Transmission Line Parameters Lecture #2

Ground Effects

(2)

III. R, L, and C PER UNIT LENGTH OF n PARALLEL CONDUCTORS WITH GROUND EFFECTS

Inductance Matrix when $\sum_{i=1}^{n} I_i = 0$ (i.e. No ground current):

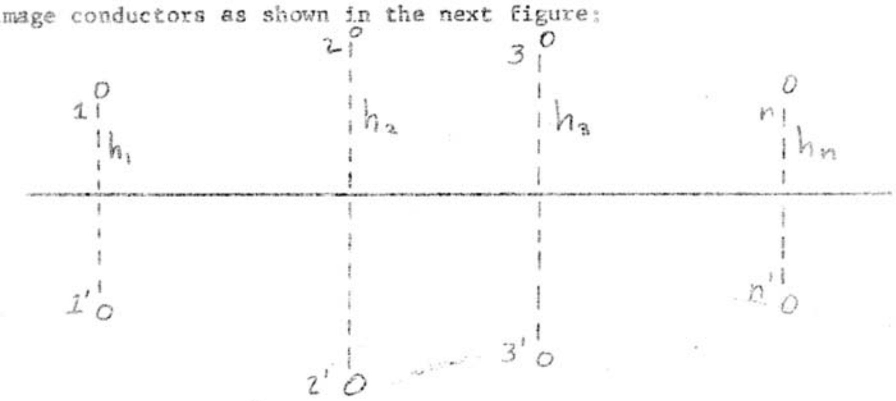
If the sum of the currents in all conductor measured in the same direction is zero, the earth will have little influence on impedances to conductor currents, and hence can be neglected, i.e. the self and mutual inductances can be computed as in Part II.

Note that this situation arises in either of the following cases:

1) Three-phase ungrounded transmission lines (delta circuits or ungrounded Y- circuits).
2) Grounded 3-phase transmission lines carrying balanced 3-phase currents.

Inductance Matrix Assuming Perfect Ground:

If the earth had infinite conductivity, its potential at all points would be the same, i.e. the ground plane will be an equipotential surface. In this case, inductance and electrostatic coefficient matrices can be computed by using the method of images, i.e. the earth would be replaced by image conductors as shown in the next figure:



Above figure represents n straight, parallel, non-magnetic, solid cylindrical wires in uniform non-magnetic medium. Also, assume that frequency is low enough for the current distribution to be uniform over the cross section of the conductor and that the flux beyond the ends of the wires to be negligible. Using the equation for flux linkage given in Part II for the total flux linking conductor j due to currents I_j flowing

②

in the n conductors and the image currents $-I_j$ in the image conductors (note that the ground current $= -\sum_{i=1}^{n} I_i$ = sum of image currents).

$$\psi_i = 2 \times 10^{-7} \left[\sum_{j=1}^{n} I_j \ln \frac{1}{D_{ij}} - \sum_{j=1}^{n} I_j \ln \frac{1}{D_{ij'}} \right]$$

$$= 2 \times 10^{-7} \left[\sum_{j=1}^{n} I_j \ln \frac{D_{ij'}}{D_{ij}} \right] \text{ weber-turns/met}$$

where:

D_{ii} = self G.M.D. of conductor i

D_{ij} = Distance between conductors i and j

$D_{ij'}$ = Distance between conductor i and the image of conductor j

$D_{ii'}$ = Distance between conductor i and its image

= $2h_i$ = twice the light of conductor i over ground.

Hence:

$$\psi_i = \sum_{j=1}^{n} L_{ij} I_j$$

where:

L_{ii} = self inductance of conductor i assuming perfect ground return.

$$= 2 \times 10^{-7} \ln \frac{2h_i}{r_i'} \text{ henrys/mt.}$$

$$= .7411 \log_{10} \frac{2h_i}{r_i'} \text{ mh/mile}$$

L_{ij} = Mutual inductance between conductors i and j assuming perfect ground return

$$= 2 \times 10^{-7} \ln \frac{D_{ij'}}{D_{ij}} \text{ henrys/mt}$$

$$= .7411 \log_{10} \frac{D_{ij'}}{D_{ij}} \text{ mh/mile}$$

Inductance Matrix of n Conductors with Earth Return (Carson's Formulas):

In 1926, Dr. John R. Carson* gave equations for the self-impedance of a conductor with earth return and the mutual impedance between two conductors with common earth return. The equations are based on an earth of uniform resistivity ρ and semi-infinite in extent.

Carson's formulas for self-inductance with earth return and mutual inductance with common earth return are:

$$L_{ii} = .7411 \log_{10} \frac{2h_i}{r_i'} + Q(k_{ii}) \quad mh/mile$$

$$L_{ij} = .7411 \log_{10} \frac{D_{ij'}}{D_{ij}} + Q(k_{ij})$$

Q is an infinite series of Bessel functions which can be approximated by:

$$Q(k) = K_1 + \log \frac{K_2}{k} + K_3 k \frac{h_i + h_j}{D_{ij'}} + \Delta Q$$

where K_1, K_2 and K_3 are constants and $k_{ii} = h_i \sqrt{f/\rho}$ $\quad k_{ij} = \frac{D_{ij'}}{2}\sqrt{f/\rho}$

Substituting in the formulas for self and mutual inductances with earth return we get:

$$L_{ii} = .7411 \log_{10} \frac{1}{r_i'} + .7411 \log_{10} 2162.5361 \sqrt{\rho/f} + .26\ h_i \sqrt{f/\rho} - \ldots \quad mh/mile$$

$$L_{ij} = .7411 \log_{10} \frac{1}{D_{ij}} + .7411 \log_{10} 2162.5361 \sqrt{\rho/f} + .26 \frac{(h_i + h_j)}{2} \sqrt{f/\rho} - \ldots \quad mh/mile.$$

Note that the first term in each of the above equations represents the corresponding inductance with ground effect neglected. The second term only represents a first order earth return correction which neglects the effects of conductor hight above ground and is the most commonly used. The third term in each equation represents a second order earth return correction and allows for conductors hight above ground.

---

*"Wave Propagation in Overhead Wires with Ground Return," John R. Carson, Bell System Tech. J., Vol. 5, 1926, pp. 539-554.

Carson's correction series are convergent and when trancated after their first term (first order correction) the earth return correction value is too small while if trancated after the second term (second order correction) the earth return correction is too large. In other words the true earth return correction lies between these two bounds.

Resistance Matrix of n Conductors with Earth Return (Carson's Formulas):

If ground return was neglected the resistance matrix would be diagonal, i.e.

R_{ii} = resistance of conductor i in ohms/mile

$= r_{ii}$

and $R_{ij} = 0 \quad ; \; i \neq j$

With ground return included and using Carson's formulas we get:

$R_{ii} = r_{ii} + \Delta r_{ii}$

and $R_{ij} = \Delta r_{ij}$

where:

Δr_{ii} = Carson's earth return correction for self resistance of conductor i.

$= .09530327 \left(\frac{f}{60} \right) - .09798 \times 10^{-3} \left(\frac{f}{60} \right) h_i \sqrt{\rho/f} + \cdots \; \Omega/\text{mile}$

and Δr_{ij} = Carson's earth return correction for mutual resistance between conductors i and j.

$= .09530327 \left(\frac{f}{60} \right) - .09798 \times 10^{-3} \left(\frac{f}{60} \right) \frac{h_i + h_j}{2} \sqrt{\rho/f} + \cdots \; \Omega/\text{mile}$

Potential Coefficients Matrix of n Conductors Assuming Perfect Ground:

Assumptions:

1. The surface of the earth is an equipotential plane of zero potential. The method of images is then used.
2. Charges on the conductors are uniformly distributed on their surfaces.
3. Potential of each conductor is the same throughout its length, i.e. the effect of resistance and inductance of the conductor is neglected.

Using the formula for voltage between 2 conductors given on page II.7; the voltage of conductor i with respect to its image, due to charges q_j on the n conductors and charges $q_{j'} = -q_j$ on the image conductors:

$$V_{i-i'} = \frac{1}{\pi k}\left[\sum_{j=1}^{n} q_j \ln \frac{\sqrt{D_{ji'} D_{i'i}}}{\sqrt{D_{ij} D_{i'i'}}} + \sum_{j'=1'}^{n} q_{j'} \ln \frac{\sqrt{D_{j'i'} D_{i'i}}}{\sqrt{D_{ij'} D_{i'i'}}}\right.$$

But $q_{j'} = -q_j$

$D_{ji'} = D_{ij'}$ and $D_{ij} = D_{i'j'}$

$$\therefore V_{i-i'} = \frac{1}{\pi k} \sum_{j=1}^{n} q_j \ln \frac{D_{ij'}}{D_{ij}}$$

But the potential of conductor i with respect to the ground surface is half of V_{i-i}

$$\therefore V_i = \frac{1}{2\pi k} \sum_{j=1}^{n} q_j \ln \frac{D_{ij'}}{D_{ij}}$$

or

$$V_i = \sum_{j=1}^{n} P_{ij} q_j$$

where:

P_{ii} = self potential coefficient w.r.t. ground

$$= \frac{1}{2\pi k} \ln \frac{D_{ii'}}{D_{ii}}$$

$$= \frac{1}{2\pi k} \ln \frac{2h_i}{r_i} \text{ (Farad)}^{-1}\text{. mt}$$

$$= \frac{1}{.0388} \log_{10} \frac{2h_i}{r_i} (\mu.f)^{-1} \text{ mile}$$

and P_{ij} = Mutual capacitance between conductors i and j while ground is reference

$$P_{ij} = \frac{1}{2\pi k} \ln \frac{D_{ij'}}{D_{ij}} \text{ Farad}^{-1}\text{. mt}$$

$$= \frac{1}{.0388} \log_{10} \frac{D_{ij'}}{D_{ij}} (\mu.f)^{-1}\text{.mile}$$

IV. TRANSMISSION LINE PARAMETERS PROGRAM
(TRANS2)
PEREC REPORT NUMBER IV (1965)

1. Purpose of the Program

TRANS2 (Transmission line coefficients #2) is a program for calculating at any single frequency the electromagnetic (series distributed impedance and/or admittance per mile) and electrostatic coefficients (shunt distributed impedance and/or admittance per mile) for any geometric configuration of up to 90 conductors on the same right-of-way.

The program will also correct the coefficients for ground wire and earth return effects, and has the capability to reduce several parallel conductors to a single equivalent conductor (bundling).

The program was written for a 32 K digital computer (IBM 7090/94) using FORTRAN IV as described in "IBM 7090/94 Programming Systems FORTRAN IV Language" C28-6274-3.

The electromagnetic coefficients will be referred to as the matrix Z and the electrostatic coefficients, P. The voltage equation for these two matrices are:

$$e = Z\ i \qquad v = P\ q$$

where e is the voltage drop per mile and v is the potential of each wire (conductor) with respect to ground.

2. Description of Program

The program will calculate any selected combination of the below listed matrices and any combination of their inverses.

P ncond	matrix of electrostatic coefficients for a geometric configuration of n conductors.
P 3-ph	matrix of electrostatic coefficients with ground wires eliminated and parallel conductors reduced.
P symext	exact symmetrical components transformation of P 3-ph.
P symapr	approximate symmetrical components transformation of P 3-ph
Z ncond	matrix of electromagnetic coefficients for a geometric configuration of n conductors.
Z 3-ph	matrix of electromagnetic coefficients with ground wires eliminated and parallel conductors reduced.
Z symext	exact symmetrical components transformation of Z 3-ph
Z symapr	approximate symmetrical components transformation of Z 3-ph.

The user can select the output to be either in per unit values or in ohms and mhos and either calculated on a per mile basis or have the coefficients calculated for a nominal pi representation.

The user can also obtain punched card output of the calculated matrices which can be used as input data for other programs (e.g. Equivalent Pi Program).

The program consists of a main program and 13 subroutines.

a. Main Program TRANS2:

TRANS2 handles all input and some of the output duties, controls the logic flow of the problem, and calls all needed subroutines.

Matrix symmetry is taken advantage of in storage allocation in order to conserve space. The program also has destructive output for the same reason. In other words, when a user required matrix has been transferred to an output tape, it is replaced by the next required matrix. Also, arrays no longer needed for calculations in the program are used for temporary work storage.

b. Subroutine CPUNCH:

CPUNCH handles punched card output of complex matrices if punched card output is desired by the user.

c. Subroutine IPUNCH:

IPUNCH handles punched card output of imaginary matrices if punched card output is desired by the user.

d. Subroutine CARSON:

CARSON corrects the Z ncond matrix for earth return effects. Three degrees of correction are available to the user; these being

1. No earth return correction, (NEARTH = 0)
2. 1st order earth return correction (NEARTH = 1)
3. 2nd order earth return correction (NEARTH = 2)

e. Subroutine INVRTC:

INVRTC is called by the main program (TRANS2) whenever the inverse of a complex matrix is required.

f. Subroutine INVRTI:

INVRTI is called by the main program (TRANS2) whenever the inverse of an imaginary matrix is required.

g. Subroutine BUNDLE:

BUNDLE reduces parallel conductors to single equivalent conductors and eliminates ground wires.

h. Subroutine SYM:

SYM transforms the $\underline{Z}$ 3-ph and $\underline{P}$ 3-ph matrices to their exact symmetrical components representation.

i. Subroutine IWRITE:

IWRITE transfers the imaginary part of a complex lower triangular matrix from the computer to output tape.

j. Subroutine CWRITE:

CWRITE transfers a complex lower triangular matrix from the computer to output tape.

k. Subroutine IAPSYM:

IAPSYM transforms $\underline{P}$ symext or $\underline{P}^{-1}$ symext into $\underline{P}$ symapr or $\underline{P}^{-1}$ symapr, writes the result on output tape, and punches the output on cards if desired.

l. Subroutine CAPSYM:

CAPSYM transforms $\underline{Z}$ symext or $\underline{Z}^{-1}$ symext into $\underline{Z}$ symapr or $\underline{Z}^{-1}$ symapr, writes the result on output tape, and punches the output on cards if desired.

m. Function DIMAGE:

DIMAGE computes the distance between conductor i and the image of conductor j.

n. Function DACTUL:

DACTUL computes the distance between conductor i and conductor j. $i \neq j$.

3. Equations Used by TRANS2:

$$P_{ii} = -(4.099)(10^6)\ (1/f)\ \log_{10} 4(y_i/d_i)$$

$$P_{ij} = -(4.099)(10^6)\ (1/f)\ \log_{10}(D'_{ij}/D_{ij})$$

$$X_{ij} = 0.27942(f/60)\ \log_{10} D_{ij}$$

$$Z_{ii} = \mathrm{Re}(ZSELF(i)) + j\ \mathrm{Im}(ZSELF(i))\ (f/60) \quad \text{ohms/mile}$$

$$Z_{ij} = -j\ X_{ij} \quad \text{ohms/mile}$$

$$P_{ii} = jP_{ii} \quad \text{ohms/mile}$$

$$P_{ij} = jP_{ij} \quad \text{ohms/mile}$$

$$D_{ij} = ((x_i - x_j)^2 + (y_i - y_j)^2)^{\frac{1}{2}} = \text{DACTUL}(i,j)$$

$$D'_{ij} = ((x_i - x'_j)^2 + (y_i - y'_j)^2)^{\frac{1}{2}} = \text{DIMAGE}(i,j)$$

f = system frequency

x_i, y_i = co-ordinates of conductor i in feet

x'_i, y'_i = co-ordinates of the image of conductor i in feet

4. CARSON's Earth Return Corrections:

CARSON corrects the Z ncond matrix for earth-return effects. These correction terms are shown below.

$$\Delta r_{ii} = .09530327(f/60) - .09798(f/60)(y_i/10^3)\sqrt{f/rho} + \ldots$$

$$\Delta r_{ik} = .09530327(f/60) - .09798(f/60)(y_i+y_k)\ \sqrt{f/rho}/2000 + \ldots$$

$$\Delta X_{ii} = .27942(f/60)\log_{10}2162.5361\sqrt{rho/f} + .09798(f/60)(y_i/10^3)\ \sqrt{f/rho} - \ldots$$

$$\Delta X_{ik} = .27942(f/60)\log_{10}2162.5361\ \sqrt{rho/f} + .09798(f/60)(y_i+y_k)\ \sqrt{f/rho}/2000 - \ldots$$

Carson's solution for the earth-return problem results in an infinite series of Bessel functions. (Ref.2) The above terms have been extracted from an equivalent, but less formidable, infinite series given in Ref. 3.

This subroutine allows for three alternatives. These are

1. No correction (no earth-return)
2. First order correction (only first term of each series is used)
3. Second order correction (first two terms of each series used)

The first order correction neglects the effects of conductor height above ground and is the one most commonly used. The second order correction allows for conductor height above ground and that of adjacent conductors and may be significant when circuits are located on adjacent towers or rights-of-way. Within limitations listed later, the above series are convergent. Therefore, the first order correction is too large for r_{ii} and r_{ik}, and too small for X_{ii} and X_{ik}. The second order correction terms overcompensate this condition. It follows that the true correction lies between these two bounds.

The subroutine uses the correction terms in the following manner:

$$Z_{ii} = Z_{ii} + \Delta r_{ii} + j\ \Delta X_{ii}$$

$$Z_{ik} = Z_{ik} + \Delta r_{ik} + j\ \Delta X_{ik}$$

Convergence Limitations:

The infinite series shown earlier is convergent as long as k is less than or equal to unity, where

$$k_{ii} = 1.713 \times 10^{-3} \times Y(I) \sqrt{f/rho}$$

$$k_{ij} = 1.713 \times 10^{-3} \times DIMAGE(I,J) \sqrt{f/rho/2}$$

5. Elimination of Ground Wires from Z & P Matrices:

Ground wires are considered to be solidly grounded and have no source emf. The ground wire is then eliminated by ordinary matrix elimination.

Example:

In the following matrix equation, consider the fourth conductor to be a ground wire.

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{1gw} \\ Z_{21} & Z_{22} & Z_{23} & Z_{2gw} \\ Z_{31} & Z_{32} & Z_{33} & Z_{3gw} \\ Z_{gw1} & Z_{gw2} & Z_{gw3} & Z_{gwgw} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_{gw} \end{bmatrix}$$

Solving for i_{gw}:

$$i_{gw} = -(Z_{gwgw})^{-1} \ (Z_{gw1} i_1 + Z_{gw2} i_2 + Z_{gw3} i_3)$$

Substituting this value into the first three equations, a reduced system is obtained as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} Z'_{11} & Z'_{12} & Z'_{13} \\ Z'_{21} & Z'_{22} & Z'_{23} \\ Z'_{31} & Z'_{32} & Z'_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

where

$$Z'_{ik} = Z_{ik} - Z_{igw}(Z_{gwgw})^{-1} Z_{gwk} \quad i \neq gw \text{ and } k \neq gw$$

The same procedure is used on the P ncond matrix where $v = P\,q$.

5. Combining Parallel Conductors into One Equivalent Conductor (BUNDLING):

The following example illustrates the method.

Example:

Assume that conductor 4 is in parallel with conductor 1. Then, $e_4 = e_1$ in the following matrix equation.

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \quad (1)$$

Subtracting row 1 from row 4 results in:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41}-Z_{11} & Z_{42}-Z_{12} & Z_{43}-Z_{13} & Z_{44}-Z_{14} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

To restore symmetry, subtract column 1 from column 4:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14}-Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24}-Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34}-Z_{31} \\ Z_{41}-Z_{11} & Z_{42}-Z_{12} & Z_{43}-Z_{13} & Z_{44}-Z_{14}-Z_{41}+Z_{11} \end{bmatrix} \begin{bmatrix} i_1+i_4 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}$$

The matrix equation shown above is equivalent to (1) shown at the beginning of the example. Also, it is of the same form as the matrix equation shown in the elimination example and conductor 4 is eliminated in the same manner. The same procedure is used on the P ncond matrix where v = P q

7. References

1. M. M. Hesse, "Electromagnetic and Electrostatic Transmission Line Parameters by Digital Computer," CP 62-1387, AIEE General Meeting, Chicago, October 1962.

2. M. M. Hesse, "Electromagnetic and Electrostatic Transmission Line Parameters by Digital Computer," Power App. & Syst. IEEE Trans. No. 66, June 1963, pp. 282-91.] *

3. Edith Clarke, Circuit Analysis of A-C Power Systems, John Wiley & Sons, Inc., New York, New York, Vol. I, 1948, pp. 372-384.