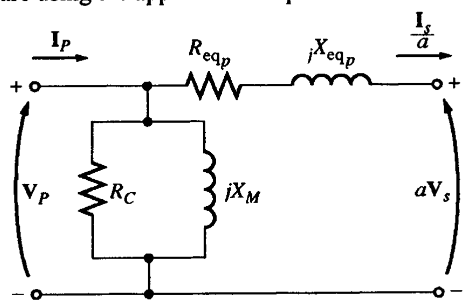
EE341 Homework #3 Solutions

2-1. The secondary winding of a transformer has a terminal voltage of $v_s(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 50:200 (a = 0.25). If the secondary current of the transformer is $i_s(t) = 7.07 \sin(377t - 36.87°)$ A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$R_{eq} = 0.05\ \Omega$ $\quad$ $R_C = 75\ \Omega$

$X_{eq} = 0.225\ \Omega$ $\quad$ $X_M = 20\ \Omega$

SOLUTION The equivalent circuit of this transformer is shown below. (Since no particular equivalen circuit was specified, we are using the approximate equivalent circuit referred to the primary side.)



The secondary voltage and current are

$$\mathbf{V}_S = \frac{282.8}{\sqrt{2}}\angle 0°\text{ V} = 200\angle 0°\text{ V}$$

$$\mathbf{I}_S = \frac{7.07}{\sqrt{2}}\angle -36.87°\text{ A} = 5\angle\text{-}36.87°\text{ A}$$

The secondary voltage referred to the primary side is

$$\mathbf{V}_S' = a\mathbf{V}_S = 50\angle 0°\text{ V}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S' = \frac{\mathbf{I}_S}{a} = 20\angle\text{-}36.87°\text{ A}$$

The primary circuit voltage is given by

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S'\left(R_{eq} + jX_{eq}\right)$$
$$\mathbf{V}_P = 50\angle 0°\text{ V} + \left(20\angle -36.87°\text{ A}\right)\left(0.05\,\Omega + j0.225\,\Omega\right) = 53.6\angle 3.2°\text{ V}$$

The excitation current of this transformer is

$$\mathbf{I}_{EX} = \mathbf{I}_C + \mathbf{I}_M = \frac{53.6\angle 3.2°\text{ V}}{75\,\Omega} + \frac{53.6\angle 3.2°\text{ V}}{j20\,\Omega} = 0.7145\angle 3.2° + 2.679\angle -86.8°$$
$$\mathbf{I}_{EX} = 2.77\angle -71.9°$$

Therefore, the total primary current of this transformer is

$$\mathbf{I}_P = \mathbf{I}_S' + \mathbf{I}_{EX} = 20\angle -36.87° + 2.77\angle -71.9° = 22.3\angle -41.0° \text{ A}$$

The voltage regulation of the transformer at this load is

$$\text{VR} = \frac{V_P - aV_S}{aV_S} \times 100\% = \frac{53.6 - 50}{50} \times 100\% = 7.2\%$$

The input power to this transformer is

$$P_{\text{IN}} = V_P I_P \cos\theta = (53.6 \text{ V})(22.3 \text{ A})\cos(3.2° - (-41.0°))$$
$$P_{\text{IN}} = (53.6 \text{ V})(22.3 \text{ A})\cos 44.2° = 857 \text{ W}$$

The output power from this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (200 \text{ V})(5 \text{ A})\cos(36.87°) = 800 \text{ W}$$

Therefore, the transformer's efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{800 \text{ W}}{857 \text{ W}} \times 100\% = 93.4\%$$

2-3. A 2000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
$V_{OC} = 230$ V	$V_{SC} = 13.2$ V
$I_{OC} = 0.45$ A	$I_{SC} = 6.0$ A
$P_{OC} = 30$ W	$P_{SC} = 20.1$ W

All data given were taken from the primary side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION

(a) OPEN CIRCUIT TEST:

$$|Y_{\text{EX}}| = |G_C - jB_M| = \frac{0.45 \text{ A}}{230 \text{ V}} = 0.001957$$

$$\theta = \cos^{-1}\frac{P_{\text{OC}}}{V_{\text{OC}} I_{\text{OC}}} = \cos^{-1}\frac{30 \text{ W}}{(230 \text{ V})(0.45 \text{ A})} = 73.15°$$

$$Y_{\text{EX}} = G_C - jB_M = 0.001957\angle -73.15° \text{ mho} = 0.000567 - j0.001873 \text{ mho}$$

$$R_C = \frac{1}{G_C} = 1763 \ \Omega$$

$$X_M = \frac{1}{B_M} = 534 \ \Omega$$

SHORT CIRCUIT TEST:

$$|Z_{EQ}| = |R_{EQ} + jX_{EQ}| = \frac{13.2\text{ V}}{6.0\text{ A}} = 2.20\ \Omega$$

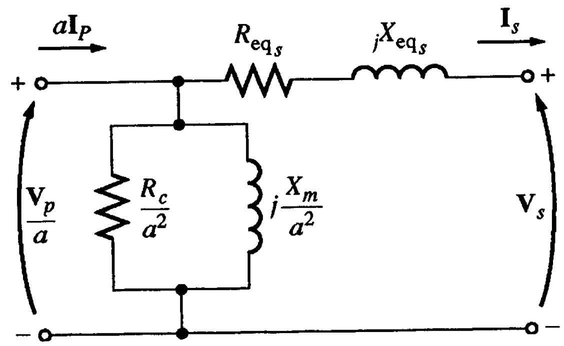
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{20.1\text{ W}}{(13.2\text{ V})(6\text{ A})} = 75.3°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 2.20\angle 75.3°\ \Omega = 0.558 + j2.128\ \Omega$$

$$R_{EQ} = 0.558\ \Omega$$

$$X_{EQ} = j2.128\ \Omega$$

To convert the equivalent circuit to the secondary side, divide each impedance by the square of the turns ratio ($a = 230/115 = 2$). The resulting equivalent circuit is shown below:



$R_{EQ,s} = 0.140\ \Omega$ $\qquad$ $X_{EQ,s} = j0.532\ \Omega$

$R_{C,s} = 441\ \Omega$ $\qquad$ $X_{M,s} = 134\ \Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{ V}} = 8.70\text{ A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) **0.8 PF Lagging:**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\ \Omega)(8.7\angle\text{-}36.87°\text{ A})$$

$$\mathbf{V}_P' = 118.8\angle 1.4°\text{ V}$$

$$\text{VR} = \frac{118.8 - 115}{115} \times 100\% = 3.3\%$$

(2) **1.0 PF:**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\ \Omega)(8.7\angle 0°\text{ A})$$

$$\mathbf{V}_P' = 116.3\angle 2.28°\text{ V}$$

$$\text{VR} = \frac{116.3 - 115}{115} \times 100\% = 1.1\%$$

(3) **<u>0.8 PF Leading:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0° \text{ V} + (0.140 + j0.532\ \Omega)(8.7\angle 36.87° \text{ A})$$

$$\mathbf{V}_P' = 113.3\angle 2.24° \text{ V}$$

$$\text{VR} = \frac{113.3 - 115}{115} \times 100\% = -1.5\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (115 \text{ V})(8.7 \text{ A})(0.8) = 800 \text{ W}$$

The copper and core losses of this transformer are

$$P_{\text{CU}} = I_S^{\,2} R_{\text{EQ},S} = (8.7 \text{ A})^2 (0.140\ \Omega) = 10.6 \text{ W}$$

$$P_{\text{core}} = \frac{\left(V_P'\right)^2}{R_C} = \frac{(118.8 \text{ V})^2}{441\ \Omega} = 32.0 \text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{800 \text{ W}}{800 \text{ W} + 10.6 \text{ W} + 32.0 \text{ W}} = 94.9\%$$

(2-18, page 138) Three 25-kVA 24,000/277-V distribution transformers are connected in Δ-Y. The open-circuit test was performed on the low-voltage side of this transformer bank, and the following data were recorded:

$$V_{line,OC} = 480 \text{ V} \qquad I_{line,OC} = 4.10 \text{ A} \qquad P_{3\phi,OC} = 945 \text{ W}$$

The short-circuit test was performed on the high-voltage side of this transformer bank, and the following data were recorded:

$$V_{line,SC} = 1400 \text{ V} \qquad I_{line,SC} = 1.80 \text{ A} \qquad P_{3\phi,SC} = 912 \text{ W}$$

(a) Find the per-unit equivalent circuit of this transformer bank.
(b) Find the voltage regulation of this transformer bank at the rated load and 0.90 PF lagging.
(c) What is the transformer bank's efficiency under these conditions?

SOLUTION *(a)* **The equivalent of this transformer bank can be found just like the equivalent circuit of single-phase transformer if we work on a per-phase bases. The open-circuit test data on the low-voltag side can be used to find the excitation branch impedances referred to the secondary side of th transformer bank. Since the secondary is Y-connected, the per-phase open-circuit quantities are:**

$$V_{\phi,OC} = 277 \text{ V} \qquad I_{\phi,OC} = 4.10 \text{ A} \qquad P_{\phi,OC} = 315 \text{ W}$$

The excitation admittance is given by

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{4.10 \text{ A}}{277 \text{ V}} = 0.01483 \text{ mho}$$

The admittance angle is

$$\theta = -\cos^{-1}\left(\frac{P_{\phi,OC}}{V_{\phi,OC}\, I_{\phi,OC}}\right) = -\cos^{-1}\left(\frac{315 \text{ W}}{(277 \text{ V})(4.10 \text{ A})}\right) = -73.9°$$

Therefore,

$$Y_{EX} = G_C - jB_M = 0.01483\angle -73.9° = 0.00411 - j0.01425$$
$$R_C = 1/G_C = 243\,\Omega$$
$$X_M = 1/B_M = 70.2\,\Omega$$

The *base impedance* referred to the secondary side is

$$Z_{\text{base},S} = \frac{(V_{\phi,S})^2}{S_\phi} = \frac{(277 \text{ V})^2}{25 \text{ kVA}} = 3.069\,\Omega$$

so the excitation branch elements can be expressed in per-unit as

$$R_C = \frac{243\,\Omega}{3.069\,\Omega} = 79.2 \text{ pu} \qquad X_M = \frac{70.2\,\Omega}{3.069\,\Omega} = 22.9 \text{ pu}$$

The short-circuit test data can be used to find the series impedances referred to the primary side, since the short-circuit test data was taken on the primary side. Note that the primary is Δ-connected, so $V_{\phi,SC} = V_{SC} = 1400\text{ V}$, $I_{\phi,SC} = I_{SC}/\sqrt{3} = 1.039\text{ A}$, and $P_{\phi,SC} = P_{SC}/3 = 304\text{ W}$.

$$|Z_{EQ}| = \frac{V_{\phi,SC}}{I_{\phi,SC}} = \frac{1400\text{ V}}{1.039\text{ A}} = 1347\ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{\phi,SC}}{V_{\phi,SC}\, I_{\phi,SC}}\right) = \cos^{-1}\left(\frac{304\text{ W}}{(1400\text{ V})(1.039\text{ A})}\right) = 77.9°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1347\angle 77.9° = 282 + j1371\,\Omega$$

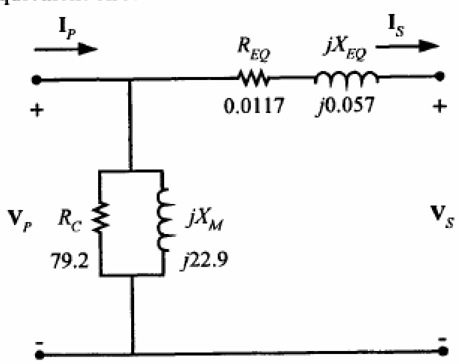
The *base impedance* referred to the secondary side is

$$Z_{\text{base},P} = \frac{(V_{\phi,P})^2}{S_\phi} = \frac{(24{,}000\text{ V})^2}{25\text{ kVA}} = 24{,}040\,\Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{282\,\Omega}{24{,}040\,\Omega} = 0.0117\text{ pu} \qquad X_{EQ} = \frac{1371\,\Omega}{24{,}040\,\Omega} = 0.057\text{ pu}$$

The per-unit, per-phase equivalent circuit of the transformer bank is shown below:



(b) If this transformer is operating at rated load and 0.90 PF lagging, then current flow will be at an angle of $-\cos^{-1}(0.9)$, or $-25.8°$. The voltage at the primary side of the transformer will be

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}_S Z_{EQ} = 1.0\angle 0° + (1.0\angle -25.8°)(0.0117 + j0.057) = 1.037\angle 2.65°$$

The voltage regulation of this transformer bank is

$$\text{VR} = \frac{1.037 - 1.0}{1.0} \times 100\% = 3.7\%$$

(c) The output power of this transformer bank is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (1.0)(1.0)(0.9) = 0.9\text{ pu}$$

The copper losses are

$$P_{\text{CU}} = I_S^{\,2} R_{\text{EQ}} = (1.0)^2(0.0117) = 0.0117\text{ pu}$$

The core losses are

$$P_{\text{core}} = \frac{V_P^{\,2}}{R_C} = \frac{(1.037)^2}{79.2} = 0.014 \text{ pu}$$

Therefore, the total input power to the transformer bank is

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}} = 0.9 + 0.0117 + 0.014 = 0.9257$$

and the efficiency of the transformer bank is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{0.9}{0.9257} \times 100\% = 97.2\%$$