A. Keyhani

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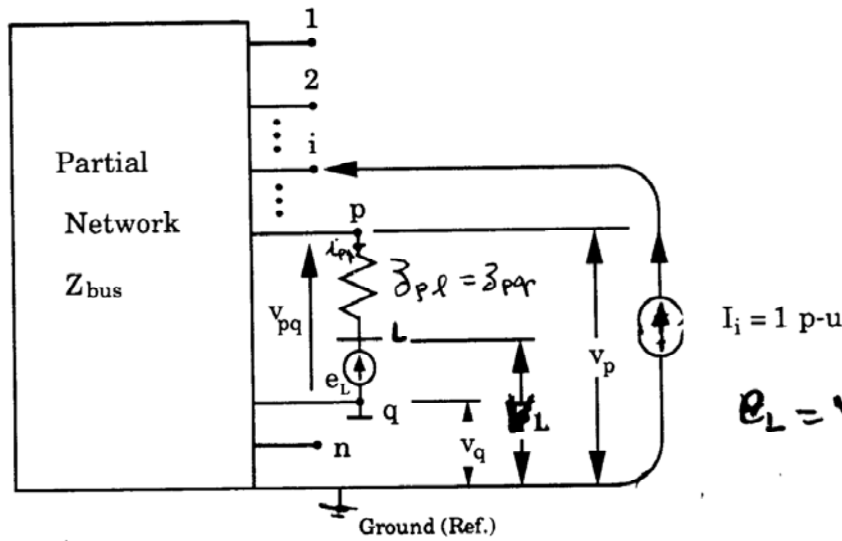
EE740
lecture 16.

Z_{bus} impedance matrix algorithm

Addition of a link from [redacted] existing bus "p" to the existing bus "q"



$e_L = V_L - V_q$

- step 1. • Add a radial line from bus "p" to [redacted] fictitious bus "ℓ". This bus will be eliminated later.
  - Place a voltage source e_L between bus "ℓ" and "q" such that $i_{pq} = 0$
  - Inject one amp at bus i ($I_i = 1$) and calculate the voltage at the ℓth node with respect to bus q.
- Step2. Eliminate bus "ℓ"

(B)

$$
\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_i \\ v_p \\ \vdots \\ v_q \\ v_n \\ \hline e_L \end{bmatrix}
=
\begin{array}{c|cccccc|c|}
 & 1 & i & p & & n & & \ell \\
1 & Z_{11} & Z_{1i} & Z_{1p} & \cdots & Z_{1n} & & Z_{1\ell} \\
 & Z_{21} & Z_{2i} & Z_{2p} & & Z_{2n} & & Z_{2\ell} \\
 & Z_{i1} & Z_{ii} & Z_{ip} & & Z_{in} & & Z_{i\ell} \\
p & Z_{p1} & Z_{pi} & Z_{pp} & & Z_{pn} & & Z_{p\ell} \\
 & \vdots & \vdots & \vdots & & \vdots & & \vdots \\
q & & Z_{qi} & & & & & \\
n & Z_{n1} & Z_{ni} & Z_{np} & & Z_{nn} & & Z_{n\ell} \\
\hline
\ell & Z_{\ell 1} & Z_{\ell i} & Z_{\ell p} & & Z_{\ell n} & & Z_{\ell\ell}
\end{array}
\begin{bmatrix} 0 \\ 0 \\ I_i = 1\text{ A} \\ 0 \\ \\ \\ 0 \\ \hline 0 \end{bmatrix}
$$

$v_1 = Z_{1i} I_i$

$v_2 = Z_{2i} I_i$

$\vdots$

$v_i = Z_{ii} I_i$

$\vdots$

$v_n = Z_{ni} I_i$

$e_L = Z_{\ell i} I_i$

$\Longrightarrow \quad v_K = Z_{Ki} I_i \quad K = 1, \ldots p, \ldots q, n$

$e_L = Z_{\ell i} I_i$

note

$$e_L = v_{pq} - v_{p\ell} = v_p - v_q - v_{p\ell}$$

since e_L is selected s.t $i_{pq} = 0$, then the element p to ℓ can be treated as a radial line

(c)

$$i_{pl} = i_{pq} = 0 = y_{pl}\, v_{pl}$$

$$\therefore\ y_{pl} = \frac{1}{z_{pq}} \neq 0 \qquad \therefore\ v_{pl} = 0$$

$$e_L = v_p - v_q$$

$$e_L = Z_{li}\, I_i \qquad \text{since } I_i = 1 \qquad Z_{li} = e_L$$

$$v_p = Z_{pi}\, I_i \qquad\qquad Z_{pi} = v_p$$

$$v_q = Z_{qi}\, I_i \qquad\qquad Z_{qi} = v_q$$

$$Z_{li} = Z_{pi} - Z_{qi} \qquad i = 1, 2 \ldots\ n+1,\ i \neq l$$

The element Z_{LL} can be calculated by injecting a current at the lth bus with the bus q as reference and calculating the voltage at the lth bus with respect to bus q, $I_l = 1$ $I_i = 0$ $i = 1, \cdots n$

$$\begin{bmatrix} v_1 \\ \vdots \\ \vdots \\ \vdots \\ v_n \\ \hdashline e_L \end{bmatrix} = \begin{array}{c} \\ \\ p \\ q \\ \\ l \end{array}\!\!\begin{array}{c} \begin{array}{ccc} & p & \end{array} \\ \begin{bmatrix} Z_{11} & & Z_{1l} \\ & & Z_{2l} \\ & Z_{pp} & \\ & Z_{qp} & \vdots \\ & & Z_{nl} \\ Z_{l1} & & Z_{ll} \end{bmatrix} \end{array} \begin{bmatrix} 0 \\ \\ \\ \\ \\ \hdashline I_l = 1 \end{bmatrix}$$

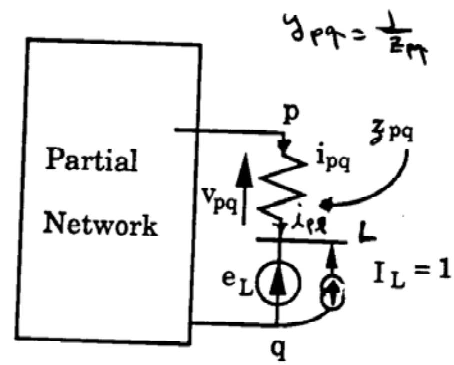
$$\begin{aligned} v_1 &= Z_{1l}\, I_L \\ &\vdots \\ v_n &= Z_{nl}\, I_L \\ e_L &= Z_{LL}\, I_L \end{aligned} \quad \Rightarrow \quad \begin{cases} v_K = Z_{Kl}\, I_L & K = 1, \ldots\ n \\ e_L = Z_{LL}\, I_L \end{cases}$$

The current in the element p–ℓ is

$$i_{p\ell} = -I_\ell = -1$$

$$i_{p\ell} = y_{pq} v_{p\ell} = -1$$

$$v_{p\ell} = -\frac{1}{y_{pq}} = -z_{pq}$$



Recall

$$e_L = v_p - v_q - v_{p\ell}$$

$$\downarrow$$

$$e_L = Z_{LL} \cancel{I_L}^{1}$$

$$v_p = Z_{p\ell} \cancel{I_L}^{1}$$

$$v_q = Z_{q\ell} \cancel{I_L}^{1}$$

$$v_{p\ell} = -z_{pq}$$

$$\therefore \quad \boxed{Z_{LL} = Z_{p\ell} - Z_{q\ell} + z_{pq}}$$

Note $\boxed{Z_{\ell i} = Z_{pi} - Z_{qi}}$

$\therefore \quad Z_{p\ell} = Z_{pp} - Z_{qp} \qquad Z_{p\ell} - Z_{q\ell} = Z_{pp} - Z_{qp} - Z_{pq} + Z_{qq}$

$\quad\;\; Z_{q\ell} = Z_{pq} - Z_{qq} \qquad\qquad\qquad\quad = Z_{pp} + Z_{qq} - 2Z_{pq}$

$$\boxed{\therefore \quad Z_{LL} = Z_{pp} + Z_{qq} - 2Z_{pq} + z_{pq}}$$

E

Now, eliminate bus "l"

$$\begin{bmatrix} V_1 \\ \\ v_n \\ e_L=0 \end{bmatrix} = \begin{array}{c} \\ 1 \\ \\ n \\ l \end{array} \underbrace{\begin{array}{c} \begin{array}{cc} 1 \quad\quad n & l \end{array} \\ \begin{bmatrix} Z_{nn} & Z_{nl} \\ & \\ Z_{ln} & Z_{ll} \end{bmatrix} \end{array}}_{Z_{\text{original bus}}} \begin{bmatrix} I_1 \\ \\ I_n \\ I_L \end{bmatrix}$$

$(n+1)\times(n+1)$

$$\underset{n\times n}{Z_{\text{Bus (without } l)}} = \underset{n\times n}{Z_{nn}} - \underset{nl}{Z_{nl}}\, \underset{1\times 1}{Z_{ll}^{-1}}\, \underset{1\times n}{Z_{ln}}$$

F

Exampl of bus elemenation using Z_{BUS}

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \left[\begin{array}{cc|c} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ \hline Z_{31} & Z_{32} & Z_{33} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Elemenate bus 3, i.e $V_3 = 0$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3$$

$$0 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3$$

$$I_3 = \frac{-(Z_{31} I_1 + Z_{32} I_2)}{Z_{33}}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 - \frac{Z_{13}}{Z_{33}} (Z_{31} I_1 + Z_{32} I_2)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 - \frac{Z_{23}}{Z_{33}} (Z_{31} I_1 + Z_{32} I_2)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} - \frac{Z_{13} Z_{31}}{Z_{33}} & Z_{12} - \frac{Z_{13} Z_{32}}{Z_{33}} \\ Z_{21} - \frac{Z_{23} Z_{31}}{Z_{33}} & Z_{22} - \frac{Z_{23} Z_{32}}{Z_{33}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{new} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} - \underset{2 \times 1}{\begin{bmatrix} Z_{13} \\ Z_{23} \end{bmatrix}} \underset{1 \times 1}{[Z_{33}]^{-1}} \underset{1 \times 2}{[Z_{31} \quad Z_{32}]}$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} - \begin{bmatrix} \frac{Z_{13}}{Z_{33}} \\ \frac{Z_{23}}{Z_{33}} \end{bmatrix} [Z_{31} \quad Z_{32}]$$

$$Z_{new} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} - \begin{bmatrix} \frac{Z_{13} Z_{31}}{Z_{33}} & \frac{Z_{13} Z_{32}}{Z_{33}} \\ \frac{Z_{23} Z_{31}}{Z_{33}} & \frac{Z_{23} Z_{32}}{Z_{33}} \end{bmatrix}$$

$$= \begin{bmatrix} Z_{11} - \frac{Z_{13} Z_{31}}{Z_{33}} & Z_{12} - \frac{Z_{13} Z_{32}}{Z_{33}} \\ Z_{21} - \frac{Z_{23} Z_{31}}{Z_{33}} & Z_{22} - \frac{Z_{23} Z_{32}}{Z_{33}} \end{bmatrix}$$