~~EE 740~~

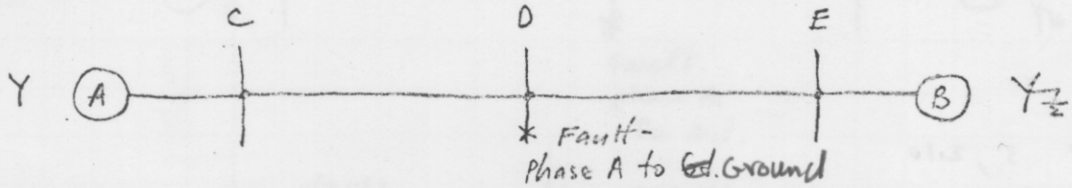
ALI. KEYHANI

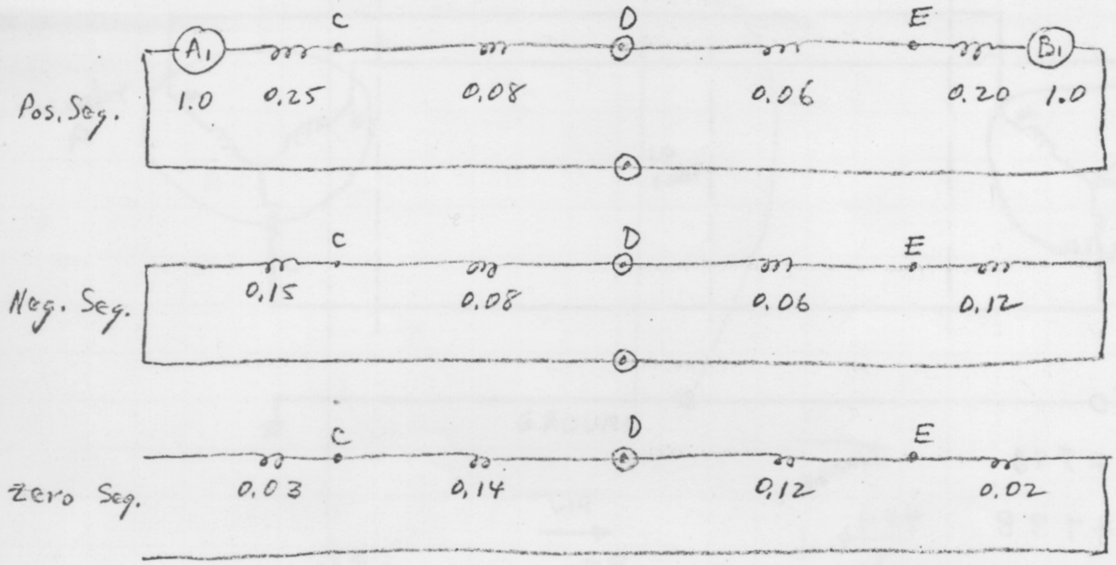
Special Problem

12/69

39

Single Line to Ground Fault.





$$\begin{array}{r} .12 \\ .02 \\ \hline .14 \end{array}$$

a) Find the actual phase voltages at the 3 buses. Show the phasor diagrams.

b) Find actual generator line currents.

c) Compare the currents in (b) with those for a 3-phase fault at bus D.

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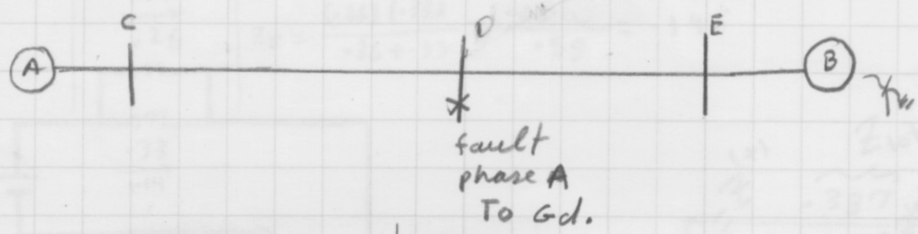

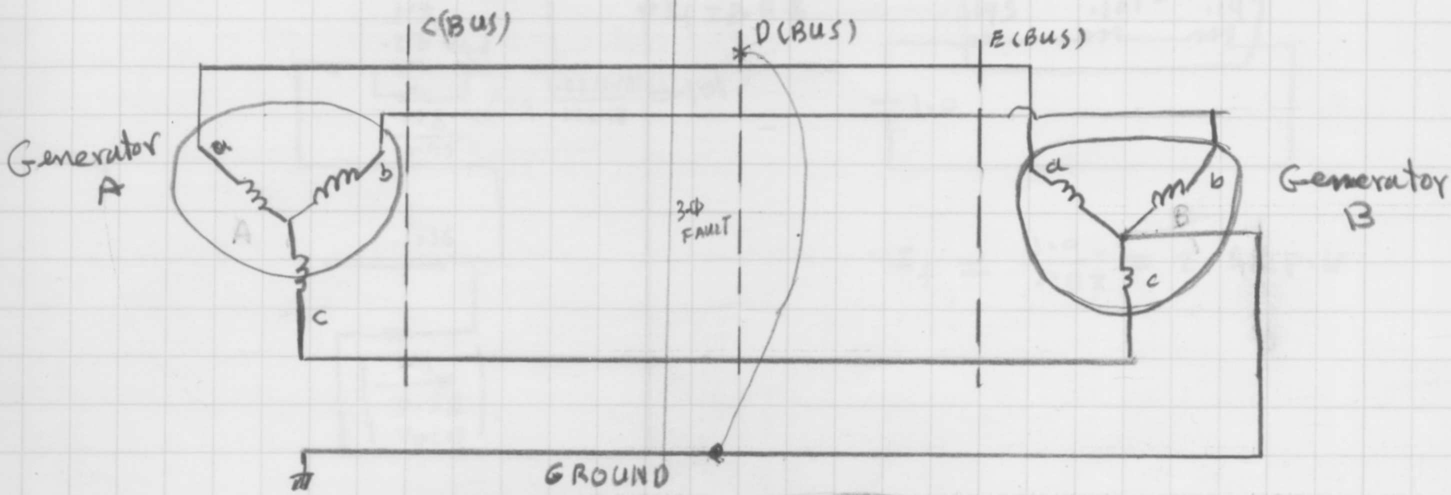

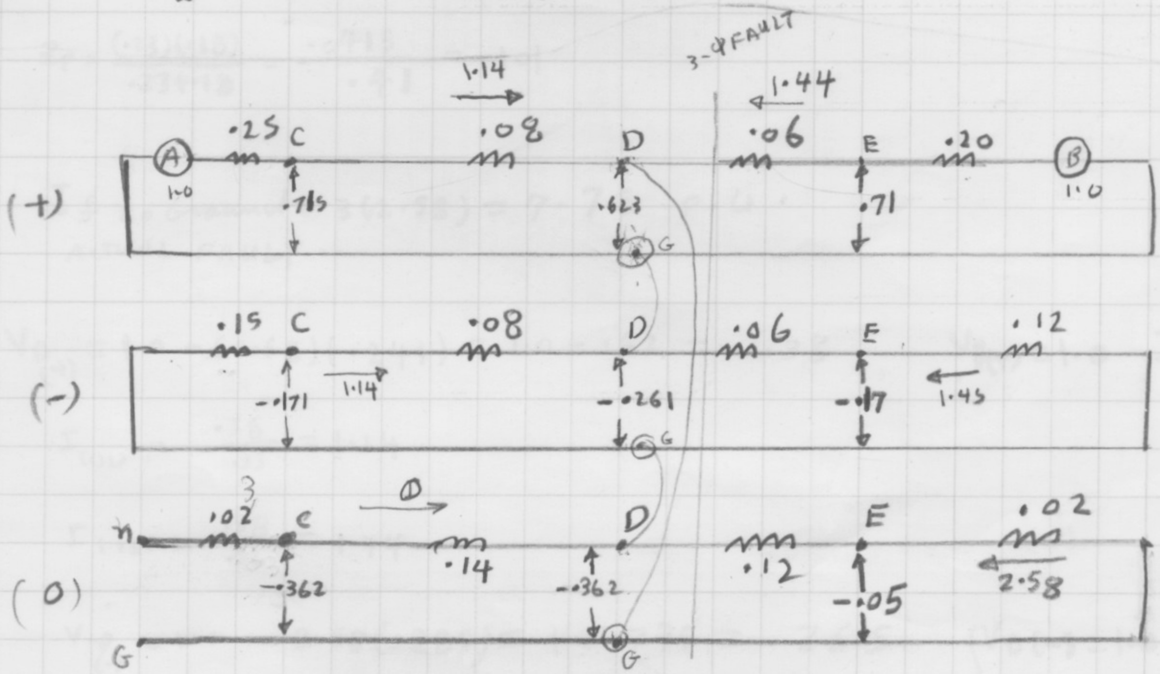

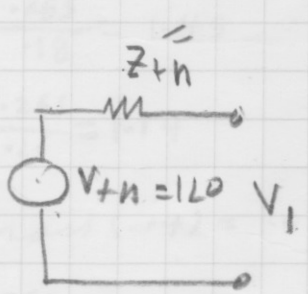

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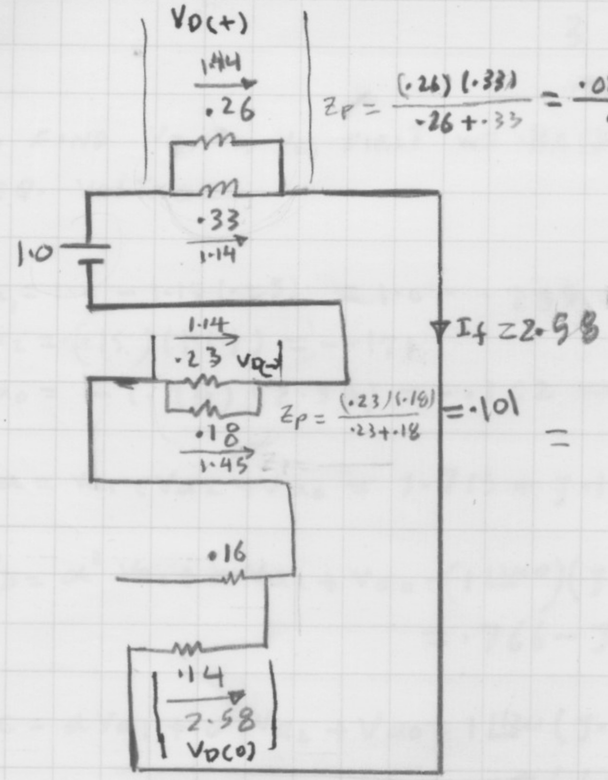
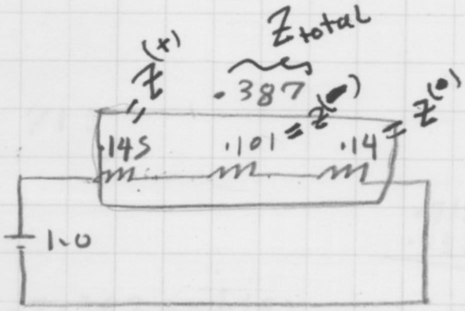
a)



$Z_p = \frac{(.26)(.33)}{.26+.33} = \frac{.086}{.59} = .145$



$I_f = \frac{1.0}{.387} = 2.58$ p.u

$Z_p = \frac{(.23)(.18)}{.23+.18} = \frac{.0415}{.41} = .101$

F

I_f to Ground $= 3(2.58) = 7.75$ p.u
ACTUAL FAULT..

$V_{D(+)} = 1.0 - (2.58)(.241) = 1.0 - .62 = .38$

$V_{D(+)} = 1.0 - I_f(\underbrace{\overset{.14}{Z^0} + \overset{.101}{Z^-}}_{.241})$

$I_{(.33)} = \frac{.38}{.33} = 1.14$

$I_{(.26)} = \frac{.38}{.26} = 1.44$

$V_{D(-)} = 1.0 - 2.58(.285) = 1 - .735 = .265$

$V_{D(-)} = 1.0 - I_f(\underbrace{\overset{.14}{Z^0} + \overset{.145}{Z^+}}_{.285})$

$I_{(.18)} = \frac{.265}{.18} = 1.45$

$I_{(.23)} = \frac{.265}{.23} = 1.14$

(0) $V_{D(0)}$ = drop cross $(.14) = 1.0 - (2.58)(.246) = 1 - .645 = .355$

$I_{(.14)} = \frac{.355}{.14} = 2.58$

$V_{D(0)} = 1.0 - I_f(\underbrace{\overset{.101}{Z^-} + \overset{.145}{Z^+}}_{.246})$

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TO FIND V_a, V_b, V_c FIRST WE HAVE TO FIND THE (+), (-), (0) ZEQ. VOLTAGES.

BUS C

$$V_{a_1} = 1.0 - 1.14(.25) = 1.0 - .285 = .715$$
$$V_{a2} = -(.15)(1.14) = -.171$$
$$V_{a0} = -(.14)(2.58) = -.362$$

$$\begin{array}{r} .715 \\ .533 \\ \hline \end{array}$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = J.715 - J.171 - J.362 = J.182$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} = (1\angle 240)(J.715) + 1\angle 120(-J.171) + (-J.362)$$
$$= .766 - J.634$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} = 1\angle 120(J.715) + 1\angle 240(-J.171) - J.362$$
$$= -.766 - J.634$$

BUS D

$$V_{a_1} = 1.0 - 1.14(.33) = 1.0 - .376 = .624$$
$$V_{a2} = -.171 - (1.14)(.08) = -.261$$
$$V_{a0} = -.362$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = +J.624 - J.261 - J.362 = 0$$ (IT IS SHORTED TO GROUND AND SHOULD BE ZERO)

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} = 1\angle 240(J.624) + 1\angle 120(-J.261) - J.362$$
$$= +.766 - J.543$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0}$$
$$= 1\angle 120(J.624) + 1\angle 240(-J.261) + (-J.362)$$

$$V_c = -.766 - J.543$$

BUS E

$$V_{a1} = 1.0 - (.2)(1.44) = 1 - .28 = .71$$
$$V_{a2} = -(.12)(1.45) = -.174$$

$$V_{a0} = -(.02)(2.58) = -.0516$$

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a) CONT.

BUS E

$$V_a = V_{a1} + V_{a2} + V_{a0} = j.71 - j.174 - j.05$$
$$= j.487$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} = 1\angle 240\,(j.71) + (1\angle 120)(-j.17) + (-j.05)$$
$$= .766 - j.32$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} = (1\angle 120)(j.71) + 1\angle 240(-j.19) + (-j.05)$$
$$= -.766 - j.32$$

... to Ground Fault.

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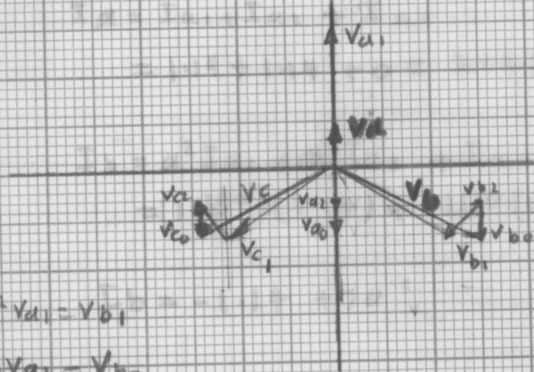
BUS C



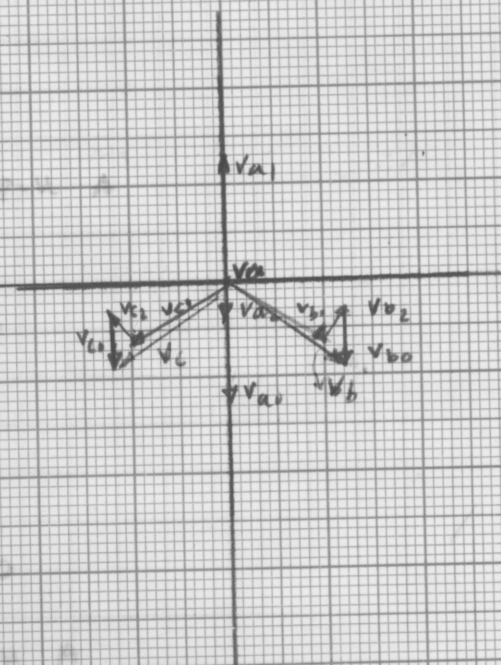
$a^2 V_{a1} = V_{b1}$

$a V_{a2} = V_{b2}$

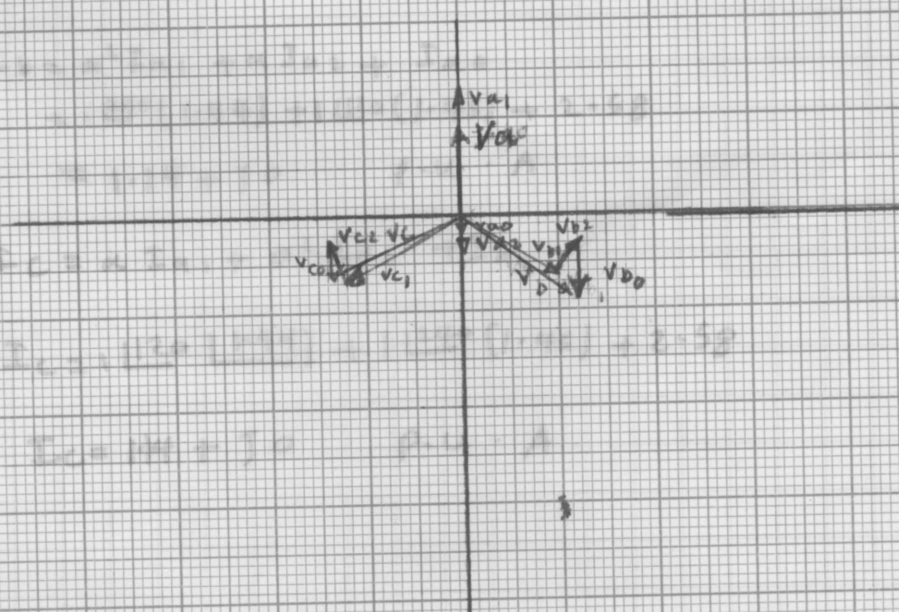
$a V_{a1} = V_{c1}$

$a^2 V_{a2} = V_{c2}$

BUS D



BUS E



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b) TO FIND THE ACTUAL GENERATOR LINE CURRENTS FIRST WE HAVE TO FIND POSITIVE, NEGATIVE AND ZERO SEQ. CURRENTS.

GEN. A

$I_a = I_{a1} + I_{a2} + I_{a0}$

$= 1.14 + 1.14 + 0 = 2.28$ p.u A

$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$

$= (1\angle 240)(1.14) + 1\angle 120(1.14) + 0$

$I_b = -1.14 + j0$ p.u A

$I_c = a I_{a1} + a^2 I_{a2} + I_{a0}$

$I_c = 1\angle 120(1.14) + 1\angle 240(1.14) + 0$

$I_c = -1.14 + j0$ p.u A

GEN. B

$I_a = I_{a1} + I_{a2} + I_{a0}$

$I_a = 1.44 + 1.45 + 2.58 = 5.47$ p.u A

$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$

$= 1\angle 240(1.44) + 1\angle 120(1.45) + 2.58$

$= 1.14 + j0$ p.u. A

$I_c = a I_{a1} + a^2 I_{a2} + I_{a0}$

$I_c = 1\angle 120\,(1.44) + 1\angle 240(1.45) + 2.58$

$I_c = 1.14 + j0$ p.u. A.

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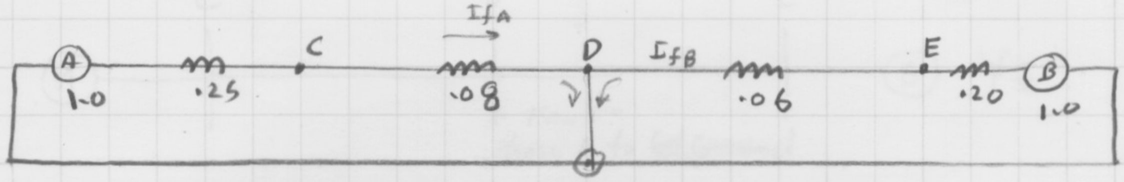
c) 3 - Φ FAULT



$I_{fA} = \frac{1.0}{.33} = 3.03$ A. PU

3.03 > 2.28

$I_{fB} = \frac{1.0}{.26} = 3.85$ A-PU

but -

5.47 > 3.85

$I_{fT} = 3.85 + 3.03 = 6.88$ P.U.A

Max values

$I_{L-G_A} = 1.14$ 2.28

$I_{BL-G} = 1.44$ 5.47

$I_{fL-G} = 2.\cancel{5}8$ 7.75

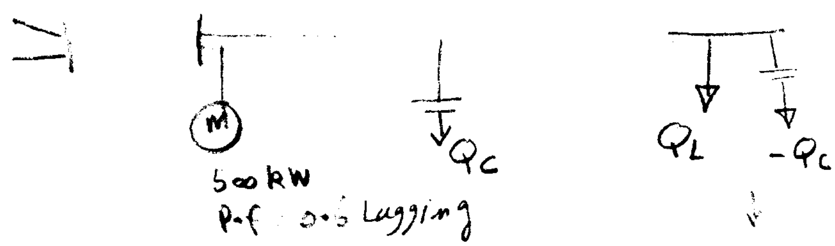
3 - Φ FAULT CURRENT IS GREATHER THAN SINGLE-LINE TO GROUND.

Not always -

EE 740
solution set #1
(1)

A. Jaynew

2.8 An industrial plant consisting primarily of induction motor loads absorbs 500 kW at 0.6 power factor lagging. (a) Compute the required kVA rating of a shunt capacitor to improve the power factor to 0.9 lagging. (b) If a synchronous motor rated 500 hp with 90% efficiency operating at rated load and at unity power factor is added to the plant instead of the capacitor, calculate the resulting power factor. Assume constant voltage. (1 hp = 0.747 kW)



a) $P_L = VI\cos\theta = S\cos\theta$ $\qquad S_L = P_L + jQ_L$

$\theta = \cos^{-1}(p.f) = \cos^{-1}(.6) = 53.13^\circ$

$$S_L = \frac{P_L}{\cos\theta} = \frac{500\text{ kW}}{0.6} = 833.33 \text{ KVA}$$

$Q_L = VI\sin\theta = 833.33 \sin(53.13^\circ) = 666.7$ kvars

After capacitor is in service

$P_L = VI\cos\theta \qquad \cos\theta = .9 \qquad \theta = 25.84^\circ$

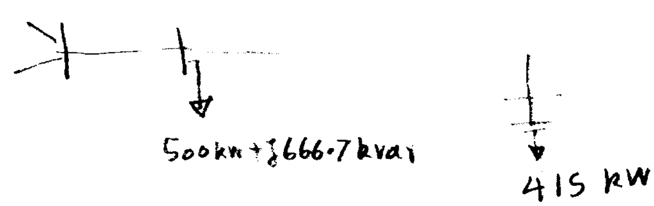
$500 = S_{new}(0.9) \qquad S_{new} = \frac{500}{0.9} = 555.56$ KVA

$Q_{new} = S_{new}\sin\theta$

$Q_{new} = 555.56 \sin(25.84) = 242.2$ kvar

$Q_{new} = -Q_C + Q_L \qquad Q_C = 666.7 - 242.2 = 424.5$ kva

b)

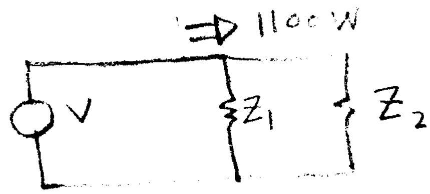


$P_M = \frac{500 \times .746}{.9} = 415$ kW

$S_{net} = 500 + 415_{kw} + j666.7\text{ kva} = 914.44 + j666.7$ KVa

$\tan^{-1}\frac{666.7}{914.44}$

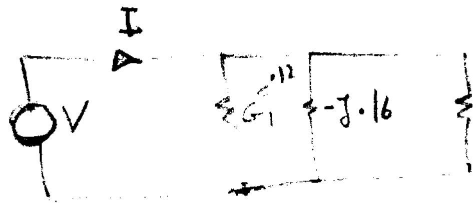
(2.9) The real power delivered by a source to two impedances, $Z_1 = 3 + j4\,\Omega$ and $Z_2 = 10\,\Omega$, connected in parallel, is 1100 W. Determine (a) the real power absorbed by each of the impedances and (b) the source current.



$$Y_1 = \frac{1}{Z_1} = \frac{1}{3+j4} = \frac{1}{5\angle 53.13}$$

$$Y_1 = 0.2\angle -53.13^\circ = 0.12 - j.16\ \Omega$$

$$Y_1 = G_1 - jB \qquad G_1 = .12\ \mho$$



$$G_2 = \frac{1}{Z_2} = \frac{1}{10} = 0.1\ \mho$$

a) $$P = V^2(G_1 + G_2) \qquad V = \sqrt{\frac{P}{(G_1+G_2)}} = \sqrt{\frac{1100}{(.12+.1)}} =$$

$$V = 70.71\ V$$

$$P_1 = V^2 G_1 = (70.71)^2(.12) = 600\ W$$

$$P_2 = V^2 G_2 = (70.71)^2(0.10) = 500\ W$$

b) $$Y_{eq} = Y_1 + Y_2 = (.12 - j.16) + .1 = 0.22 - j.16$$

$$I_s = V\,Y_{eq} = (70.71)(.2720) = 19.23\ A$$

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3

2.11 For the circuit shown in Figure 2.20, convert the voltage sources to equivalent current

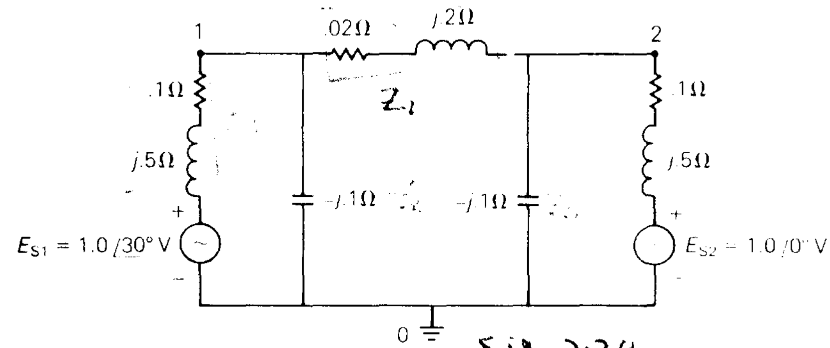
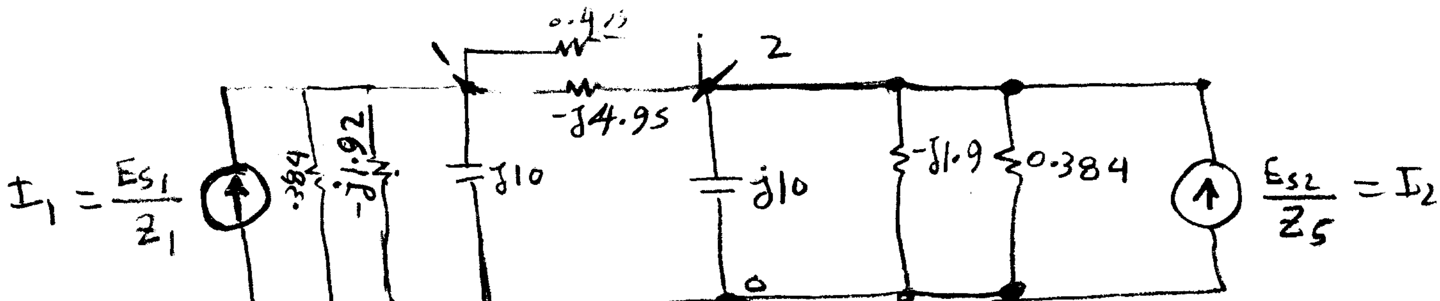


Fig. 2.20

sources and write nodal equations in matrix format using bus 0 as the reference bus. Do not solve the equations.



Admittance Values are shown

$$Y_1 = \frac{1}{Z_1} = \frac{1}{.1 + j.5} = G_1 - jB_1 = .384 - j1.72$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{.02 + j.2} = G_2 - jB_2 = .495 - j4.95$$

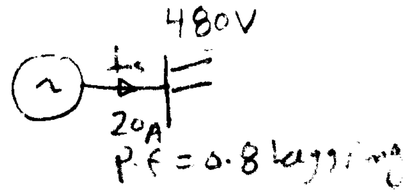
$$Y_3 = \frac{1}{Z_3} = j10 \qquad Y_4 = \frac{1}{Z_4} = j10$$

$$Y_5 = \frac{1}{Z_5} = \frac{1}{.1 + j.5} = .384 - j1.92$$

$$I_1 = 1.96 \angle -48.6^\circ \qquad I_2 = 1.96 \angle -78.69^\circ$$

$$\begin{bmatrix} +.8796 + j3.12 & -.495 + j4.95 \\ -.495 + j4.95 & 0.879 + j3.12 \end{bmatrix} \begin{bmatrix} V_{10} \\ V_{20} \end{bmatrix} = \begin{bmatrix} 1.961 \angle -48.69 \\ 1.961 \angle -78.69^\circ \end{bmatrix}$$

2.16 A three-phase 25-kVA, 480-V, 60-Hz alternator, operating under balanced steady-state conditions, supplies a line current of 20 A per phase at a 0.8 lagging power factor and at rated voltage. Determine the power triangle for this operating condition.

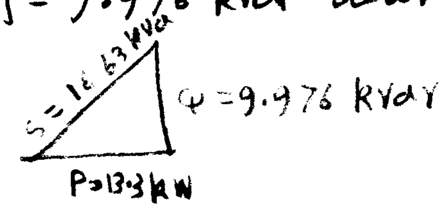


$$S_{3\phi} = \sqrt{3}\, V_{LL} I_L \angle \cos^{-1}(P.f) = \sqrt{3}(480)(20) \angle \cos^{-1}(.8)$$

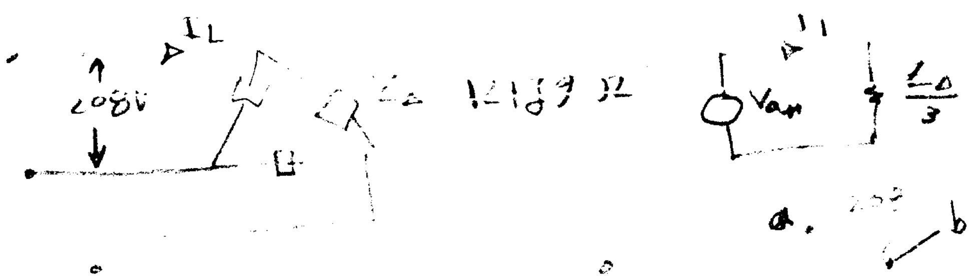
$$= 16.627\times10^{3} \angle 36.87^{\circ} = 13.3\times10^{3} + j9.976\times10^{3}$$

$P_{3\phi} = \mathrm{Re}\{S_{3\phi}\} = 13.30$ KW delivered

$Q_{3\phi} = \mathrm{Im}\{S_{3\phi}\} = 9.976$ kvar delivered



2.17 A balanced Δ-connected impedance load with $(12 + j9)\Omega$ per phase is supplied by a balanced three-phase 60-Hz, 208-V source. (a) Calculate the line current, the total real and reactive power absorbed by the load, the load power factor, and the apparent load power. (b) Sketch a phasor diagram showing the line currents, the line-to-line source voltages, and the Δ-load currents. Assume positive sequence and use V_{ab} as the reference.



$$V_{an} = \frac{208\angle -30^{\circ}}{\sqrt{3}} \qquad Z_Y = \frac{Z_\Delta}{3} = 4 + j3 = 5\angle 36.87^{\circ}\ \Omega$$

$V_{an} = 120.1\angle -30^{\circ}$

$$I_a = \frac{V_{an}}{Z_Y} = \frac{120.1\angle -30^{\circ}}{5\angle 36.87} = 24.02\angle -66.87^{\circ}$$

$$S_{3\phi} = 3 V_{an} I_a^{*} = 3(120.1\angle -30^{\circ})(24.02\angle 66.87^{\circ})^{*}$$

$$S_{3\phi} = 8654\angle 36.87^{\circ} = 6923 + j5192$$

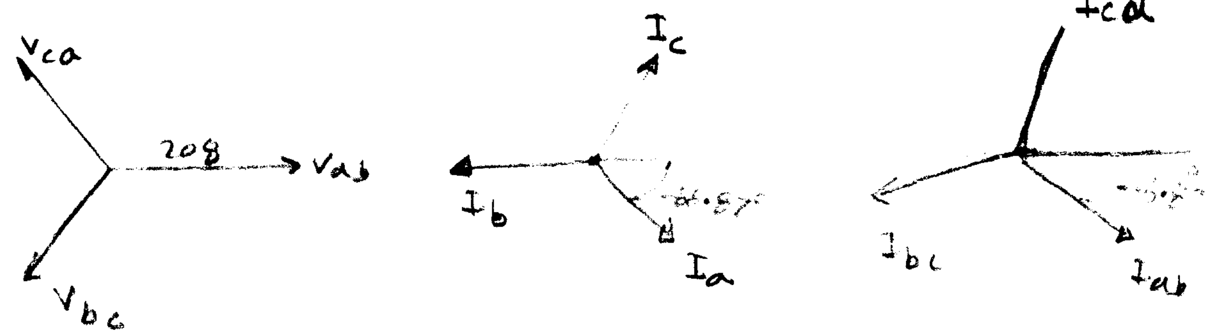
$P_{3\phi} = 6923$ W $\quad Q_{3\phi} = 5192$ } absorbed by the load

$S_{3\phi} = |S_{3\phi}| = 8654$ VA

(5)

2.17 cont.

b)



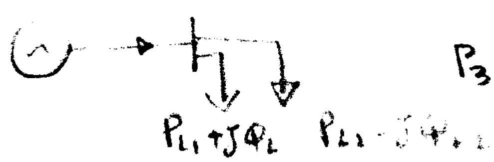
$V_{ab} = 208\angle 0$

$I_a = 24.02 \angle -66.87°$ $\qquad I_a = \sqrt{3}\, I_{AB} \angle -30°$

$$I_{AB} = \frac{I_a}{\sqrt{3}\angle -30°} = \frac{24.02\angle -66.87°}{\sqrt{3}\angle -30°}$$

$$I_{AB} = 13.88 \angle -36.87°$$

(2.18) Two balanced Y-connected loads, one drawing 10 kW at 0.8 p.f. lagging and the other 15 kW at 0.9 p.f. leading, are connected in parallel and supplied by a balanced three-phase Y-connected, 480-V source. (a) Determine the source current. (b) If the load neutrals are connected to the source neutral by a zero-ohm neutral wire through an ammeter, what will the ammeter read?



$P_{3\phi} = S_{3\phi} \cos\theta$

$$S_{3\phi_1} = \frac{10 \times 10^3}{0.8} = 12.500 \text{ KVA} \qquad S_{3\phi_2} = \frac{15 \times 10^3}{.9} = 16.67 \text{ KVA}$$

$\theta_1 = \cos^{-1}(.8) = 36.86 \qquad \theta_2 = \cos^{-1}(.9) = 25.84$

$Q_1 = S_{3\phi_1} \sin\theta_1 = 7.498 \qquad S_T = P_1 + P_2 + j(Q_1 - Q_2)$

$Q_2 = S_{3\phi_2} \sin\theta_2 = 7.265$

$S_T = 25 \text{ kW} + j.233 \text{ kvar}$

$$|I_s| = \frac{S_T}{\sqrt{3}\, V_{LL}} = \frac{25.001 \times 10^3}{\sqrt{3}(480)} = 30.07 \text{ A}$$

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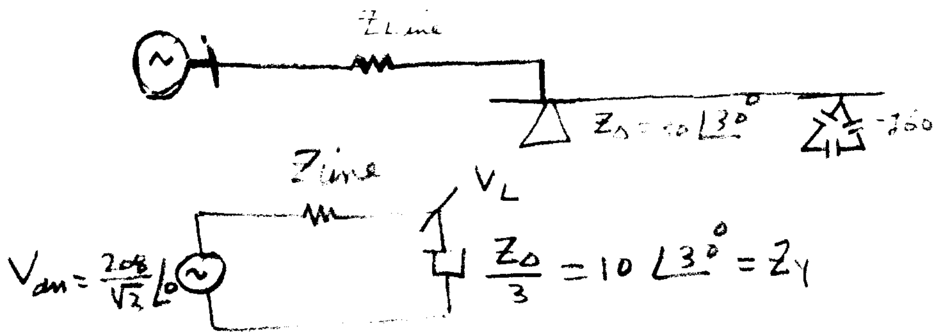
2.19 Three identical impedances $Z_\Delta = 30\angle 30^\circ\ \Omega$ are connected in Δ to a balanced three-phase 208-V source by three identical line conductors with impedance $Z_L = (0.8 + j0.6)\ \Omega$ per line. (a) Calculate the line-to-line voltage at the load terminals. (b) Repeat part (a) when a Δ-connected capacitor bank with reactance $(-j60)\ \Omega$ per phase is connected in parallel with the load.

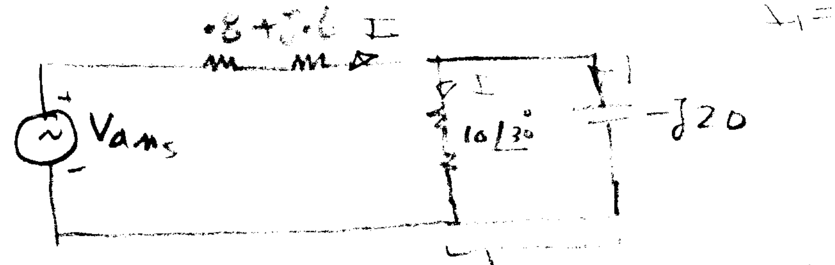


$$V_{an} = V_{an}\frac{Z_Y}{Z_Y + Z_{line}} = \frac{208\angle 0}{\sqrt{3}}\ \frac{10\angle 30}{10\angle 30 + (.8+j.6)}$$

$$= \frac{(120.09)(10\angle 30^\circ)}{9.46+j5.6} = \frac{1200.9\angle 30^\circ}{10.99\angle 30.62} = 109.3\angle -.62^\circ\ V$$

Load Voltage : $V_{AB} = \sqrt{3}\,(109.3) = 189.3\ V$ (line-to-line)

b)



$$Z_{eq} = 10\angle 30^\circ \,//\, (-j20) = \frac{200\angle -60^\circ}{8.66 - j15}$$

$$Z_{eq} = 11.547\angle 0\ \Omega$$

$$V_{an_L} = V_{an_s}\frac{Z_{eq}}{Z_{eq} + Z_{line}}$$

$$= \frac{(120.09)(11.547)}{12.347 + j.6} = \frac{1386.7}{12.362\angle 2.78^\circ}$$

$V_{an_L} = 112.2\angle -2.78^\circ$

The load voltage is $V_{AB} = \sqrt{3}(112.2) = 194.3\ V$ (line-to-line)

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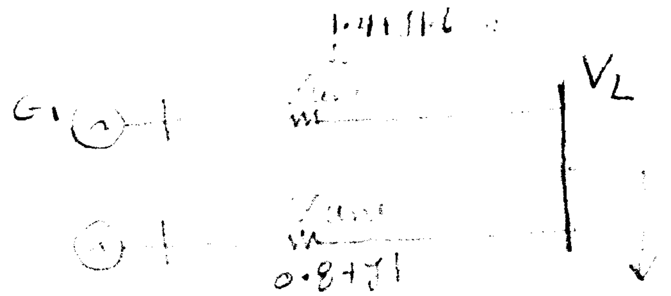
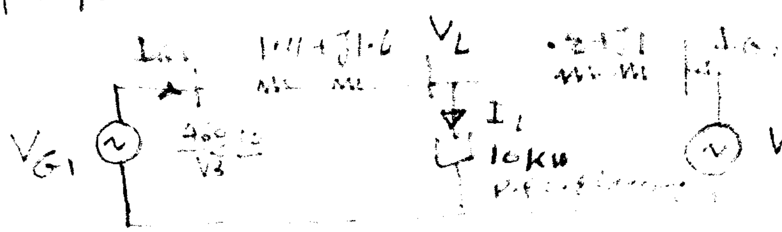
(2.20) Two three-phase generators supply a three-phase load through separate three-phase lines. The load absorbs 30 kW at 0.8 p.f. lagging. The line impedance is $(1.4 + j1.6)\,\Omega$ per phase between generator G1 and the load, and $(0.8 + j1)\,\Omega$ per phase between generator G2 and the load. If generator G1 supplies 15 kW at 0.8 p.f. lagging, with a terminal voltage of 460 V line-to-line, determine: (a) the voltage at the load terminals; (b) the voltage at the terminals of generator G2; and (c) the real and reactive power supplied by generator G2. Assume balanced operation.



$P_{L_{3\phi}}$ = 30 kW p.f = 0.8 lagging

P_{G1} = 15 kW p.f 0.8 lagging

$V_{t1} = 460$ (L-L)



$I_{G1} = \frac{P_{G1}}{\sqrt{3}\,(V_{G1\,L-L})(p.f)} = \frac{15 \times 10^3}{(1.73)(460)(0.8)} \angle -36.87° = 23.53 \angle -36.87°$

$V_{L_1} = V_{G1} - Z_{line} I_{G1} = 216.9 \angle$ [illegible]

$V_{L-L} = \sqrt{3}\,(216.9) = 375.7$ V (L-L)

b) $S_{L_{3\phi}} = \frac{P_{3\phi}}{p.f} \angle \cos^{-1} p.f = \frac{30 \times 10^3}{.8} \angle 36.87°$

$S_{L3\phi} = 37.5$ kVA [illegible]

[illegible] $I_L = \frac{37.5 \times 10^3 \angle -36.87}{3(216.9)}$ [illegible]

$I_L = 57.63 \angle -39.6$ A

$I_{G2} = I_L - I_{G1} = 34.14 \angle -41.98$ A

$V_{G2} = V_L + Z_{line} I_{G2} = 259.7 \angle .62°$ [illegible]

c) $S_{G2} = 3 V_{G2} I^*_{G2} = 20.12$ [illegible] $P_{G2} = 20.12$ kW, Q_{G2} [illegible]

A. Keyhani

EE

Solution set #2

3.3 Find the phase voltages V_{an}, V_{bn}, and V_{cn} whose sequence components are: $V_0 = 50\underline{/80^\circ}$, $V_1 = 100\underline{/0^\circ}$, $V_2 = 50\underline{/90^\circ}$.

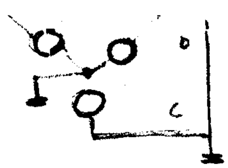
$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 50\underline{/80} \\ 100\underline{/0} \\ 50\underline{/90} \end{bmatrix}$$

$$= 50 \begin{bmatrix} 1\underline{/80} + 2\underline{/0} + \underline{/90} \\ 1\underline{/80} + 2\underline{/240} + 1\underline{/210} \\ 1\underline{/80} + 2\underline{/120} + 1\underline{/330} \end{bmatrix} = 50 \begin{bmatrix} 2.174 + j1.985 \\ -1.692 - j1.247 \\ 0.0397 + j2.217 \end{bmatrix}$$

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 147.2\underline{/42.40^\circ} \\ 105.1\underline{/216.4^\circ} \\ 110.9\underline{/88.97^\circ} \end{bmatrix}$$

3.4 One line of a three-phase generator is open-circuited, while the other two are short-circuited to ground. The line currents are $I_a = 0$, $I_b = 1000\underline{/150^\circ}$, and $I_c = 1000\underline{/30^\circ}$ A. Find the symmetrical components of these currents. Also find the current into the ground.

$I_a = 0 \qquad I_b = 1000\underline{/150^\circ} \qquad I_c = 1000\underline{/30^\circ}$



$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ 1000\underline{/150^\circ} \\ 1000\underline{/30^\circ} \end{bmatrix} = \frac{1000}{3} \begin{bmatrix} 1\underline{/150} + 1\underline{/30} \\ 1\underline{/270} + 1\underline{/270} \\ 1\underline{/30} + 1\underline{/150} \end{bmatrix}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 333.3\underline{/90^\circ} \\ 666.7\underline{/270^\circ} \\ 333.3\underline{/90^\circ} \end{bmatrix} \text{ A}$$

current into ground $= I_n = 3I_0 = 1000\underline{/90^\circ}$

EE 740

2

3.5 Given the line-to-ground voltages $V_{ag} = 280\angle 0°$, $V_{bg} = 250\angle -110°$, and $V_{cg} = 290\angle 130°$ volts, calculate (a) the sequence components of the line-to-ground voltages, denoted V_{Lg0}, V_{Lg1}, and V_{Lg2}; (b) line-to-line voltages V_{ab}, V_{bc}, and V_{ca}; and (c) sequence components of the line-to-line voltages V_{LL0}, V_{LL1}, and V_{LL2}. Also, verify the following general relation: $V_{LL0} = 0$, $V_{LL1} = \sqrt{3}V_{Lg1}\angle +30°$, and $V_{LL2} = \sqrt{3}V_{Lg2}\angle -30°$ volts.

a)

$$\begin{bmatrix} \bar{V}_{Lg0} \\ \bar{V}_{Lg1} \\ \bar{V}_{Lg2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 280\angle 0 \\ 250\angle -110 \\ 290\angle 130 \end{bmatrix}$$

$$= \begin{bmatrix} 5.039\angle -57.65° \\ 272.4\angle 6.58° \\ 27.82\angle -76.05° \end{bmatrix} \quad V$$

b)

$$\begin{bmatrix} \bar{V}_{ab} \\ \bar{V}_{bc} \\ \bar{V}_{ca} \end{bmatrix} = \begin{bmatrix} \bar{V}_{ag} - \bar{V}_{bg} \\ \bar{V}_{bg} - \bar{V}_{cg} \\ \bar{V}_{cg} - \bar{V}_{ag} \end{bmatrix} = \begin{bmatrix} 280\angle 0 - 250\angle -110 \\ 250\angle -110 - 290\angle 130 \\ 290\angle 130 - 280\angle 0 \end{bmatrix}$$

$$= \begin{bmatrix} 434.5\angle 32.73° \\ 468.1\angle -77.55° \\ 516.6\angle 154.5° \end{bmatrix}$$

Note

$\tilde{V}_{LL}^{+} = \sqrt{3}\tilde{V}_{Lg}^{+}\angle 30$

$\tilde{V}_{LL}^{-} = \sqrt{3}\tilde{V}_{Lg}^{-}\angle -30$

c)

$$\begin{bmatrix} \bar{V}_{LL0} \\ \bar{V}_{LL1} \\ \bar{V}_{LL2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \bar{V}_{ab} \\ \bar{V}_{bc} \\ \bar{V}_{ca} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{V}_{LL0} \\ \tilde{V}_{LL1} \\ \tilde{V}_{LL2} \end{bmatrix} = \begin{bmatrix} 0 \\ 471.8\angle 36.58° \\ 48.35\angle -166.2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}\,\hat{V}_{Lg1}\angle 30° \\ \sqrt{3}\,\tilde{V}_{Lg2}\angle -30° \end{bmatrix}$$

$\tilde{V}_{LL1} = \sqrt{3}\hat{V}_{Lg1}\angle 30° = \sqrt{3}\,272.4\angle 6.58°\angle 30° = 471.8\angle 36.58°$

$\angle -30°$

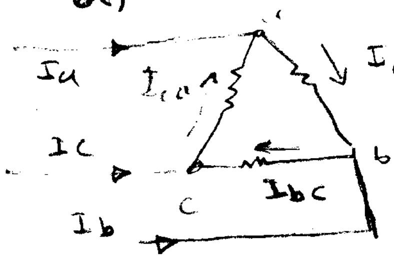
EE740

(3)

3.6 **Section 3.2**

The currents in a Δ load are $I_{ab} = 10\underline{/0°}$, $I_{bc} = 15\underline{/-90°}$, and $I_{ca} = 20\underline{/90°}$ A. Calculate (a) the sequence components of the Δ-load currents, denoted $I_{\Delta 0}$, $I_{\Delta 1}$, $I_{\Delta 2}$; (b) the line currents I_a, I_b, and I_c, which feed the Δ load; and (c) sequence components of the line currents I_{L0}, I_{L1}, and I_{L2}. Also, verify the following general relation: $I_{L0} = 0$, $I_{L1} = \sqrt{3} I_{\Delta 1}\underline{/-30°}$, and $I_{L2} = \sqrt{3} I_{\Delta 2}\underline{/+30°}$ A.

a)



$$\begin{bmatrix} I_{\Delta 0} \\ I_{\Delta 1} \\ I_{\Delta 2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

$I_{ab} = 10\angle 0$

$I_{bc} = 15\angle -90$ A

$I_{ca} = 20\angle 90$

$$\begin{bmatrix} I_{\Delta 0} \\ I_{\Delta 1} \\ I_{\Delta 2} \end{bmatrix} = \begin{bmatrix} 3.727\angle 26.56° \\ 13.47\angle -3.55° \\ 6.821\angle 187.0° \end{bmatrix}$$

b)

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} I_{ab} - I_{ca} \\ I_{bc} - I_{ab} \\ I_{ca} - I_{bc} \end{bmatrix} = \begin{bmatrix} 10\angle 0 - 20\angle 90° \\ 15\angle -90 - 10\angle 0 \\ 20\angle 90 - 15\angle -90 \end{bmatrix} = \begin{bmatrix} 22.36\angle -63.43 \\ 18.03\angle 236.3 \\ 35\angle 90° \end{bmatrix}$$

c)

$$\begin{bmatrix} I_{L0} \\ I_{L1} \\ I_{L2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 \\ 23.32\angle -33.56 \\ 11.81\angle 217.° \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}\tilde{I}_{\Delta 1}\angle -30° \\ \sqrt{3}\tilde{I}_{\Delta 2}\angle 30° \end{bmatrix}$$

$$\hat{I}_{L1} = \sqrt{3}\,\tilde{I}_{\Delta 1}\angle -30 = \sqrt{3}\; 13.47\angle -3.56° \times \angle -30$$
$$= 23.47\angle -33.56$$

Note:

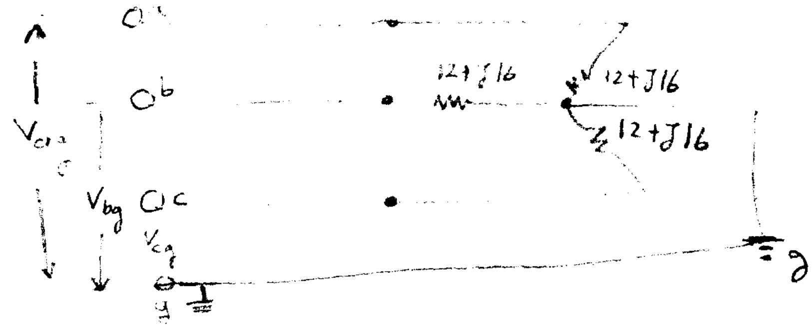
" For a balanced – Δ Load "

Positive sequence $\hat{I}_L^{+} = \hat{I}_\Delta^{+}\angle -30$

Negative sequence $I_L^{-} = I_\Delta^{-}\angle 30$

$$\hat{I}_{L2} = \sqrt{3}\,\tilde{I}_{\Delta 2}\angle -30$$
$$= (\sqrt{3})\,6.821\angle 187.0\ \angle +30$$
$$= 11.81\angle 217$$

3-7 The voltages given in Problem 3.5 are applied to a balanced-Y load consisting of $(12 + j16)$ ohms per phase. The load neutral is solidly grounded. Draw the sequence networks and calculate I_0, I_1, and I_2, the sequence components of the line currents. Then calculate the line currents I_a, I_b, and I_c.



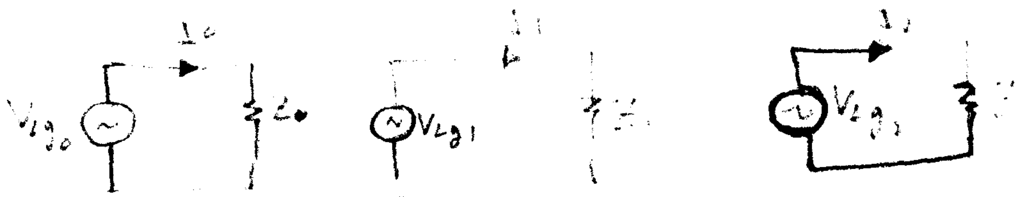
$V_{ag} = 280\angle 0^\circ$ V

$V_{bg} = 250\angle -110$ V

$V_{cg} = 290\angle 130$ V

From Prob. 3.5 $V_{Lg_0} = 5.039\angle -57.65^\circ$ $V_{Lg_1} = 272.4\angle 6.59^\circ$

$V_{Lg_2} = 27.82\angle -16.05^\circ$ $Z_L = 12 + j16 = 20\angle 53.13^\circ = Z_0 = Z_1 = Z_2$



$I_0 = \frac{V_{Lg_0}}{Z_0} = \frac{5.039}{20\angle 53.13^\circ} = 0.252\angle -110.78^\circ$ $I_1 = \frac{V_{Lg_1}}{Z_1} = 13.62\angle -46.58^\circ$ A

$I_2 = \frac{V_{Lg_2}}{Z_2} = 1.391\angle -129.18^\circ$ A

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.252\angle -110.78^\circ \\ 13.62\angle -46.54^\circ \\ 1.391\angle -129.18^\circ \end{bmatrix} = \begin{bmatrix} 14.0\angle -53.13^\circ \\ 12.5\angle -163.1^\circ \\ 14.5\angle 76.37^\circ \end{bmatrix}$$

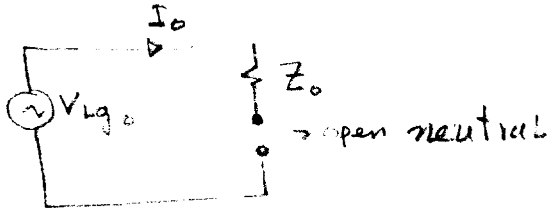
Since the source and load neutrals are connected

$$I_a = \frac{V_{ag}}{Z_Y} = \frac{280\angle 0}{20\angle 53.13} = 14.0\angle -53.13^\circ \text{ A}$$

$$I_b = \frac{V_{bg}}{Z_Y} = \frac{250\angle -110}{20\angle 53.13} = 12.5\angle -163.1^\circ \text{ A}$$

$$I_c = \frac{V_{cg}}{Z_Y} = \frac{290\angle 130}{20\angle 53.13} = 14.5\angle 76.87^\circ \text{ A}$$

3.8 Repeat Problem 3.7 with the load neutral open



$I_0 = 0$

$I_1 = 13.62 \angle -46.54^\circ$ A

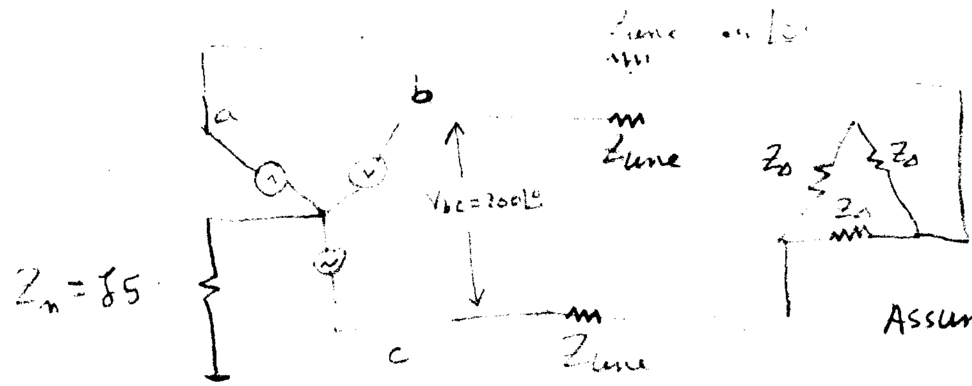
I_2 [illegible] A

From Prob. 3.7

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 13.62 \angle -46.54^\circ \\ 1.391 \angle -129.18^\circ \end{bmatrix} = \begin{bmatrix} 13.86 \angle -52.24^\circ \\ 12.35 \angle 195.9^\circ \\ 14.75 \angle 76.74^\circ \end{bmatrix}$$

3.13 ~~Section 3.4~~

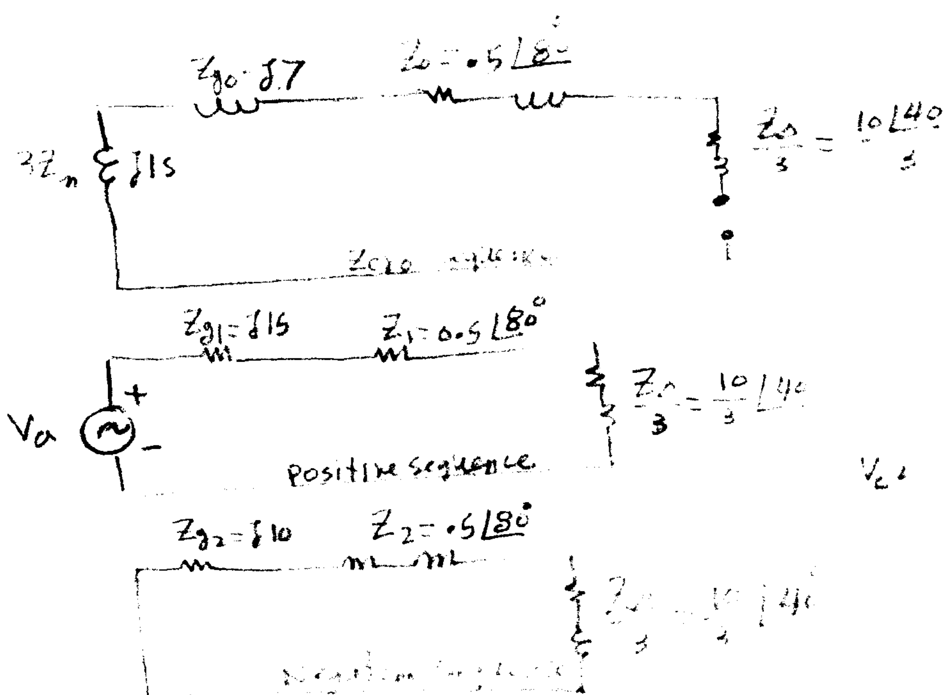
A balanced-Y-connected generator with terminal voltage $V_{bc} = 200\angle 0^\circ$ volts is connected to a balanced-Δ load whose impedance is $10\angle 40^\circ$ ohms per phase. The line impedance between the source and load is $0.5\angle 80^\circ$ ohm for each phase. The generator neutral is grounded through an impedance of $j5$ ohms. The generator sequence impedances are given by $Z_{g0} = j7$, $Z_{g1} = j15$, and $Z_{g2} = j10$ ohms. Draw the sequence networks for this system and determine the sequence components of the line currents.

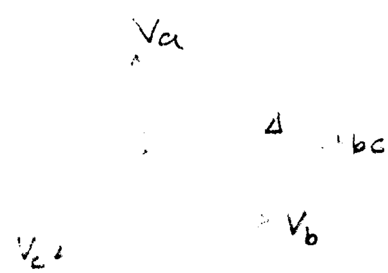


$Z_\Delta = 10\angle 40^\circ$

Assume $Z_{line} = Z_0 = Z_1 = Z_2 = .5\angle 80^\circ$

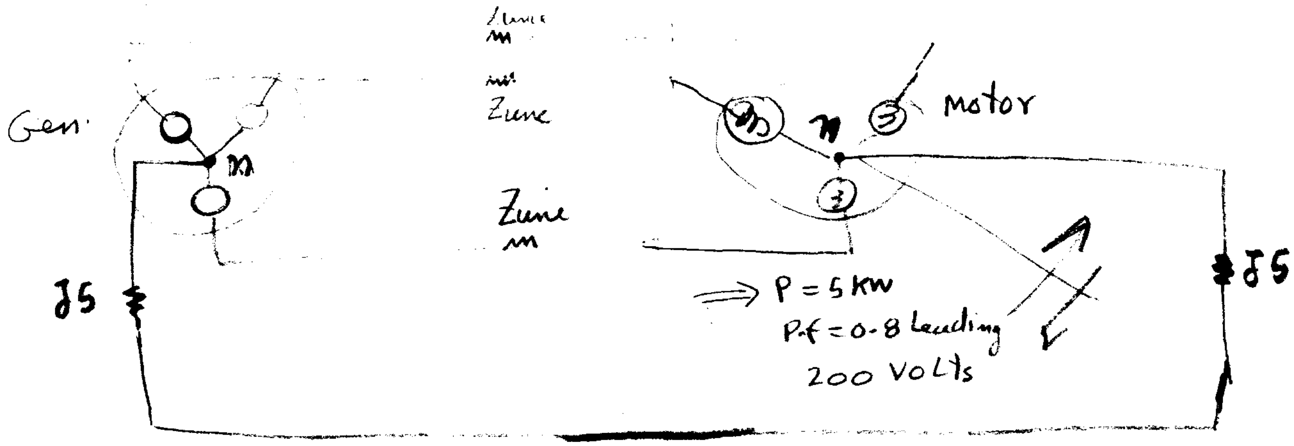
$Z_{g0} = j7$, $Z_{g1} = j15$, $Z_{g2} = j10$ Ω





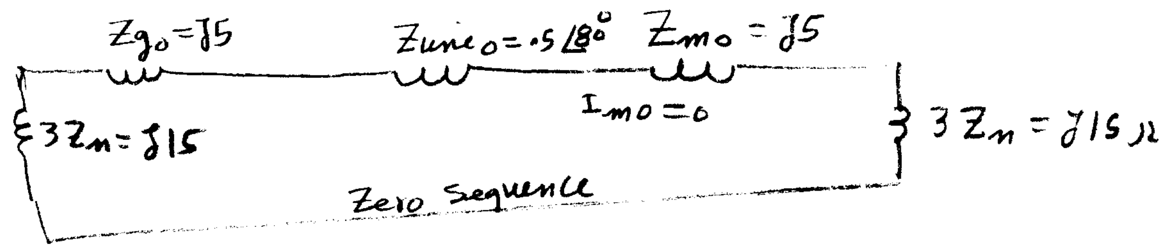
$$I_{line} = I_{line,seq} \qquad I_{seq} = \frac{V_a}{Z_{line} + \frac{Z_\Delta}{3}} = \frac{\frac{200}{\sqrt{3}}\angle 90^\circ}{.5\angle 80^\circ + \frac{10}{3}\angle 40^\circ} = 30.96\angle 45.05^\circ$$

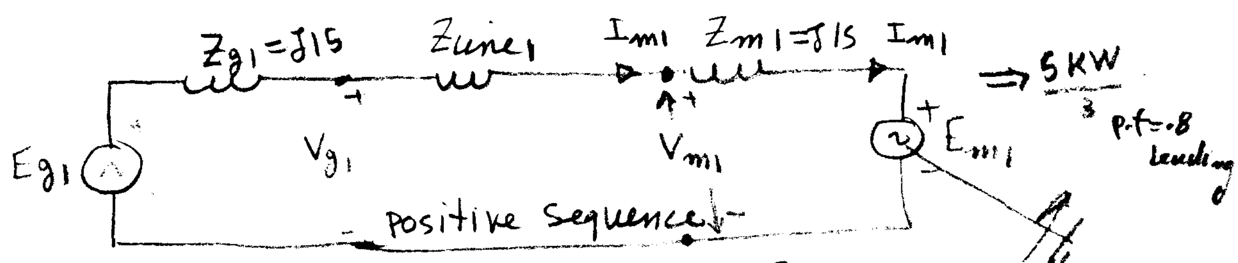
3.14 In a three-phase system, a synchronous generator supplies power to a 200-volt synchronous motor through a line having an impedance of $0.5\angle 80°$ ohm per phase. The motor draws 5 kW at 0.8 p.f. leading and at rated voltage. The neutrals of both the generator and motor are grounded through impedances of $j5$ ohms. The sequence impedances of both machines are $Z_0 = j5$, $Z_1 = j15$, and $Z_2 = j10$ ohms. Draw the sequence networks for this system and find the line-to-line voltage at the generator terminals. Assume balanced three-phase operation.

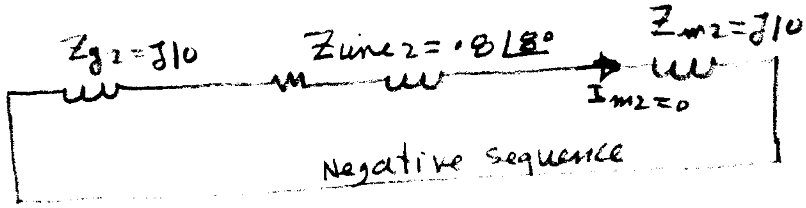


Assume $Z_{line} = Z_0 = Z_1 = Z_2$

Solution







$$\tilde{V}_{gan} = \tilde{V}_{g1} = \tilde{V}_{m1} + \hat{Z}_{line\,1}\tilde{I}_{m1} \qquad I_{m1} = \frac{5000\angle \cos^{-1}.8}{200\sqrt{3}(.8)} = 18.04\angle 36.8°$$

$$\bar{V}_{gan} = \frac{200}{\sqrt{3}}\angle 0 + (.5\angle 80)(18.04\angle 36.87) = 111.7\angle 4.13°$$

$$V_g = \sqrt{3}\,(111.7) = 193.5\ \text{V} \quad (\text{line-to-line})$$

Solution
Problem set #4

1

(4.12) Consider the three single-phase two-winding transformers shown in Figure 4.32. The high-voltage windings are connected in Y. (a) For the low-voltage side, connect the windings in Δ, place the polarity marks, and label the terminals a, b, and c in accordance with the American standard. (b) Relabel the terminals a', b', and c' such that V_{AN} is 90° out of phase with $V_{a'n}$ for positive sequence.

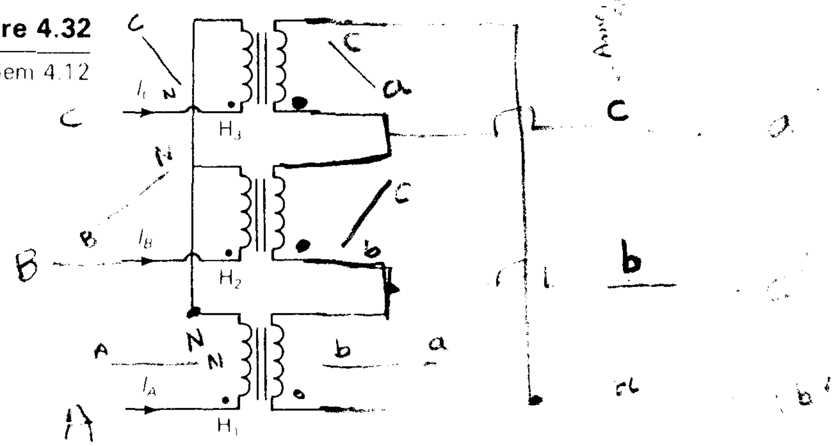
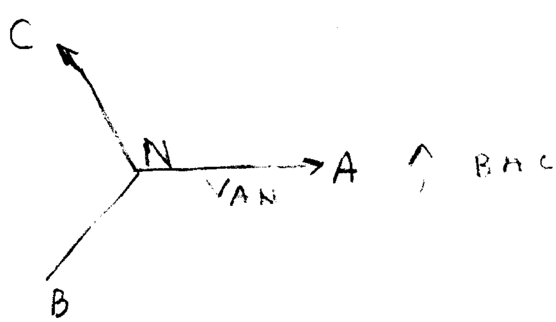
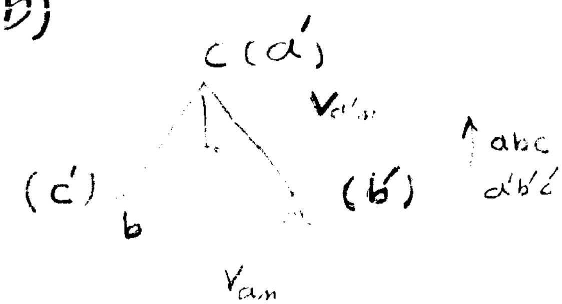


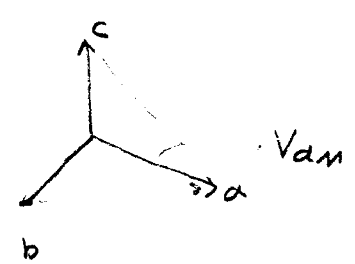
Figure 4.32
Problem 4.12

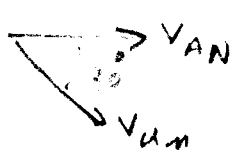
a)



b)







V_{AN} lead V_{an} by 30°

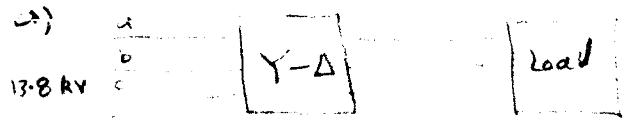
high-voltage ... place the polarity marks, ... terminals a', b', and ...

Problemset #4

2

4.13 Consider a bank of three single-phase two-winding transformers whose high-voltage terminals are connected to a three-phase, 13.8-kV feeder. The low-voltage terminals are connected to a three-phase substation load rated 2.1 MVA and 2.3 kV. Determine the required voltage, current, and MVA ratings of both windings of each transformer, when the high-voltage/low-voltage windings are connected (a) Y–Δ, (b) Δ–Y, (c) Y–Y, and (d) Δ–Δ.

a)



3-ϕ (13.8/2.3 kV, 2.1 MVA

2.1 MVA, $V_{L-L} = 2.3$ kV

Single-phase voltage $= \frac{2.1}{3} = 0.7$ MVA

Single-phase voltage rating $\frac{13.8}{\sqrt{3}}/2.3$ kV

b) Δ Y Three phase; 13.8 Δ / 2.3 Y kV rating 2.1 MVA

Single-phase rating; $\frac{2.1}{3} = 0.7$ MVA

Single-phase voltage rating $13.8\text{ kV}/\frac{2.3}{\sqrt{3}}$ or $13.8/1.33$ kV

c) Y – Y Three-phase rating 2.1 MVA 13.8 kV Y/2.3 kV Y

Single-phase rating 0.7 MVA 7.97/1.33 kV

d) Δ Δ Three-phase rating 2.1 MVA

13.8 kV Δ / 2.3 kV Δ

Single-phase rating: 0.7 MVA 13.8/2.3 kV

Problem set # 4

3

4.17 Consider the single-line diagram of the power system shown in Figure 4.33. Equipment ratings are:

generator 1:	1000 MVA, 18 kV, X″ = 0.2 per unit
generator 2:	1000 MVA, 18 kV, X″ = 0.2
synchronous motor 3:	1500 MVA, 20 kV, X″ = 0.2
3-phase Δ-Y transformers T_1, T_2, T_3, T_4:	1000 MVA, 500 kV Y/20 kV Δ, X = 0.1
3-phase Y-Y transformer T_5:	1500 MVA, 500 kV Y/20 kV Y, X = 0.1

Neglecting resistance, transformer phase shift, and magnetizing reactance, draw the positive-sequence reactance diagram. Use a base of 1000 MVA and 500 kV for the 50-ohm line. Determine the per-unit reactances.

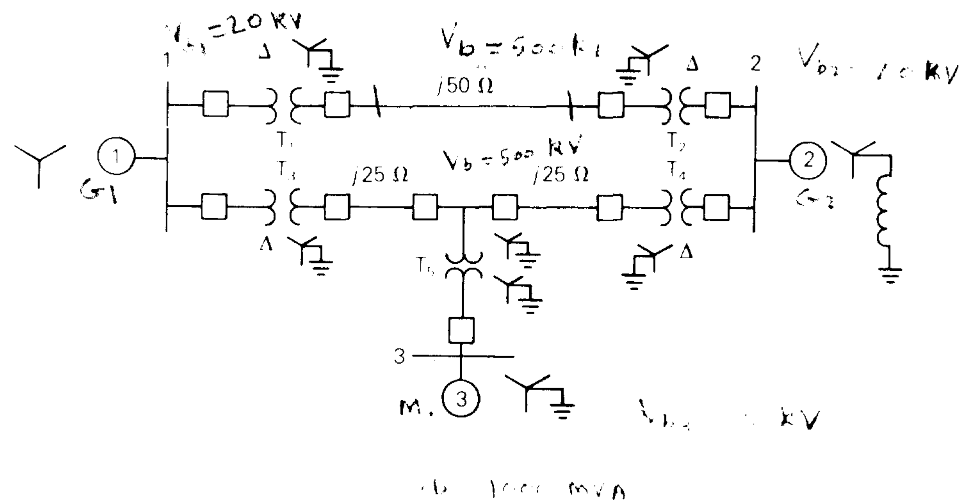


Figure 4.33
Problems 4.17, 4.18, 4.19

$V_{b1} = \frac{20}{500}(500) = 20$ kV $\qquad V_{b2} = \frac{20}{500}(500) = 20$ kV

$X''_{g1} = .2\left(\frac{18}{20}\right)^2\left(\frac{1000}{1000}\right) = .162$ p.u $\qquad X''_{g2} = X''_{g1} = 0.162$ p.u

$X''_{m3} = 0.2\left(\frac{20}{20}\right)^2\left(\frac{1000}{1500}\right) = 0.1333$ p.u

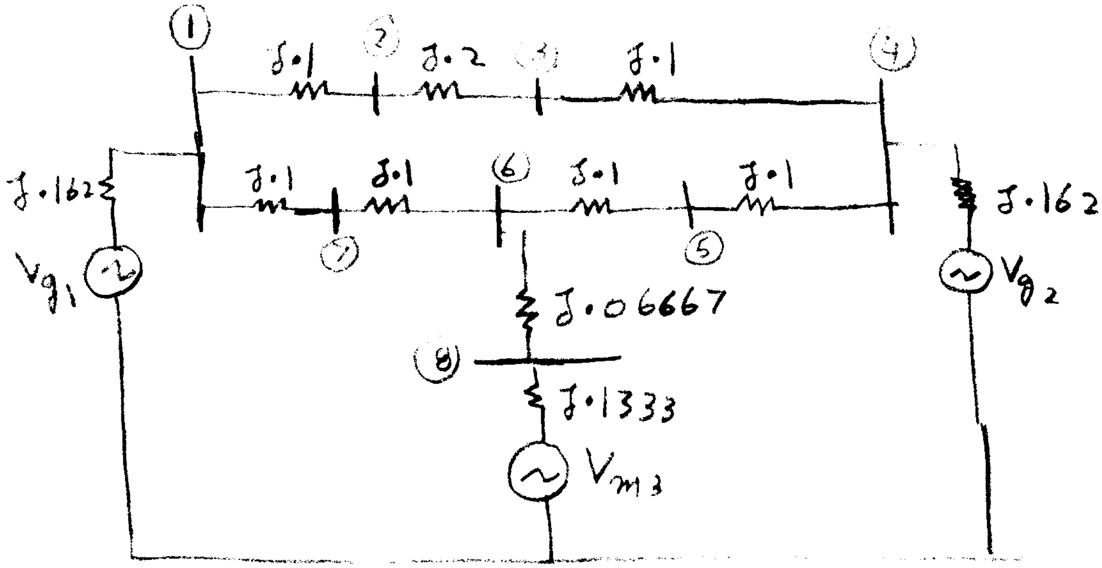
$X_{T_1} = X_{T_2} = X_{T_3} = X_{T_4} = 0.10$ p.u

$X_{T_5} = 0.1\left(\frac{20}{20}\right)^2\left(\frac{1000}{1500}\right) = 0.0667$ p.u

$Z_{base_{500}} = \frac{(500)^2}{1000} = 250.\ \Omega$ $\qquad X_{line\,50} = \frac{50}{250} = 0.20$ p.u

$X_{line\,25} = \frac{25}{250} = 0.10$ p.u

4.17 cont.


1
2
3
4
j.1
j.2
j.1
j.162
j.1
j.1
6
j.1
j.1
j.162
7
5
V_{g_1}
V_{g_2}
j.06667
8
j.1333
V_{m_3}

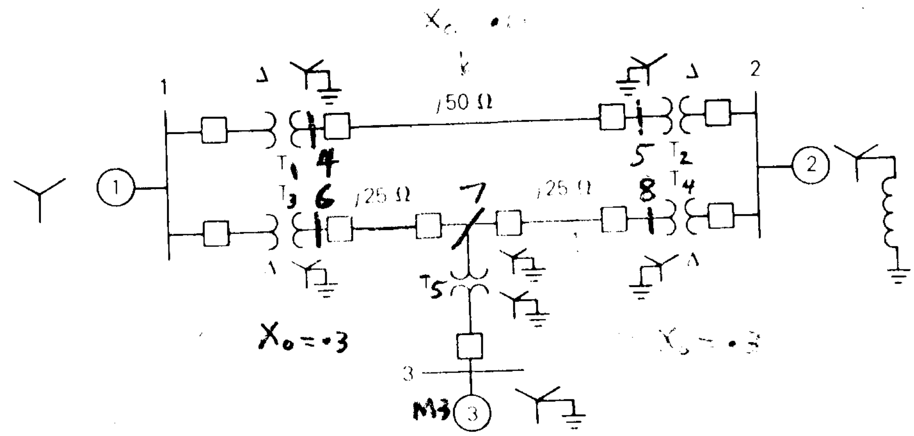
Set #4

5

4.19 Draw the zero-sequence reactance diagram for the power system shown in Figure 4.33. The zero-sequence reactance of each generator and of the synchronous motor is 0.05 per unit based on equipment ratings. Generator 2 is grounded through a neutral reactor of 0.06 per unit on a 100-MVA, 18-kV base. The zero-sequence reactance of each transmission line is assumed to be three times its positive-sequence reactance. Use the same base as in Problem 4.17.

Figure 4.33

Problems 4.17, 4.18, 4.19



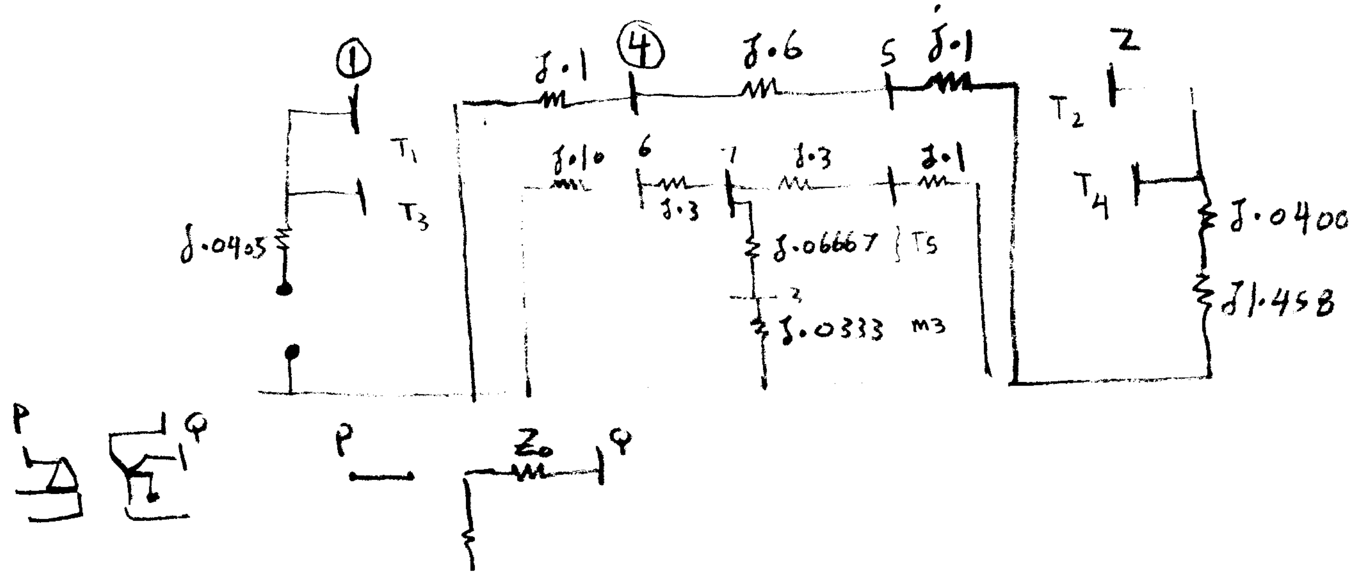
S_b ... MVA

$X_{T1\,0} = X_{T2\,0} = X_{T3\,0} = X_{T4\,0} = 0.1$ $\qquad X_{T5\,0} = .06667$

$X_{g1-0} = (.05)\left(\frac{18}{20}\right)^2 = 0.0405$ p.u $= X_{g2-0}$

$X_{m3-0} = (.05)\left(\frac{1000}{1500}\right) = 0.0333$ p.u

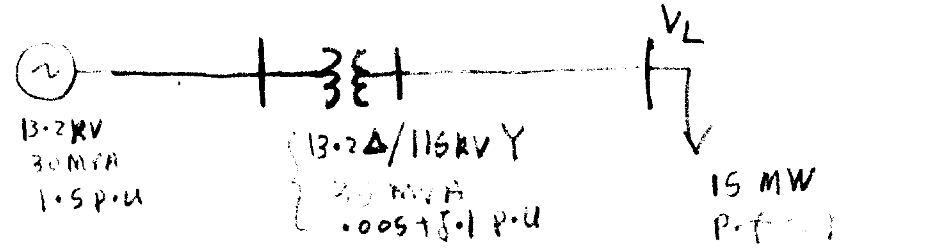
$X_{n2} = (.06)\left(\frac{18}{20}\right)\left(\frac{1000}{100}\right) = .486$ p.u $\qquad 3X_{n2} = 1.458$ p.u



Problem set #4

1.

4.21 A 30-MVA, 13.2-kV three-phase generator, which has a positive-sequence reactance of 1.5 per unit on the generator base is connected to a 35-MVA, 13.2Δ/115Y-kV step-up transformer with a series impedance of $(0.005 + j0.1)$ per unit on its own base. (a) Calculate the per-unit generator reactance on the transformer base. (b) The load at the transformer terminals is 15 MW at unity power factor and at 115 kV. Choosing the transformer high-side voltage as the reference phasor, draw a phasor diagram for this condition. (c) For the condition of part (b), find the transformer low-side voltage and the generator internal voltage behind its reactance. Also compute the generator output power and power factor.

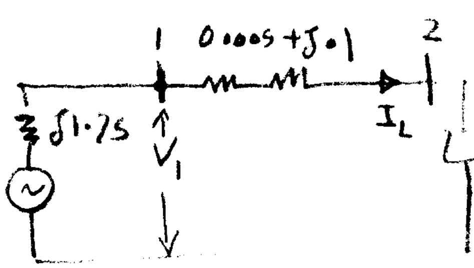


$V_L = 115$ KV

a) $S_{b_{new}} = 35$ MVA

$$X_{T_{new}} = 1.5\left(\frac{13.2}{13.2}\right)^2\left(\frac{35}{30}\right) = 1.75 \text{ p.u}$$

p.u Eq. CKT

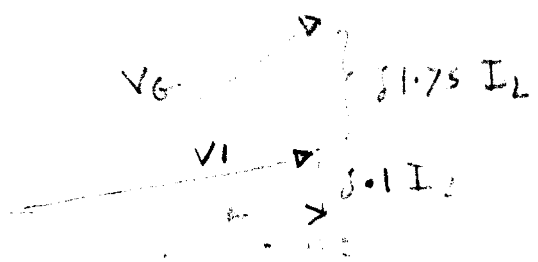


$V_2 = 1\angle 0^\circ$ p.u

$S_{p.u\ load} = \frac{15}{35} = .4286\angle 0^\circ$

V_1 : Generator terminal voltage

$$I_L = \left(\frac{S}{V_2}\right)^* = \frac{.4286\angle 0^\circ}{1\angle 0} = .4286\angle 0^\circ$$



4.21 cont.

$$V_1 = I_L (.005 + j.1) I_L + V_2$$

$$V_1 = 0.42865 (.005 + j.1) + 1\angle 0 = 1.0021 \angle .12^\circ$$

$$V_G = I_L (j1.75 + .005 + j.1) + V_2$$

$$V_G = 1.793 \angle 0.067 \text{ p.u}$$

$$V_{1\,actual} = (1.0021)(13.2) = 13.23 \text{ kV}$$

$$V_G = (1.793)(13.2) = 23.67 \text{ kV}$$

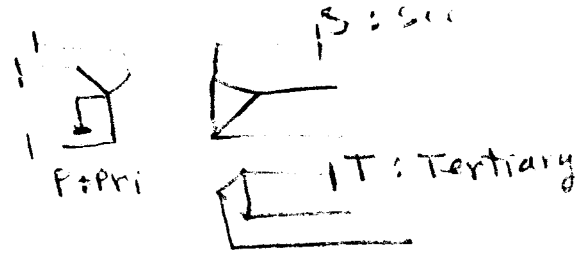
$$S_1 = (V_1 I_L^*) = 0.4295 \angle .12^\circ \text{ p.u}$$

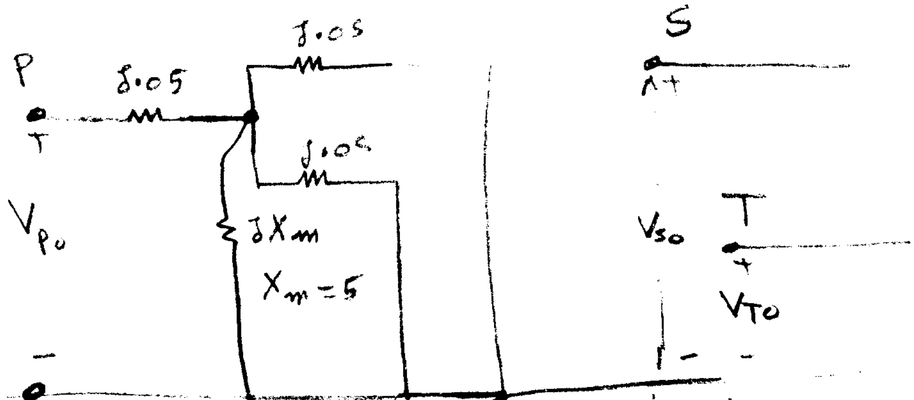
$$S_{1\,actual} = (S_1) * S_b = 15.03 \text{ MW} + j.03 \text{ MVars}$$

$$p.f = \cos(.12^\circ) = 0.999 \text{ lagging}$$

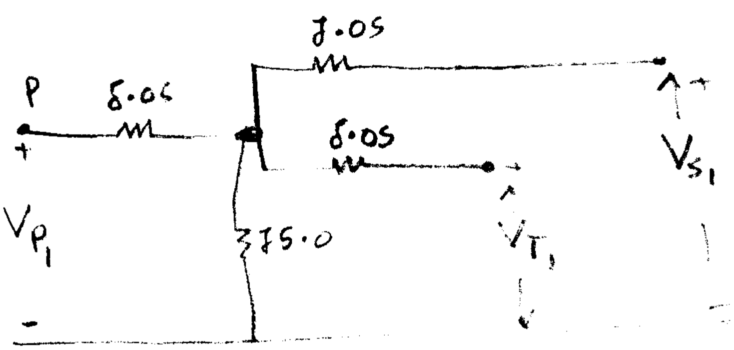
Section 4.6

4.22 A single-phase three-winding transformer has the following parameters: $Z_1 = Z_2 = Z_3 = 0 + j0.05$, $G_c = 0$, and $B_m = 0.2$ per unit. Three identical transformers, as described, are connected with their primaries in Y (solidly grounded neutral) and with their secondaries and tertiaries in Δ. Draw the per-unit sequence networks of this transformer bank.





P.U. Zero Sequence Network



Per Unit positive (or Negative) Sequence Network