Homework #5 Solutions

1. A 100-kVA, 240/120-V Y-Y three-phase power transformer has a per-unit resistance of 0.01 pu and a per-unit reactance of 0.035 pu. The excitation branch elements are R_C = 80 pu and X_M = 20 pu.
   (a) If this transformer supplies a rated load at 0.8 PF lagging, draw the phasor diagram of one phase of the transformer.
   (b) What is the voltage regulation of the transformer bank under these conditions?
   (c) Sketch the equivalent circuit referred to the low-voltage side of one phase oh this transformer. Calculate all the transformer impedances referred to the low-voltage side.

Solution:

Select $S_{base} = 100kVA$, $V_{P,base} = 240V$, $V_{S,base} = 240V$

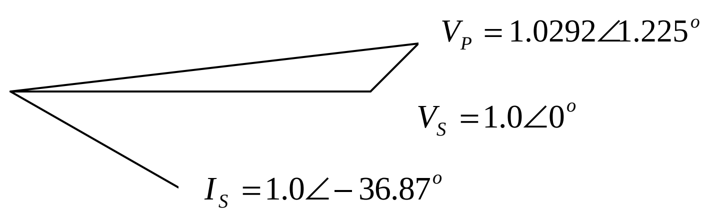
(a) $S_{load} = 1.0\angle\cos^{-1}(0.8) = 1.0\angle 36.87^o\ pu$

$V_S = 1.0\angle 0^o$

$$I_S = \left(\frac{S_{load}}{V_S}\right)^* = \left(\frac{1.0\angle 36.87^o}{1.0\angle 0^o}\right)^* = 1.0\angle -36.87^o\ pu$$

$V_P = V_S + I_S Z_{eq} = 1.0\angle 0^o + 1.0\angle -36.87^o (0.01 + j0.035) = 1.0292\angle 1.225^o\ pu$

The phasor diagram is shown as below:



(b) The voltage regulation under these conditions is:

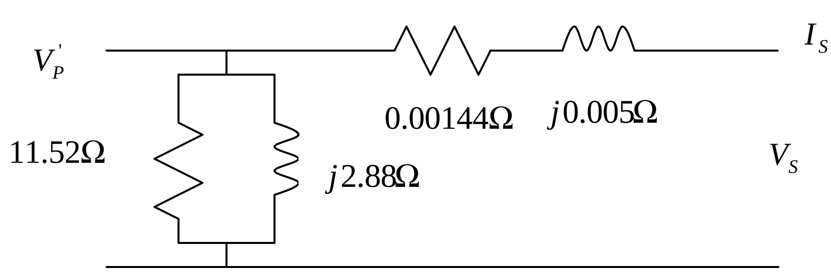
$$VR = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\% = \frac{1.0292 - 1.0}{1.0} \times 100\% = 2.92\%$$

(c) The base impedance of the LV side is:

$$Z_{S,base} = \frac{V_{S,base}^2}{S_{basel}} = \frac{120^2}{100{,}000} = 0.144\Omega$$

So $R_{eq} = 0.01 \times Z_{S,base} = 0.00144\Omega$, $X_{eq} = 0.005\Omega$,

$R_C = 11.52\Omega$, $X_M = 2.88\Omega$. The equivalent circuit referred to LV side is:



2. Problem 2-19 on textbook (Chapman)
A 20-kVA, 20,000/480-V 60-Hz distribution transformer is tested with the following results.

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
$V_{OC} = 480V$	$V_{SC} = 1130V$
$I_{OC} = 1.51A$	$I_{SC} = 1.00A$
$P_{OC} = 271W$	$P_{SC} = 260W$

(a) Find the per-unit equivalent circuit for this transformer at 60 Hz.
(b) What would the rating of this transformer be if it were operated on a 50-Hz power system? (Hint: If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60.)
(c) Sketch the equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz*. (Hint: when frequency changes, the resistances will be unaffected, but the reactances will be changed because $X = wL = 2pfL$.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(20{,}000\text{ V})^2}{20\text{ kVA}} = 20\text{ k}\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{\text{base},S} = \frac{(V_S)^2}{S} = \frac{(480\text{ V})^2}{20\text{ kVA}} = 11.52\,\Omega$$

The open circuit test yields the values for the excitation branch (referred to the *secondary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{1.51\text{ A}}{480\text{ V}} = 0.00315\text{ mho}$$

$$\theta = -\cos^{-1}\left(\frac{P_{OC}}{V_{OC}\,I_{OC}}\right) = -\cos^{-1}\left(\frac{271\text{ W}}{(480\text{ V})(1.51\text{ A})}\right) = -68°$$

$$Y_{EX} = G_C - jB_M = 0.00315\angle -68° = 0.00118 - j0.00292$$

$$R_C = 1/G_C = 847\,\Omega$$

$$X_M = 1/B_M = 342\,\Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{847\,\Omega}{11.52\,\Omega} = 73.5\text{ pu} \qquad X_M = \frac{342\,\Omega}{11.52\,\Omega} = 29.7\text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{1130\text{ V}}{1.00\text{ A}} = 1130\ \Omega$$

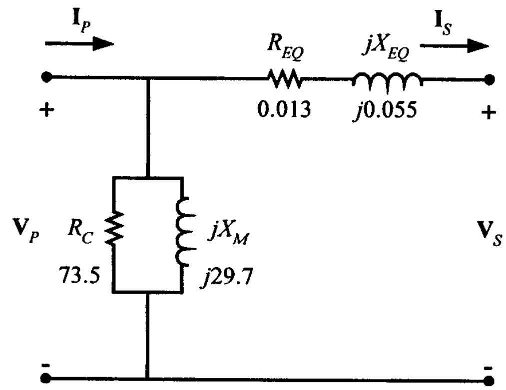
$$\theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC}\, I_{SC}}\right) = \cos^{-1}\left(\frac{260\text{ W}}{(1130\text{ V})(1.00\text{ A})}\right) = 76.7°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1130\angle 76.7° = 260 + j1100\,\Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{260\,\Omega}{20{,}000\,\Omega} = 0.013\text{ pu} \qquad X_{EQ} = \frac{1100\,\Omega}{20{,}000\,\Omega} = 0.055\text{ pu}$$

The per-unit equivalent circuit is



(b) If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60, so its ratings would be 16.67 kVA, 16,667/400 V, and 50 Hz.

(c) The transformer parameters referred to the primary side *at 60 Hz* are:

$$R_C = Z_{base} R_{C,pu} = (20\text{ k}\Omega)(73.5) = 1.47\text{ M}\Omega$$
$$X_M = Z_{base} X_{M,pu} = (20\text{ k}\Omega)(29.7) = 594\text{ k}\Omega$$
$$R_{EQ} = Z_{base} R_{EQ,pu} = (20\text{ k}\Omega)(0.013) = 260\,\Omega$$
$$X_{EQ} = Z_{base} X_{EQ,pu} = (20\text{ k}\Omega)(0.055) = 1100\,\Omega$$

At 50 Hz, the resistance will be unaffected but the reactances are reduced in direct proportion to the decrease in frequency. At 50 Hz, the reactances are

$$X_M = 495\text{ k}\Omega$$
$$X_{EQ} = 917\,\Omega$$

The resulting equivalent circuit referred to the primary at 50 Hz is shown below: