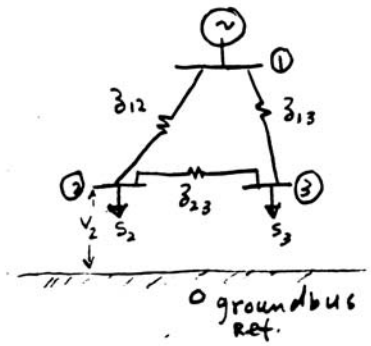
Z-BUS Lecture #1

EE 740
Lecture #15

A. Keyhani

(1)

Z - bus building Algorithm



- ALL voltages are measured respect to ground.
- replace the generator with its internal reactance
- replace the loads by their impedance model.
- Z_{BUS} model is used for short circuit studies.

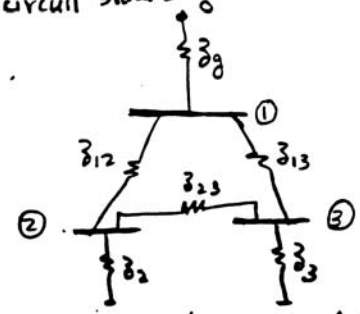
Recall

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

or

$$[Y_{BUS}][V_{BUS}] = [I_{BUS}]$$

Network model — Bus voltage — Injected bus current



Network model for short circuit studies

$$[V_{BUS}] = [Y_{BUS}]^{-1}[I_{BUS}] \quad \text{or} \quad [V_{BUS}] = [Z_{BUS}][I_{BUS}]$$

Bus voltage — Injected Bus current

$$[Z_{BUS}] = [Y_{BUS}]^{-1}$$

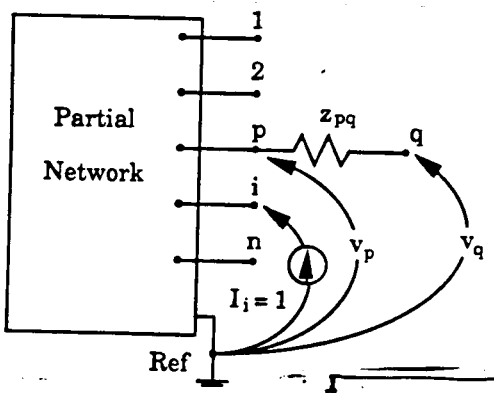
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

methods of finding Z_{BUS}

- Find the inverse of Y_{BUS} (can not be performed for very large network (20,000 buses).
- construct Z_{BUS} directly using Z_{BUS} building algorithm.

Z_{bus} algorithm

- addition of a radial line from bus p to bus q with impedance z_{pq}



p: existing bus

q: new bus

$$\begin{matrix} 1 \\ \\ \\ \\ n \\ q \end{matrix} \begin{bmatrix} V_1 \\ V_2 \\ \\ V_p \\ \\ V_n \\ V_q \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ \\ p \\ \\ n \\ q \end{matrix} \begin{bmatrix} 1 & 2 & & p & i & n & q \\ Z_{11} & Z_{12} & \cdots & Z_{1p} & Z_{1i} & Z_{1n} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & & \cdots & & Z_{2q} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & Z_{pi} & Z_{pn} & Z_{pq} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{ip} & Z_{ii} & Z_{in} & Z_{iq} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{np} & Z_{ni} & Z_{nn} & Z_{nq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & Z_{qi} & Z_{qn} & Z_{qq} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_i \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Note

- Z_{old} = Existing Partial Network = $\begin{matrix} & 1 & & n \\ 1 & \lceil & \cdots & \rceil \\ & \vdots & & \vdots \\ n & \lfloor & \cdots & \rfloor \end{matrix}$ $n \times n$
- $Z_{new} = \begin{matrix} & 1 & & n & q \\ 1 & \lceil & & & \rceil \\ & \vdots & & & \\ n & & & & \\ q & \lfloor & & & \rfloor \end{matrix}$ $(n+1) \times (n+1)$

- The new elements of Z_{bus} are

$Z_{q1}, Z_{q2}, Z_{qp}, Z_{qi}, \cdots Z_{qn}$ and Z_{qq}

- Inject one amp at bus i ALL others = 0

$$\begin{aligned} V_1 &= Z_{1i} I_i \\ V_2 &= Z_{2i} I_i \\ V_p &= Z_{pi} I_i \\ V_i &= Z_{ii} I_i \\ V_n &= Z_{ni} I_i \end{aligned} \qquad (1)$$

$$\boxed{V_q = Z_{qi} I_i}$$

$i = 1, 2, \cdots p, n$

Since $I_i = 1$. Amp., then

$Z_{1i} = V_1 \cdots Z_{pi} = V_p \cdots Z_{ni} = V_n \cdots Z_{qi} = V_q$

For the radial line from bus p to bus q.

$$V_{pq} = V_p - V_q \qquad (4)$$

④

$$I_{pq} = \frac{V_p - V_q}{Z_{pq}} = y_{pq}(V_p - V_q)$$

Recall $I_i = 1$ ALL others $= 0$ $\therefore I_{pq} = 0$

$$y_{pq}(V_p - V_q) = 0 \qquad \therefore V_p = V_q \qquad ⑤$$

Recall

$$Z_{pi} = V_p \quad , \quad Z_{qi} = V_q$$

$$\therefore \quad Z_{qi} = Z_{pi} \qquad i = 1, \ldots n \qquad i \neq q. \qquad ⑥$$

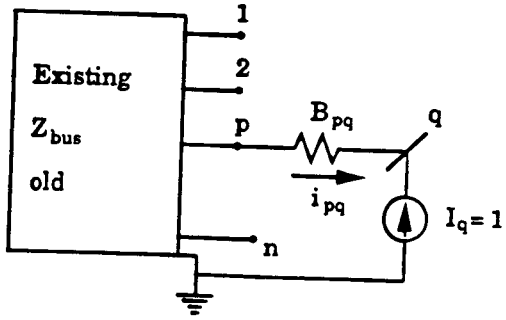
Recall q = new bus $\quad p$ = existing bus

$$Z_{q1} = Z_{p1}, \quad Z_{q2} = Z_{p2} \cdots Z_{qp} = Z_{pp}, \quad Z_{qn} = Z_{pn}$$

Z_{qq} ?

- Inject one amp in bus q
- ALL others $= 0$

$$\begin{matrix} 1 \\ p \\ n \\ q \end{matrix}\begin{bmatrix} & & & Z_{1q} \\ & Z_{old} & & Z_{pq} \\ & & & Z_{nq} \\ Z_{q1} & Z_{qp} & Z_{qn} & Z_{qq} \end{bmatrix}\begin{bmatrix} 0 \\ \vdots \\ 1_q \end{bmatrix} = \begin{bmatrix} V_1 \\ V_p \\ V_n \\ V_q \end{bmatrix}$$



$$Z_{1q} I_q = V_1$$
$$Z_{2q} I_q = V_2 \qquad (7)$$
$$\vdots$$
$$Z_{pq} I_q = V_p$$
$$Z_{qq} I_q = V_q$$

$$I_{pq} = -I_q = -1 \qquad (9)$$

$$I_{pq} = \frac{V_p - V_q}{Z_{pq}} = y_{pq}(V_{pq}) = -1$$

$$V_{pq} = -\frac{1}{y_{pq}} \qquad (10)$$

but

$$V_{pq} = V_p - V_q$$

or

$$V_q = V_p - V_{pq} \qquad (11)$$

But

$$Z_{qq} = V_q \quad \text{since } I_q = 1$$

$$Z_{pq} = V_p \quad \text{and } I = 1$$

$$\therefore \boxed{Z_{qq} = Z_{pq} + \frac{1}{y_{pq}} = Z_{pq} + Z_{pq}}$$

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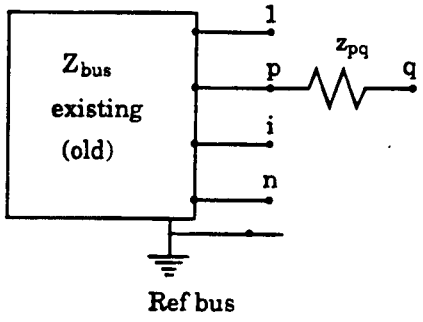
Summary of Z_{bus} impedance algorithm

• Addition of a (radial line) from bus "p" to bus "q".



- $Z_{bus_{old}}$ before addition of bus "q"
  n : number of buses $n \times n$

- $Z_{bus\ new}$ after addition of bus "q"
  number of buses : $n+1$ $(n+1)\times(n+1)$

$Z_{qi} = Z_{pi}$

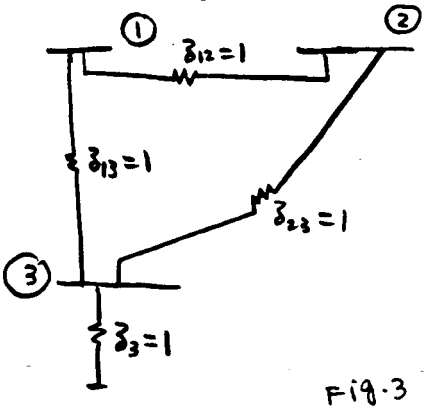
$i = 1, 2, \cdots n+1$ $i \neq q$

q : the new bus

p : existing bus (but not the ref. bus)

$Z_{qq} = Z_{pq} + z_{pq}$ or $Z_{qq} = Z_{pp} + z_{pq}$

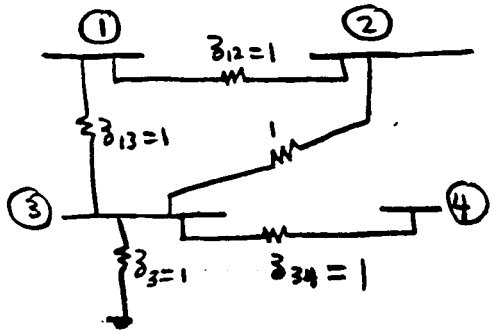
Ex.



Fig·3

$$Z_{BUS} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 5/3 & 4/3 & 1 \\ 4/3 & 5/3 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Add a radial line from bus ③ to a new bus designated as bus ④. $z_{34}=1\ \Omega$



number buses $=4$ $\qquad p=3 \qquad q=4$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 5/3 & 4/3 & 1 & 1 \\ 4/3 & 5/3 & 1 & 1 \\ 1 & 1 & \boxed{1} & \boxed{1} \\ 1 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

$Z_{qi} = Z_{pi} \quad i=1,2,3$ $\qquad Z_{41}=Z_{31} \qquad Z_{42}=Z_{32} \qquad Z_{43}=Z_{33}$

$Z_{qq} = Z_{pq} + z_{pq} \qquad Z_{44} = Z_{34} + z_{34} = 1+1 = 2$