Z-BUS Lecture #2

EE 740

Z_{bus} Lecture #17

Summary Z_{BUS} impedance matrix building Algorithms

1. Addition of a branch (radial line) from bus "p" to bus "q"

Z_{BUS} old $n \times n$

$Z_{qi} = Z_{pi}$ $\quad$ q: Added bus

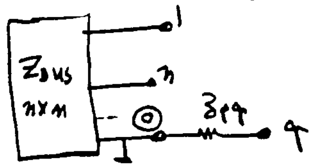
$i = 1, 2, \ldots n+1$ $\quad$ p: existing bus "p" (but not the ref. bus)

$i \neq q$

$Z_{qq} = Z_{pq} + z_{pq}$ or $Z_{qq} = Z_{pp} + z_{pq}$

Z_{BUS} new $(n+1) \times (n+1)$

2. Addition of a branch from ref. bus (i.e ground bus "0") to bus "q"



$Z_{qi} = 0$

$Z_{pq} = z_{pq}$

Z_{BUS} new $(n+1) \times n+1$

3. Addition of a branch from bus "q" to reference bus "0"

$Z_{qq} = z_{pq}$

4. Addition of a link (loop) from bus "q" to ref. bus "0"

step 1. Add a link from bus "q" to fictitious bus "ℓ"

$Z_{BUS \text{ with } \ell} \longrightarrow (n+1) \times (n+1)$

$Z_{\ell i} = - Z_{qi}$ $\quad i = 1, 2 \ldots n+1 \quad i \neq \ell$

step 2. $Z_{\ell\ell} = - Z_{q\ell} + z_{pq}$

Eliminate bus "ℓ"

$$Z_{BUS \text{ without } \ell} = Z_{nn} - Z_{n\ell} Z_{\ell\ell}^{-1} Z_{\ell n}$$

5. Addition of a link from bus "p" to bus "q".

Step 1. Add a link from bus "p" to bus "l"

$$Z_{li} = Z_{pi} - Z_{qi} \qquad i = 1, 2, \dots n+1 \quad i \neq l$$

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq}$$

or

$$Z_{ll} = Z_{pp} + Z_{qq} - 2 Z_{pq} + z_{pq}$$

Step 2. Eliminate the bus "l"

$$\mathbf{Z}_{BUS_{with\,l}} = \left[\begin{array}{c|c} \mathbf{Z}_{nn} & Z_{nl} \\ \hline Z_{ln} & Z_{ll} \end{array}\right]$$

$$Z_{Bus\ without_l} = \underbrace{\mathbf{Z}_{nn}}_{n \times n} - \underbrace{\mathbf{Z}_{nl}}_{n \times 1} \underbrace{\mathbf{Z}_{ll}^{-1}}_{1 \times 1} \underbrace{\mathbf{Z}_{ln}}_{1 \times n}$$

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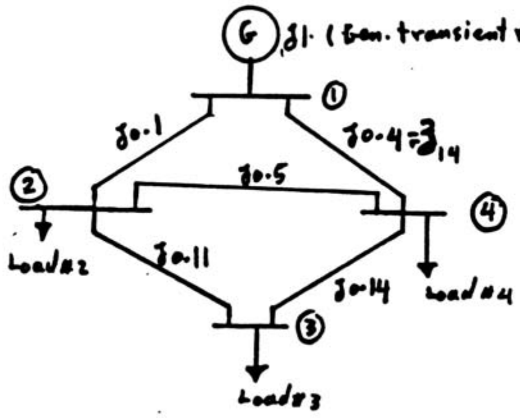
Power system modeling

1.8. Example.1 Z_{BUS} Impedance matrix model using Z_{BUS} Algorithm.

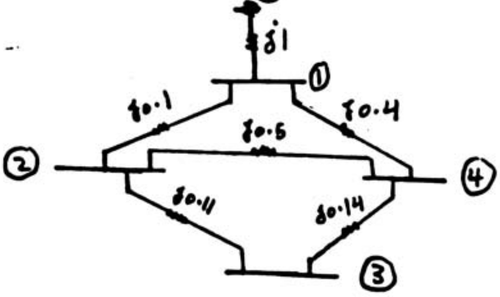
Consider the single-line diagram of a balanced system.



comments:

- Z_{BUS} matrix models are generally used for shor circuit studies. For short circuit studies:
  - Loads are either neglected or the constant Load impedance models are used.
  - The generators are represented by their Transient or sub-Transient reactances
  - Line charging capacitances could be represent but they are omitted.

Z_{BUS} model for short circuit study is:



Power system modeling

8. Example 1. Z_{Bus}. Cont.

- Radial line from bus ⓪ to bus ① (operator j is omitted)

$$Z_{bus} = \begin{array}{c|c} & 1 \\ \hline 1 & [\,1\,] \end{array}$$

- Radial line from bus ② to bus ①

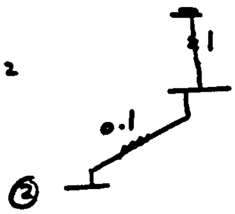
$$Z_{Bus} = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1 & 1 \\ 2 & 1. & 1.1 \end{array}$$

$Z_{qi} = Z_{pi}$ $i = 1, 2$

$p = 1$ $q = 2$

$Z_{21} = Z_{11} = 1$

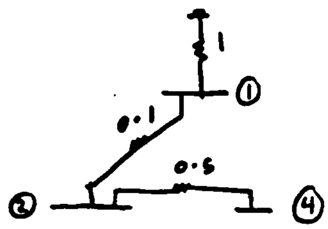
$Z_{qq} = Z_{pq} + z_{pq,pq} = 1. + .1 = 1.1$



- Radial line from bus ② to bus ④

$$Z_{Bus} = \begin{array}{c|ccc} & 1 & 2 & 4 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 1 & 1.1 & 1.1 \\ 4 & 1 & 1.1 & 1.6 \end{array}$$

$1.1 + 0.5 = 1.6$



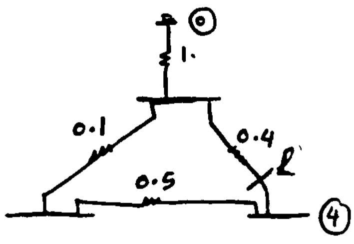
- Link from bus ④ to bus ①

$$Z_{Bus} = \begin{array}{c|cccc} & 1 & 2 & 4 & L \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1.1 & 1.1 & -0.1 \\ 4 & 1 & 1.1 & 1.6 & -0.6 \\ L & 0 & -0.1 & -0.6 & 1 \end{array}$$

$\rightarrow 1 - 1 = 0$

$\rightarrow 1 - 1.1 = -0.1$

$\rightarrow 1 - 1.6 = -0.6$



$Z_{LL} = Z_{pp} + Z_{qq} - 2Z_{pq} + z_{pq,pq}$

$Z_{LL} = 1 + 1.6 - 2(1.) + 0.4 = 1$

$Z_{li} = Z_{pi} - Z_{qi}$ $p = 1$, $q = 4$, $i = 1, 2, 4$

$Z_{l1} = Z_{p1} - Z_{41} = 1 - 1 = 0$

$Z_{l2} = Z_{12} - Z_{42} = 1 - 1.1 = -0.1$

$Z_{l4} = Z_{14} - Z_{44} = 1 - 1.6 = -0.6$

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Power system Modeling

8. Example 1. Z_{Bus} Cont.

- Eliminate bus L

$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ \ell \end{array} \!\!\begin{array}{c} \overbrace{\begin{array}{ccc} 1 & 2 & 4 \end{array}}^{n} \;\; \overset{\ell}{\ell} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 1.1 & 1.1 & -0.1 \\ 1 & 1.1 & 1.6 & -0.6 \\ 0 & -0.1 & -0.6 & 1 \end{array}\right] \end{array} \Big\} n \qquad Z_{Bus(without\ \ell)} = Z_{nn} - Z_{n\ell} Z_{\ell\ell}^{-1} Z_{\ell n}$$

$$Z_{Bus(without\ \ell)} = \underset{n\times n}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.1 & 1.1 \\ 1 & 1.1 & 1.6 \end{bmatrix}} - \underset{n\times\ell}{\begin{bmatrix} 0 \\ -0.1 \\ -0.6 \end{bmatrix}} [1] \underset{\ell\times n}{[0 \;\; -0.1 \;\; -0.6]}$$

$$\underset{(without\ \ell)}{Z_{Bus}} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.09 & 1.04 \\ 1 & 1.04 & 1.24 \end{bmatrix} \end{array}$$

- Radial line from bus 3 to bus 2

$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 3 \end{array} \!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 4 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1.09 & 1.04 & 1.09 \\ 1 & 1.04 & 1.24 & 1.04 \\ 1 & 1.09 & 1.04 & 1.2 \end{bmatrix} \end{array}$$

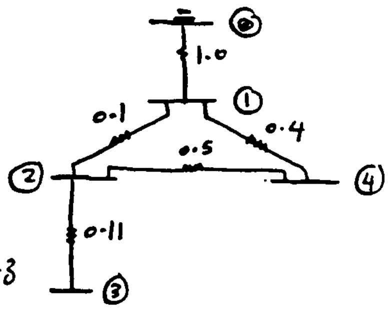
$p = 2$

$q = 3$

$Z_{qq} = Z_{pq} + z = Z_{pp} + z$

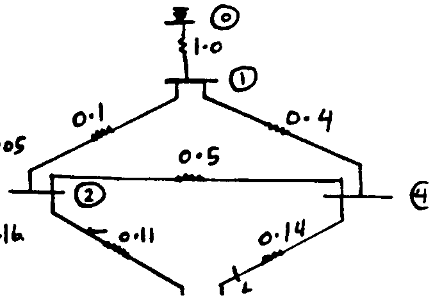
$Z_{33} = 1.09 + 0.11$

$Z_{33} = 1.20$



- Link from bus 3 to bus 4

$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 4 \\ 3 \\ L \end{array} \!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 4 & 3 & L \end{array} \\ \left[\begin{array}{ccccc} 1. & 1. & 1. & 1. & 0 \\ 1. & 1.09 & 1.04 & 1.09 & -0.05 \\ 1. & 1.04 & 1.24 & 1.04 & 0.2 \\ 1. & 1.09 & 1.04 & 1.2 & -0.16 \\ 0 & -0.05 & 0.2 & 0.04 & 0.5 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \rightarrow 1.09 - 1.09 = -0.05 \\ \rightarrow 1.24 - 1.04 = 0.2 \\ \rightarrow 1.04 - 1.2 = -0.16 \\ \\ \end{array}$$



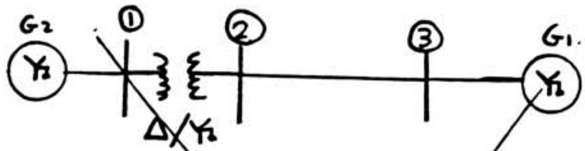
Power System Modeling

Example 1. Z_{BUS} Cont.

- Eliminate the bus 'L'

$$Z_{BUS} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1.085 & 1.06 & 1.07 \\ \text{symmetric} & & 1.16 & 1.104 \\ & & & 1.15 \end{bmatrix} \end{array}$$

Example # 2.

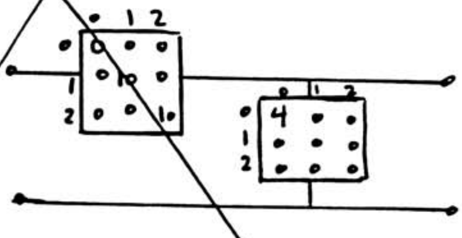


Given data:

For both generators. $$y^{012} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \end{array}$$

Transformer:

$y^0 = 4$
$y^1 = y^2 = 10$ $\Longrightarrow$



Transmission Line:

$$z^{abc} = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0.3 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \end{array} \qquad y^{abc} = \begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 4.5 & -1.5 & -.5 \\ -1.5 & 4 & -1 \\ -.5 & -1 & 3.6 \end{bmatrix} \end{array} = \left[z^{abc}\right]^{-1}$$

From this we find.

$$z^{012} = T_s^{-1} z^{abc} T_s = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \end{array}$$

$$y^{012} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \end{array}$$