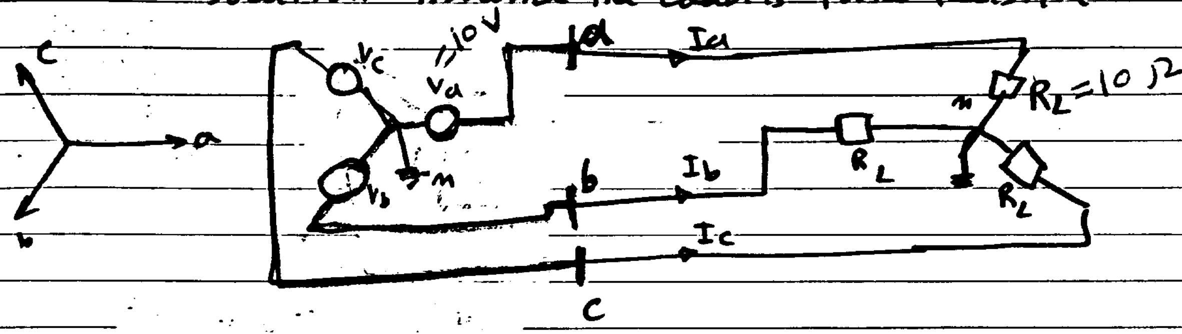

Negative Sequence




Zero Sequence

Problem 4. A balanced 3-$\phi$ voltages are impressed on a balanced 3-$\phi$ Load. Compute the $I_0$, $I_1$ and $I_2$.

solution. Assume the Load is pure resistive.



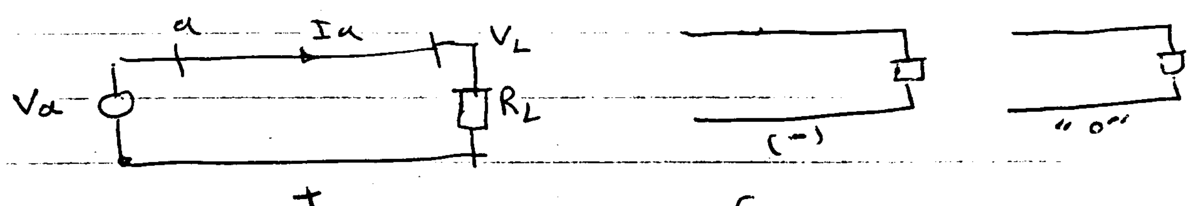

$$\begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I^0 = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I^+ = \frac{1}{3}\left[I_a + aI_b + a^2 I_c\right]$$

$$I^- = \frac{1}{3}\left[I_a + a^2 I_b + aI_c\right]$$

Balanced voltages and balanced Loads

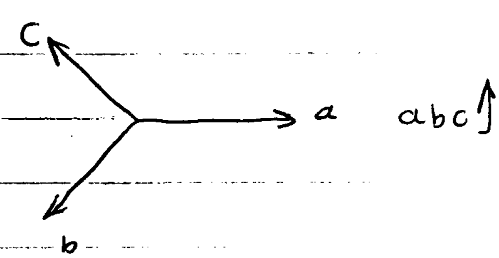



Assume

$$V_a = V\angle 0 = 10\angle 0$$

$$V_b = V_a\angle 240 = a^2 V_a = 10\angle 240$$

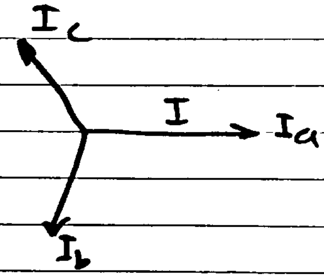
$$V_c = V_a\angle 120 = a V_a = 10\angle 120$$



$$I_a = \frac{V_{an}}{R_L} = \frac{V}{R_L}\angle 0 = I\angle 0$$

$$I_b = \frac{V}{R_L}\angle 240 = I\angle 240 = a^2 I_a$$

$$I_c = \frac{V}{R_L}\angle 120 = I\angle 120 = a\, I_a$$



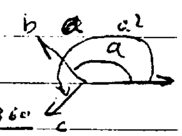
$$I_0 = \frac{1}{3}[I_a + I_b + I_c] \quad \text{but} \quad I_a + I_b + I_c = I_n$$

$\boxed{I = 0}$ $\quad I_n = 0$ for balanced 3-$\phi$ system

$$I_1 = \frac{1}{3}\left[I_a + \underbrace{a\,a^2}_{a^3 = 1 + j\cdot 0} I_a + \underbrace{a^2 \cdot a}_{a^3 = 1 + j\cdot 0} I_a\right] = \frac{1}{3} I_a [1 + 2] = I_a$$

$$I_1 = \frac{1}{3}[I_a + a^3 I_a + a^3 I_a] = \frac{1}{3} I_a [1 + 2a^3]$$

$a^3 = 1 + j\cdot 0$ since $a = 1\angle 120$ $\quad a^2 = 1\angle 240$ $\quad a^3 = 1\angle 360$

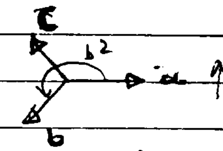


$$\therefore \boxed{I_1 = \frac{1}{3} I_a [1 + 2] = I_a = \frac{V_{an}}{R_L} = \frac{V\angle 0}{R_L}}$$

similarly,

$$I^- = \frac{1}{3}[I_a + a^2 I_b + a I_c]$$

$$I^- = \frac{1}{3}[I_a + a^2 a^2 I_a + a^2 I_a]$$



$I_b = a^2 I_a$

$I_c = a I_a$

$$I^- = \frac{1}{3} I_a [1 + a^2 + a^4]$$

$$1 + a^2 + a^4 = 0 + j\cdot 0$$

$$I^- = 0$$

Conclusions: when a balanced set of 3-$\phi$ voltages is impressed on a balanced 3-$\phi$ system, only positive sequence currents can flow. That is

* only positive sequence currents exist.
* Zero sequence currents do not exist
* Negative sequence currents " ".