1. Problem 7-5 on textbook (Chapman, page 444)
   A 50-kW, 440-V, 50-Hz, two-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 520W, and the core losses are 500W. Find the following values for full-load conditions:
   (a) The shaft speed n_m
   (b) The output power in watts
   (c) The load torque τ_{load} in newton-meters
   (d) The induced torque τ_{ind} in newton-meters
   (e) The rotor frequency in hertz

SOLUTION

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.06)(3000\text{ r/min}) = 2820\text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50\text{ kW}}{(2820\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 169.3\text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50\text{ kW} + 520\text{ W} + 500\text{ W} = 51.2\text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{51.2\text{ kW}}{(2820\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 173.4\text{ N}\cdot\text{m}$$

(e) The rotor frequency is

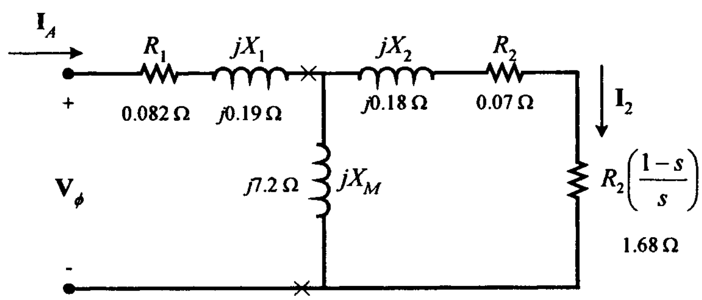
$$f_r = sf_e = (0.06)(50\text{ Hz}) = 3.00\text{ Hz}$$

2. Problem 7-14 on textbook (Chapman, page 445)
A 440-V, 50-Hz, six-pole, Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

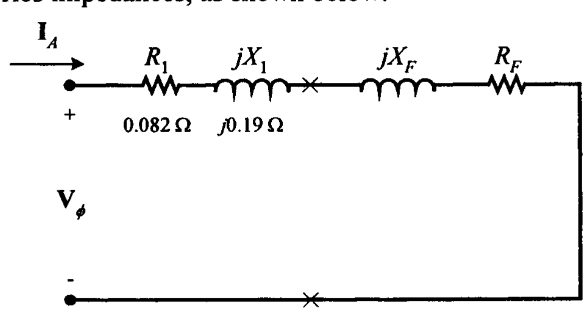
$R_1 = 0.082\Omega$	$R_2 = 0.070\Omega$	$X_M = 7.2\Omega$
$X_1 = 0.19\Omega$	$X_2 = 0.18\Omega$	
$P_{F\&W} = 1.3kW$	$P_{misc} = 150W$	$P_{core} = 1.4kW$

For a slip of 0.04, find
(a) The line current I_L
(b) The stator power factor
(c) The rotor power factor
(d) The stator copper losses P_{SCL}
(e) The air-gap power P_{AG}
(f) The power converted from electrical to mechanical form P_{conv}
(g) The induced torque τ_{ind}
(h) The load torque τ_{load}
(i) The overall machine efficiency η
(j) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j7.2\,\Omega} + \frac{1}{1.75 + j0.18}} = 1.557 + j0.550 = 1.67\angle 19.2^\circ\,\Omega$$

The phase voltage is $440/\sqrt{3} = 254$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{254\angle 0° \text{ V}}{0.082\,\Omega + j0.19\,\Omega + 1.557\,\Omega + j0.550\,\Omega}$$

$$I_L = I_A = 141\angle -24.3° \text{ A}$$

(b) The stator power factor is

$$\text{PF} = \cos 24.3° = 0.911 \text{ lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{0.18}{1.75} = 5.87°$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 5.87° = 0.995 \text{ lagging}$$

(d) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^2R_1 = 3(141\text{ A})^2(0.082\,\Omega) = 4890\text{ W}$$

(e) The air gap power is $P_{\text{AG}} = 3I_2^2\frac{R_2}{s} = 3I_A^2R_F$

(Note that $3I_A^2R_F$ is equal to $3I_2^2\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^2\frac{R_2}{s} = 3I_A^2R_F = 3(141\text{ A})^2(1.557\,\Omega) = 92.6\text{ kW}$$

(f) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.04)(92.6\text{ kW}) = 88.9\text{ kW}$$

(g) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

$$\omega_{\text{sync}} = (1000\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 104.7\text{ rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{92.6\text{ kW}}{(1000\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 884\text{ N}\cdot\text{m}$$

(h) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 88.9\text{ kW} - 1.3\text{ kW} - 1.4\text{ kW} - 300\text{ W} = 85.9\text{ kW}$$

The output speed is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.04)(1000\ \text{r/min}) = 960\ \text{r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{85.9\ \text{kW}}{(960\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 854\ \text{N}\cdot\text{m}$$

(i) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}}\times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta = \frac{85.9\ \text{kW}}{3(254\ \text{V})(141\ \text{A})\cos 24.3^\circ}\times 100\% = 87.7\%$$

(j) The motor speed in revolutions per minute is 960 r/min. The motor speed in radians per second is

$$\omega_m = (960\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 100.5\ \text{rad/s}$$