# EE341 Homework #8 Solutions

**7-2** A 220-V three-phase six-pole 50-Hz induction motor is running at a slip of 3.5 percent. Find:

*(a)* The speed of the magnetic fields in revolutions per minute

*(b)* The speed of the rotor in revolutions per minute

*(c)* The slip speed of the rotor

*(d)* The rotor frequency in hertz

SOLUTION

*(a)* The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

*(b)* The speed of the rotor is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.035)(1000\text{ r/min}) = 965\text{ r/min}$$

*(c)* The slip speed of the rotor is

$$n_{\text{slip}} = s n_{\text{sync}} = (0.035)(1000\text{ r/min}) = 35\text{ r/min}$$

*(d)* The rotor frequency is

$$f_r = \frac{n_{\text{slip}} P}{120} = \frac{(35\text{ r/min})(6)}{120} = 1.75\text{ Hz}$$

**7-4.** A three-phase 60-Hz induction motor runs at 715 r/min at no load and at 670 r/min at full load.

*(a)* How many poles does this motor have?

*(b)* What is the slip at rated load?

*(c)* What is the speed at one-quarter of the rated load?

*(d)* What is the rotor's electrical frequency at one-quarter of the rated load?

SOLUTION

*(a)* This machine has 10 poles, which produces a synchronous speed of

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{10} = 720\text{ r/min}$$

*(b)* The slip at rated load is

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\% = \frac{720-670}{720} \times 100\% = 6.94\%$$

*(c)* The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load wil

$$s = 0.25(0.0694) = 0.0174$$

The resulting speed is

$$n_m = (1-s)n_{\text{sync}} = (1-0.0174)(720\text{ r/min}) = 707\text{ r/min}$$

*(d)* The electrical frequency at ¼ load is

$$f_r = sf_e = (0.0174)(60\text{ Hz}) = 1.04\text{ Hz}$$

**7-7.** A 208-V four-pole 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.220\ \Omega$ $\quad R_2 = 0.127\ \Omega$ $\quad X_M = 15.0\ \Omega$

$X_1 = 0.430\ \Omega$ $\quad X_2 = 0.430\ \Omega$

$P_{\text{mech}} = 300\text{ W}$ $\quad P_{\text{misc}} \approx 0$ $\quad P_{\text{core}} = 200\text{ W}$

For a slip of 0.05, find

*(a)* The line current $I_L$

*(b)* The stator copper losses $P_{\text{SCL}}$

*(c)* The air-gap power $P_{\text{AG}}$

*(d)* The power converted from electrical to mechanical form $P_{\text{conv}}$

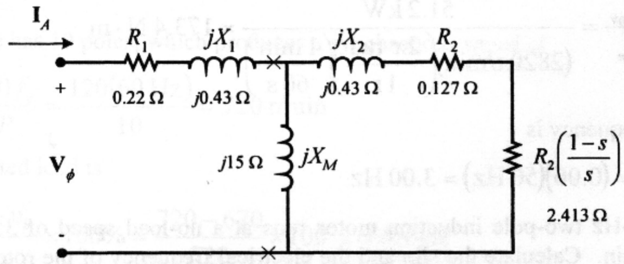
*(e)* The induced torque $\tau_{\text{ind}}$

*(f)* The load torque $\tau_{\text{load}}$

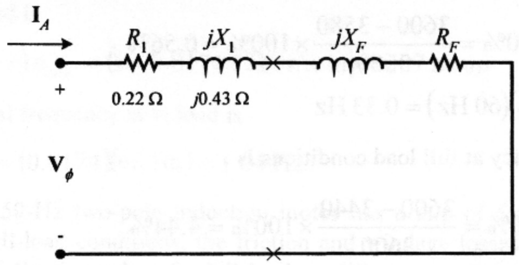
*(g)* The overall machine efficiency

*(h)* The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:




*(a)* The easiest way to find the line current (or armature current) is to get the equivalent impedance $Z_F$ of the rotor circuit in parallel with $jX_M$, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.




The equivalent impedance of the rotor circuit in parallel with $jX_M$ is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j15\ \Omega} + \frac{1}{2.54 + j0.43}} = 2.337 + j0.803 = 2.47\angle 19°\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current $I_L$ is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0° \text{ V}}{0.22\,\Omega + j0.43\,\Omega + 2.337\,\Omega + j0.803\,\Omega}$$

$$I_L = I_A = 42.3\angle -25.7° \text{ A}$$

*(b)* The stator copper losses are

$$P_{\text{SCL}} = 3 I_A{}^2 R_1 = 3(42.3 \text{ A})^2 (0.22\,\Omega) = 1180 \text{ W}$$

*(c)* The air gap power is $P_{\text{AG}} = 3 I_2{}^2 \frac{R_2}{s} = 3 I_A{}^2 R_F$

(Note that $3 I_A{}^2 R_F$ is equal to $3 I_2{}^2 \frac{R_2}{s}$, since the only resistance in the original rotor circuit was $R_2 / s$, and the resistance in the Thevenin equivalent circuit is $R_F$. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3 I_2{}^2 \frac{R_2}{s} = 3 I_A{}^2 R_F = 3(42.3 \text{ A})^2 (2.337\,\Omega) = 12.54 \text{ kW}$$

*(d)* The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1 - s) P_{\text{AG}} = (1 - 0.05)(12.54 \text{ kW}) = 11.92 \text{ kW}$$

*(e)* The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{12.54 \text{ kW}}{(1800 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 66.5 \text{ N}\cdot\text{m}$$

*(f)* The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 11.92 \text{ kW} - 300 \text{ W} - 200 \text{ W} - 0 \text{ W} = 11.42 \text{ kW}$$

The output speed is

$$n_m = (1 - s) n_{\text{sync}} = (1 - 0.05)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{11.42 \text{ kW}}{(1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 63.8 \text{ N}\cdot\text{m}$$

*(g)* The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3 V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{11.42 \text{ kW}}{3(120 \text{ V})(42.3 \text{ A})\cos 25.7°} \times 100\% = 83.2\%$$

*(h)* The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 179 \text{ rad/s}$$