Lecture # 5C EE740 A. Keyhani

Derivation of LTC Transformer (1) PI-Model
(No phase shifter) (1)

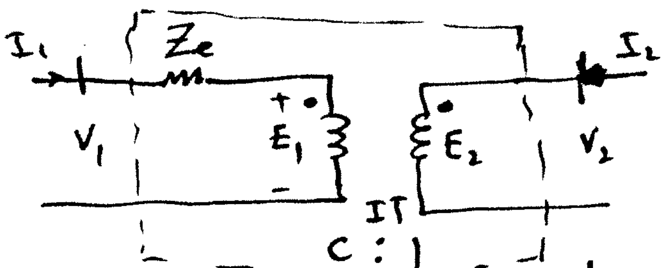

Fig. 1 p.u Eq. circuit
LTC Transf.

Note $E_1 = cE_2$

$I_1 = -\frac{c}{I_2}$

where $a = \frac{N_1}{N_2}$, $b = \frac{V_{b1}}{V_{b2}}$, $c = \frac{a}{b}$, c is real no-phase shifter

Assume two-port network

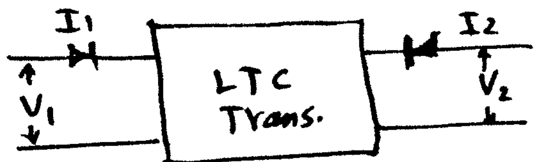

Fig. 2

$I_1 = y_{11}V_1 + y_{12}V_2$ where $y_{21} = y_{12}$ $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$I_2 = y_{21}V_1 + y_{22}V_2$

The above equation can be represented by

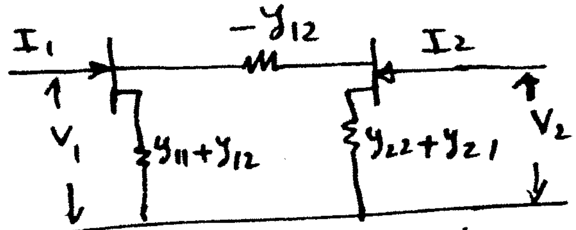

Fig. 3 PI. EQ. Circuit
For LTC Trans (No-phase shifter)

The parameters of Fig. 3 can be calculated in the following way:

$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{Z_e} = Y_e$$

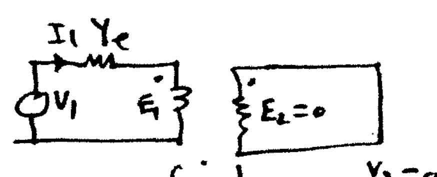

Fig. 1 when $V_2 = 0$

$V_2 = 0$, $E_2 = 0 \Rightarrow E_1 = 0$

$V_1 = I_1 Z_e$ $\therefore$ $I_1 = \frac{V_1}{Z_e}$

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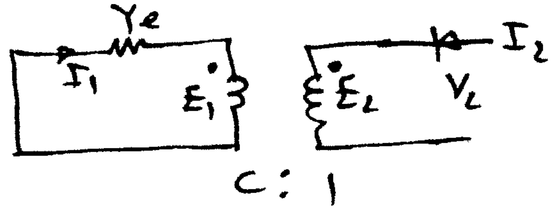
Therefore $\quad y_{11} = Y_e = \frac{1}{Z_e}$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

With $V_1 = 0$, Fig. 1 is



$E_1 = c\,E_2 \qquad E_2 = V_2 \qquad \therefore \quad E_1 = c\,V_2$

$I_1 = -E_1 Y_e$

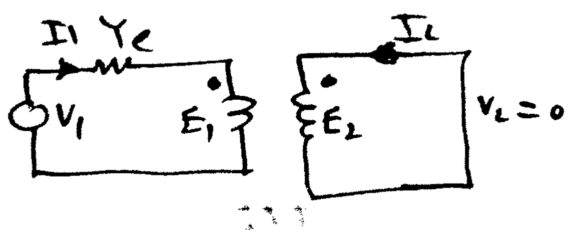
To cal. y_{12}, we need to cal. I_1 in terms of V_2

$I_1 = -c V_2 Y_e \qquad \therefore \qquad y_{12} = \frac{I_1}{V_2} = -c\,Y_e$

To cal. y_{21}, we need to short V_2.

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

With $V_2 = 0$, Fig. 1 can be represented as:



$V_2 = 0, \therefore E_2 = 0 \qquad \therefore \quad E_1 = 0$

$V_1 = I_1 Z_e = \frac{I_1}{Y_e} \qquad$ or $\qquad I_1 = V_1 Y_e$

$I_2 = -c\,I_1$

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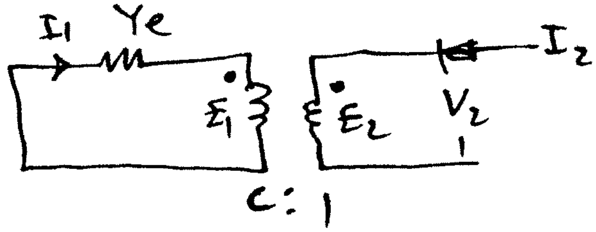
To cal. y_{21}, we need to cal. I_2 in terms of V_1

$$I_2 = -cV_1Y_e \qquad \therefore \qquad y_{21} = \frac{I_2}{V_1} = -cY_e$$

To cal. Y_{22}, we need to short V_1 (i.e $V_1 = 0$)

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

with V_1 shorted, Fig. 1 can be represented as:



$V_2 = E_2$, $E_1 = c\,E_2$ or $E_1 = cV_2$

$$I_1 = -Y_e E_1$$

Substitude for E_1. $\quad I_1 = -Y_e c V_2$

$$I_2 = -cI_1$$

substitude for I_1 from the above

$$I_2 = -c(-Y_e c V_2) = c^2 Y_e V_2$$

$$y_{22} = \frac{I_2}{V_2} = c^2 Y_e$$

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Recall our PI EQ. CKt for LTC transformer

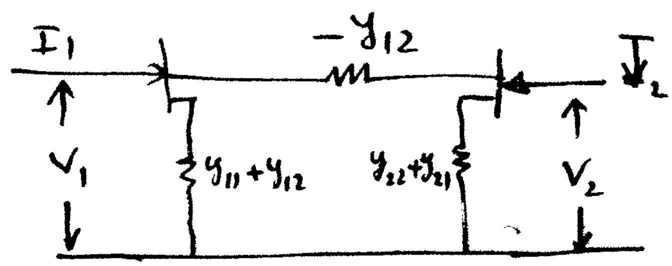

Fig. 4 P.U. PI EQ. Circuit For LTC Trans

where $y_{12} = y_{21} = -c\,Y_e$ $\qquad Y_e = \dfrac{1}{Z_e}$

$$y_{11} = Y_e$$

$$y_{22} = c^2 Y_e$$

Therefore, the parameters of Fig. 4 are as follows:

$$y_{11} + y_{12} = Y_e + (-c\,Y_e) = Y_e(1-c)$$

$$y_{22} + y_{21} = c^2 Y_e + (-c\,Y_e) = Y_e(c^2 - c)$$

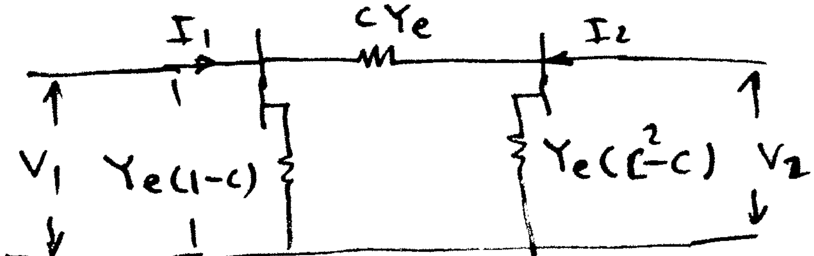

Fig. 5 P.U. PI EQ. Circuit For LTC Trans

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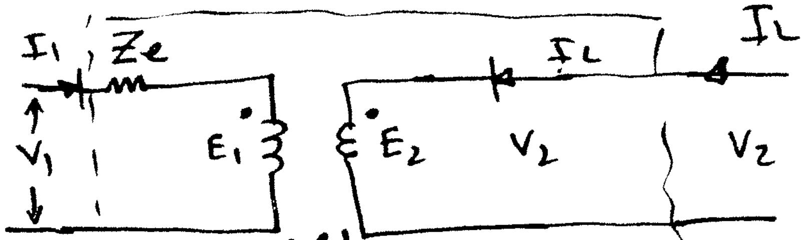
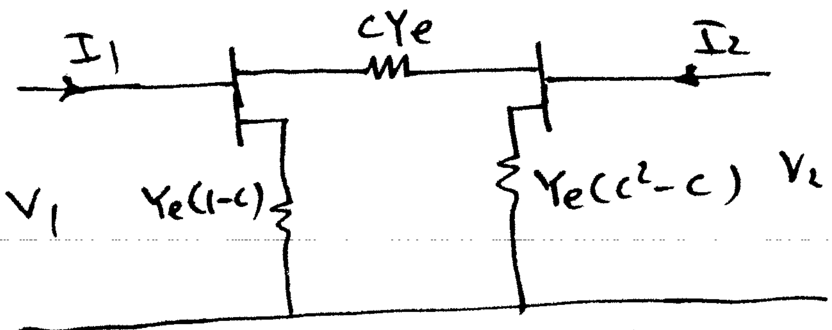
Summary

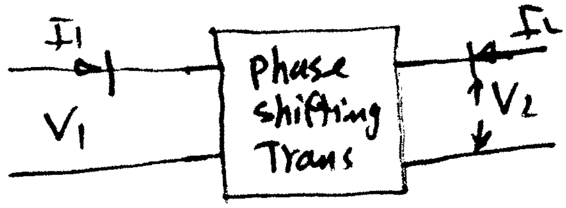


P.U EQ. circuit
For LTC Trans
"C" is real (No phase shifter)



PI. EQ. circuit
For LTC Trans.
"C" is real No phase shifter)

Note: For phase shifting transformers C is complex and $y_{12} \neq y_{21}$. We can not synthesize an equivalent circuit with real R, L, c elements. However, we ~~may~~ represent the phase shifting transformer ~~in~~ our network impedance diagram as a two-port network with two inputs and outputs.



$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$