Homework #4 Solutions

1. (2-18, page 138) Three 25-kVA 24,000/277-V distribution transformers are connected in Δ-Y. The open-circuit test was performed on the low-voltage side of this transformer bank, and the following data were recorded:

$V_{line,OC} = 480\ \text{V} \qquad I_{line,OC} = 4.10\ \text{A} \qquad P_{3\phi,OC} = 945\ \text{W}$

The short-circuit test was performed on the high-voltage side of this transformer bank, and the following data were recorded:

$V_{line,SC} = 1400\ \text{V} \qquad I_{line,SC} = 1.80\ \text{A} \qquad P_{3\phi,SC} = 912\ \text{W}$

(a) Find the per-unit equivalent circuit of this transformer bank.
(b) Find the voltage regulation of this transformer bank at the rated load and 0.90 PF lagging.
(c) What is the transformer bank's efficiency under these conditions?

SOLUTION *(a)* The equivalent of this transformer bank can be found just like the equivalent circuit of a single-phase transformer if we work on a per-phase bases. The open-circuit test data on the low-voltage side can be used to find the excitation branch impedances referred to the secondary side of the transformer bank. Since the secondary is Y-connected, the per-phase open-circuit quantities are:

$$V_{\phi,\text{OC}} = 277\ \text{V} \qquad I_{\phi,\text{OC}} = 4.10\ \text{A} \qquad P_{\phi,\text{OC}} = 315\ \text{W}$$

The excitation admittance is given by

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{4.10\ \text{A}}{277\ \text{V}} = 0.01483\ \text{mho}$$

The admittance angle is

$$\theta = -\cos^{-1}\left(\frac{P_{\phi,OC}}{V_{\phi,OC}\, I_{\phi,OC}}\right) = -\cos^{-1}\left(\frac{315\ \text{W}}{(277\ \text{V})(4.10\ \text{A})}\right) = -73.9°$$

Therefore,

$$Y_{EX} = G_C - jB_M = 0.01483\angle -73.9° = 0.00411 - j0.01425$$
$$R_C = 1/G_C = 243\ \Omega$$
$$X_M = 1/B_M = 70.2\ \Omega$$

The *base impedance* referred to the secondary side is

$$Z_{\text{base},S} = \frac{(V_{\phi,S})^2}{S_\phi} = \frac{(277\ \text{V})^2}{25\ \text{kVA}} = 3.069\ \Omega$$

so the excitation branch elements can be expressed in per-unit as

$$R_C = \frac{243\ \Omega}{3.069\ \Omega} = 79.2\ \text{pu} \qquad X_M = \frac{70.2\ \Omega}{3.069\ \Omega} = 22.9\ \text{pu}$$

The short-circuit test data can be used to find the series impedances referred to the primary side, since the short-circuit test data was taken on the primary side. Note that the primary is Δ-connected, so $V_{\phi,SC} = V_{SC} = 1400\text{ V}$, $I_{\phi,SC} = I_{SC}/\sqrt{3} = 1.039\text{ A}$, and $P_{\phi,SC} = P_{SC}/3 = 304\text{ W}$.

$$\left|Z_{EQ}\right| = \frac{V_{\phi,SC}}{I_{\phi,SC}} = \frac{1400\text{ V}}{1.039\text{ A}} = 1347\ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{\phi,SC}}{V_{\phi,SC}\,I_{\phi,SC}}\right) = \cos^{-1}\left(\frac{304\text{ W}}{(1400\text{ V})(1.039\text{ A})}\right) = 77.9°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1347\angle 77.9° = 282 + j1371\,\Omega$$

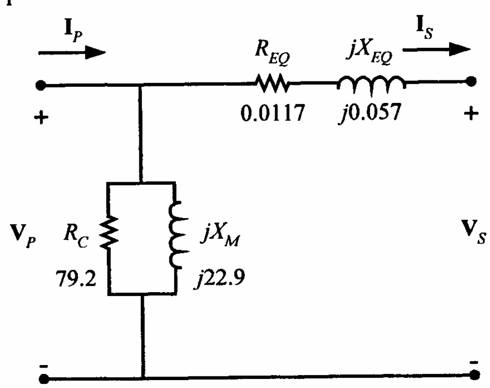
The *base impedance* referred to the secondary side is

$$Z_{\text{base},P} = \frac{(V_{\phi,P})^2}{S_\phi} = \frac{(24{,}000\text{ V})^2}{25\text{ kVA}} = 24{,}040\,\Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{282\,\Omega}{24{,}040\,\Omega} = 0.0117\text{ pu} \qquad X_{EQ} = \frac{1371\,\Omega}{24{,}040\,\Omega} = 0.057\text{ pu}$$

The per-unit, per-phase equivalent circuit of the transformer bank is shown below:



(b) If this transformer is operating at rated load and 0.90 PF lagging, then current flow will be at an angle of $-\cos^{-1}(0.9)$, or –25.8°. The voltage at the primary side of the transformer will be

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}_S Z_{EQ} = 1.0\angle 0° + (1.0\angle -25.8°)(0.0117 + j0.057) = 1.037\angle 2.65°$$

The voltage regulation of this transformer bank is

$$\text{VR} = \frac{1.037 - 1.0}{1.0} \times 100\% = 3.7\%$$

(c) The output power of this transformer bank is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (1.0)(1.0)(0.9) = 0.9\text{ pu}$$

The copper losses are

$$P_{\text{CU}} = I_S^{\,2} R_{\text{EQ}} = (1.0)^2(0.0117) = 0.0117\text{ pu}$$

The core losses are

$$P_{core} = \frac{V_P^2}{R_C} = \frac{(1.037)^2}{79.2} = 0.014 \text{ pu}$$

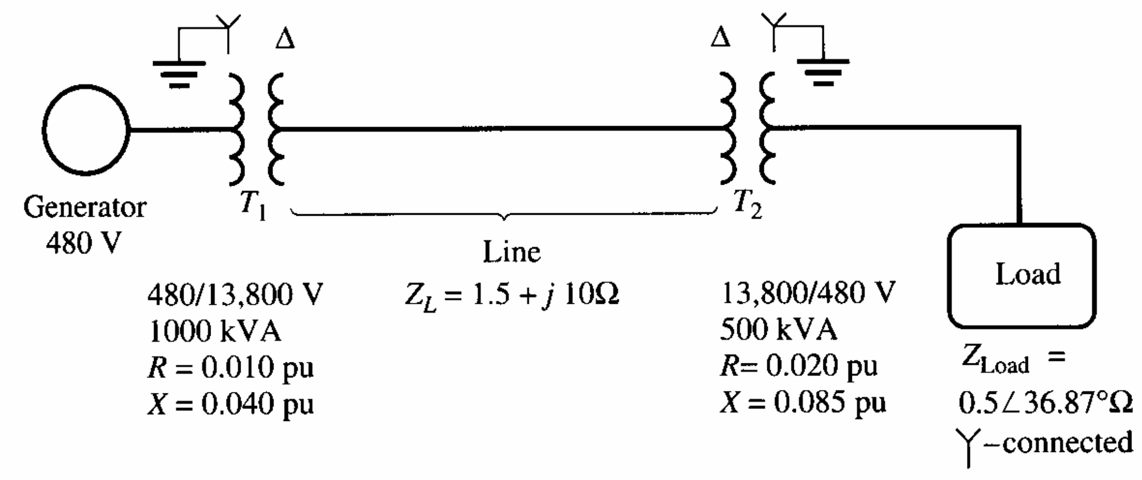
Therefore, the total input power to the transformer bank is

$$P_{IN} = P_{OUT} + P_{CU} + P_{core} = 0.9 + 0.0117 + 0.014 = 0.9257$$

and the efficiency of the transformer bank is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.9}{0.9257} \times 100\% = 97.2\%$$

2. The following figure shows a power system consisting of a three-phase 480-V 60-Hz generator supplying a load through a transmission line with a pair of transformers at either end.



(a) Sketch the per-phase equivalent circuit of this power system.
(b) Find the active power P, reactive power Q, and apparent (complex) power S supplied by the generator. What is the power factor of the generator?

Solution:

Region 1	**Region 2**	**Region 3**
$S_{base1} = 1000 \text{ kVA}$	$S_{base2} = 1000 \text{ kVA}$	$S_{base3} = 1000 \text{ kVA}$
$V_{L,base1} = 480 \text{ V}$	$V_{L,base2} = 13{,}800 \text{ V}$	$V_{L,base3} = 480 \text{ V}$

This problem can best be solved using the per-unit system of measurements. The power system can be divided into three regions by the two transformers. If the per-unit base quantities in Region 1 are chosen to be S_{base1} = 1000 kVA and $V_{L,\text{base1}}$ = 480 V, then the base quantities in Regions 2 and 3 will be as shown above. The base impedances of each region will be:

$$Z_{\text{base1}} = \frac{3V_{\phi 1}^{\ 2}}{S_{\text{base1}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

$$Z_{\text{base2}} = \frac{3V_{\phi 2}^{\ 2}}{S_{\text{base2}}} = \frac{3(7967\text{ V})^2}{1000\text{ kVA}} = 190.4\ \Omega$$

$$Z_{\text{base3}} = \frac{3V_{\phi 3}^{\ 2}}{S_{\text{base3}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

(a) To get the per-unit, per-phase equivalent circuit, we must convert each impedance in the system to per-unit on the base of the region in which it is located. The impedance of transformer T_1 is already in per-unit to the proper base, so we don't have to do anything to it:

$$R_{1,\text{pu}} = 0.010$$

$$X_{1,\text{pu}} = 0.040$$

The impedance of transformer T_2 is already in per-unit, but it is per-unit to the base of transformer T_2, so it must be converted to the base of the power system.

$$(R,\ X,\ Z)_{\text{pu on base 2}} = (R,\ X,\ Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})} \qquad (2\text{-}60)$$

$$R_{2,\text{pu}} = 0.020 \frac{(7967\text{ V})^2(1000\text{ kVA})}{(7967\text{ V})^2(500\text{ kVA})} = 0.040$$

$$X_{2,\text{pu}} = 0.085 \frac{(7967\text{ V})^2(1000\text{ kVA})}{(7967\text{ V})^2(500\text{ kVA})} = 0.170$$

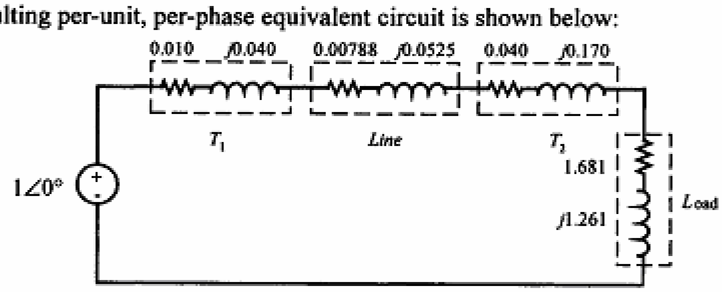
The per-unit impedance of the transmission line is

$$Z_{\text{line,pu}} = \frac{Z_{\text{line}}}{Z_{\text{base2}}} = \frac{1.5 + j10\ \Omega}{190.4\ \Omega} = 0.00788 + j0.0525$$

The per-unit impedance of Load is

$$Z_{\text{load}',\text{pu}} = \frac{Z_{\text{load1}}}{Z_{\text{base3}}} = \frac{0.5\angle 36.87°\ \Omega}{0.238\ \Omega} = 1.681 + j1.261$$

The resulting per-unit, per-phase equivalent circuit is shown below:



(b) the equivalent impedance of this circuit is

$$Z_{\text{EQ}} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + 1.681 + j1.261$$
$$Z_{\text{EQ}} = 1.7389 + j1.5235 = 2.312\angle 41.2°$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0°}{2.312\angle 41.2°} = 0.4325\angle -41.2°$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\, Z_{\text{Load}} = (0.4325\angle -41.2°)(1.681 + j1.261) = 0.909\angle -4.3°$$
$$V_{\text{Load}} = V_{\text{Load,pu}} V_{\text{base3}} = (0.909)(480\text{ V}) = 436\text{ V}$$

The power supplied to the load is

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.4325)^2(1.681) = 0.314$$
$$P_{\text{Load}} = P_{\text{Load,pu}} S_{\text{base}} = (0.314)(1000\text{ kVA}) = 314\text{ kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.4325)\cos 41.2° = 0.325$$
$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.4325)\sin 41.2° = 0.285$$
$$S_{G,\text{pu}} = VI = (1)(0.4325) = 0.4325$$
$$P_G = P_{G,\text{pu}} S_{\text{base}} = (0.325)(1000\text{ kVA}) = 325\text{ kW}$$
$$Q_G = Q_{G,\text{pu}} S_{\text{base}} = (0.285)(1000\text{ kVA}) = 285\text{ kVAR}$$
$$S_G = S_{G,\text{pu}} S_{\text{base}} = (0.4325)(1000\text{ kVA}) = 432.5\text{ kVA}$$

The power factor of the generator is

$$\text{PF} = \cos 41.2° = 0.752 \text{ lagging}$$