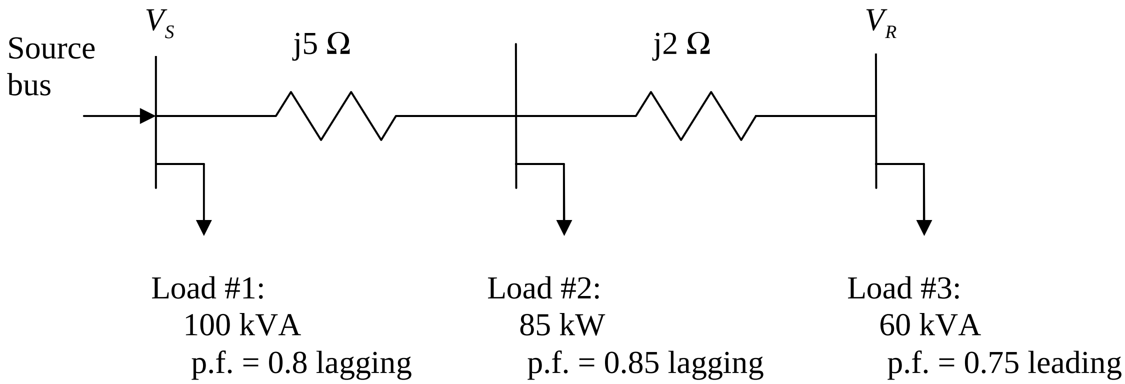
EE341 Homework #2 Solution

Problem 1: Consider a 3-ϕ distribution system as shown below:



Compute the following:

1) The source voltage V_S, if V_R is to be maintained at 2.2 kV.
2) The source current and power factor at the source.
3) The total complex power supplied by the source.
4) How much reactive power should be connected to the source bus for obtaining unity power factor at the source bus?
5) Write Matlab code to solve the above problems. Please submit the Matlab code and the results.

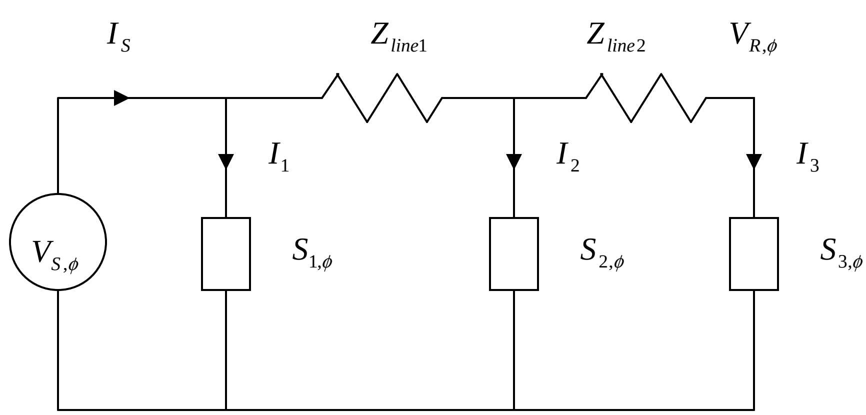
Solution:

a) load #1: $S_{1,\phi} = \dfrac{100}{3}(0.8 + j\sqrt{1-0.8^2}) = 33.33\angle 36.87^o\,kVA$

load #2: $S_{2,\phi} = \dfrac{85/0.85}{3}(0.85 + j\sqrt{1-0.85^2}) = 33.33\angle 31.79^o\,kVA$

load #3: $S_{3,\phi} = \dfrac{60}{3}(0.75 - j\sqrt{1-0.75^2}) = 20\angle -41.41^o\,kVA$

b) Single phase equivalent circuit is shown as below:



$$V_{R,\phi} = \frac{2200}{\sqrt{3}} = 1270.2\angle 0^o V$$

$$I_3 = \left(\frac{S_{3,\phi}}{V_{R,\phi}}\right)^* = \left(\frac{20\times 10^3\angle -41.41^0}{1270.2\angle 0^0}\right)^* = 15.75\angle 41.41^o A$$

$$V_{2,\phi} = V_{R,\phi} + I_3 Z_{line2} = 1270.2\angle 0^0 + 15.75\angle 41.41^0 \times j2 = 1249.6\angle 1.08^0 V$$

$$I_2 = \left(\frac{S_{2,\phi}}{V_{2,\phi}}\right)^* = \left(\frac{33.33\times 10^3\angle 31.79^0}{1249.6\angle 1.08^0}\right)^* = 26.68\angle -30.71^o A$$

$$V_{S,\phi} = V_{2,\phi} + (I_2 + I_3)Z_{line1} = 1249.6\angle 1.08^0 + (26.68\angle -30.71^0 + 15.75\angle 41.41^0)\times j5$$

$$= 1280.67\angle 8.86^0 V$$

1) So, the source voltage is $|V_{S,L-L}| = \sqrt{3}|V_{S,\phi}| = 2218.18V$

2) $I_1 = \left(\frac{S_{1,\phi}}{V_{S,\phi}}\right)^* = \left(\frac{33.33\times 10^3\angle 36.87^0}{1280.67\angle 8.86^0}\right)^* = 26.03\angle -28.01^o A$

The source current is $I_S = I_1 + I_2 + I_3 = 59.75\angle -14.96^0 A$

The power factor is $pf = \cos\theta = \cos(\theta_{VS} - \theta_{IS}) = \cos(8.86^o + 14.96^o) = 0.91$ lagging.

3) $S_{S,\phi} = V_{S,\phi} I_S^* = 1280.67\angle 8.86^0 \times 59.75\angle 14.96^0 = 76.52\angle 23.83^0 kVA$

So the total power supplied by the source is

$$S_{S,3\phi} = 3S_{S,\phi} = 229.57\angle 23.83^0 kVA = 210 + j92.74 kVA$$

4) To make *pf*=1, -j92.74 kVar reactive power should be connected to source bus.

5) Matlab code: hw2_1.m

```
% hw #2, problem 1

S1ph = 100e3 * exp(j*acos(0.8)) / 3;
fprintf('S1ph = %10.2f /_ %6.2f VA\n', abs(S1ph), angle(S1ph)*180/pi);
S2ph = 85e3/0.85 * exp(j*acos(0.85)) / 3;
fprintf('S2ph = %10.2f /_ %6.2f VA\n', abs(S2ph), angle(S2ph)*180/pi);
S3ph = 60e3 * exp(-j*acos(0.75)) / 3;
fprintf('S3ph = %10.2f /_ %6.2f VA\n', abs(S3ph), angle(S3ph)*180/pi);

Zline1 = j*5;
Zline2 = j*2;

VRph = 2.2e3 / sqrt(3);
fprintf('VRph = %10.2f /_ %6.2f V\n', abs(VRph), angle(VRph)*180/pi);
I3 = conj(S3ph / VRph);
fprintf('I3 = %10.2f /_ %6.2f A\n', abs(I3), angle(I3)*180/pi);
V2ph = VRph + I3 * Zline2;
fprintf('V2ph = %10.2f /_ %6.2f V\n', abs(V2ph), angle(V2ph)*180/pi);
I2 = conj(S2ph / V2ph);
fprintf('I2 = %10.2f /_ %6.2f A\n', abs(I2), angle(I2)*180/pi);
VSph = V2ph + (I2+I3) * Zline1;
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```
fprintf('VSph = %10.2f /_ %6.2f V\n', abs(VSph), angle(VSph)*180/pi);
VSLL = sqrt(3) * abs(VSph);
fprintf('|VSLL| = %10.2f V\n', abs(VSLL));

I1 = conj(S1ph / VSph);
fprintf('I1 = %10.2f /_ %6.2f A\n', abs(I1), angle(I1)*180/pi);
IS = I1 + I2 + I3;
fprintf('IS = %10.2f /_ %6.2f A\n', abs(IS), angle(IS)*180/pi);
pf = cos(angle(VSph) - angle(IS));
fprintf('pf = %10.2f lagging\n', pf);

SSph = VSph * conj(IS);
fprintf('SSph = %10.2f /_ %6.2f VA\n', abs(SSph), angle(SSph)*180/pi);
SS3ph = 3 * SSph;
fprintf('SS3ph = %10.2f /_ %6.2f VA\n', abs(SS3ph), angle(SS3ph)*180/pi);
fprintf('SS3ph = (%10.2f) + j*(%10.2f) VA\n', real(SS3ph), imag(SS3ph));

Sc = -imag(SS3ph);
fprintf('Sc = j*(%10.2f) Var\n', Sc);
```

Result:

```
S1ph =   33333.33 /_  36.87 VA
S2ph =   33333.33 /_  31.79 VA
S3ph =   20000.00 /_ -41.41 VA
VRph =    1270.17 /_   0.00 V
I3 =       15.75 /_  41.41 A
V2ph =    1249.56 /_   1.08 V
I2 =       26.68 /_ -30.71 A
VSph =    1280.67 /_   8.86 V
|VSLL| =    2218.18 V
I1 =       26.03 /_ -28.01 A
IS =       59.75 /_ -14.96 A
pf =        0.91 lagging
SSph =   76522.47 /_  23.83 VA
SS3ph =  229567.42 /_  23.83 VA
SS3ph = ( 210000.00) + j*(  92742.65) VA
Sc = j*( -92742.65) Var
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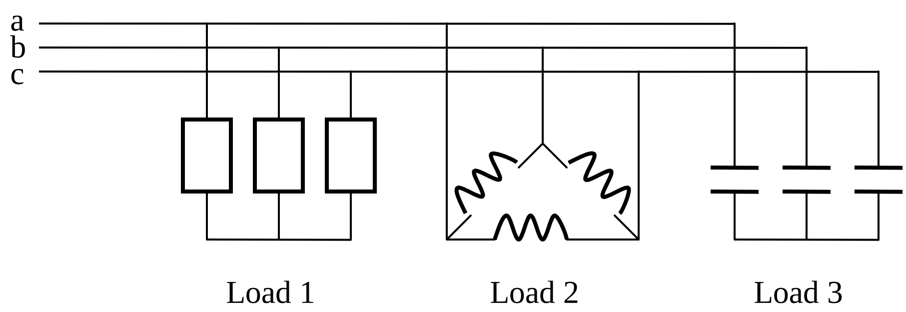
Problem 2: A balanced three-phase, three-wire feeder has three balanced loads as shown:



Load 1 are three lamps, each rated 100 watts at 120 volts. Load 2 are three inductors, each having an inductance of 0.1 Henry. Load 3 are three capacitors, each having a capacitance of 100 μF.

The line-to-line voltage on the feeder is 220 volts, and the frequency is 60 Hz. Find the source current in the feeder lines and the power delivered by the source.

Solution:

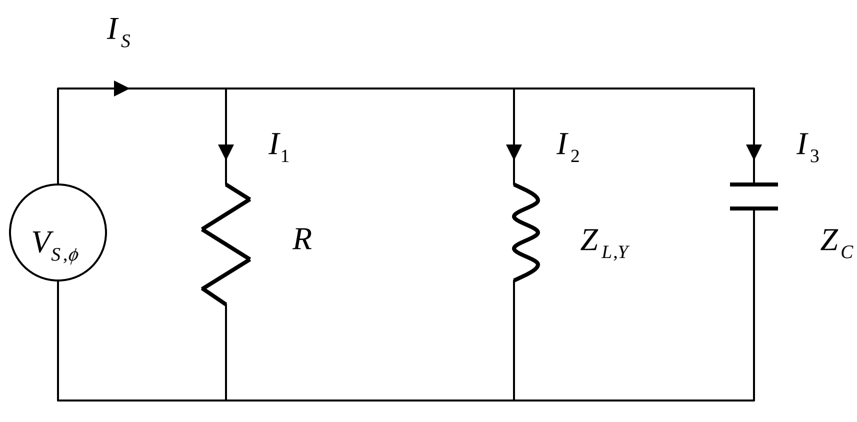
a) load 1: $R = \frac{V_{lamp}^2}{P_{lamp}} = \frac{120^2}{100} = 144\Omega$

load 2: $Z_{L,\Delta} = j\omega L = j2\pi \times 60 \times 0.1 = j37.7\Omega$

$Z_{L,Y} = Z_{L,\Delta} / 3 = j12.57\Omega$

load 3: $Z_C = -j\frac{1}{\omega C} = -j\frac{1}{2\pi \times 60 \times 100e-6} = -j26.53\Omega$

b) The single-phase equivalent circuit is shown as below:



$$V_{S,\phi} = \frac{220}{\sqrt{3}} = 127.02\angle 0^o V$$

c) $I_1 = \frac{V_{S,\phi}}{R} = \frac{127.02\angle 0^o}{144} = 0.8821\angle 0^o A$

$$I_2 = \frac{V_{S,\phi}}{Z_{L,Y}} = \frac{127.02\angle 0^o}{j12.57} = 10.11\angle -90^o\,A$$

$$I_3 = \frac{V_{S,\phi}}{Z_C} = \frac{127.02\angle 0^o}{-j26.53} = 4.788\angle 90^o\,A$$

So the source current is $I_S = I_1 + I_2 + I_3 = 5.392\angle -80.58^0\,A$

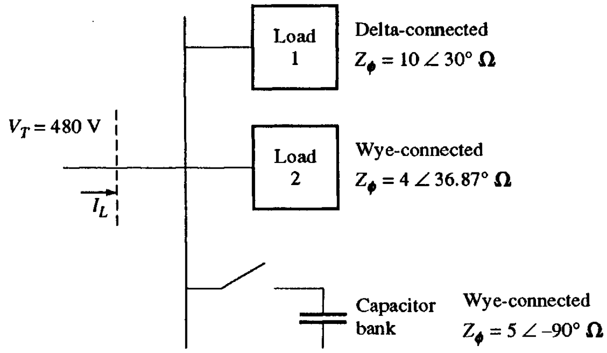
The power delivered by the source is

$$S_{S,3\phi} = 3V_{S,\phi}I_S^* = 3\times 127.02\angle 0^0 \times 5.392\angle 80.58^0 = 336.1 + j2026.9VA$$

$$= 2054.6\angle 80.58^0\,VA$$

<u>Problem 3:</u> Solve problem A-4 (textbook, page 673)

A-4. Figure A-19 shows a small 480-V distribution system. Assume that the lines in the system have zero impedance.



(a) If the switch shown is open, find the real, reactive, and apparent powers in the system. Find the total current supplied to the distribution system by the utility.

(b) Repeat part *(a)* with the switch closed. What happened to the total current supplied? Why?

SOLUTION

(a) With the switch open, the power supplied to each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(480\text{ V})^2}{10\,\Omega}\cos 30° = 59.86\text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(480\text{ V})^2}{10\,\Omega}\sin 30° = 34.56\text{ kVAR}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(277\text{ V})^2}{4\,\Omega}\cos 36.87° = 46.04\text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(277\text{ V})^2}{4\,\Omega}\sin 36.87° = 34.53\text{ kVAR}$$

$$P_{\text{TOT}} = P_1 + P_2 = 59.86\text{ kW} + 46.04\text{ kW} = 105.9\text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 = 34.56\text{ kVAR} + 34.53\text{ kVAR} = 69.09\text{ kVAR}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^2 + Q_{\text{TOT}}^2} = 126.4 \text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{69.09 \text{ kVAR}}{105.9 \text{ kW}}\right] = 0.838 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\, V_T \text{ PF}} = \frac{105.9 \text{ kW}}{\sqrt{3}\,(480 \text{ V})(0.838)} = 152 \text{ A}$$

(b) With the switch closed, P_3 is added to the circuit. The real and reactive power of P_3 is

$$P_3 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(277 \text{ V})^2}{5\,\Omega}\cos(-90°) = 0 \text{ kW}$$

$$P_3 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(277 \text{ V})^2}{5\,\Omega}\sin(-90°) = -46.04 \text{ kVAR}$$

$$P_{\text{TOT}} = P_1 + P_2 + P_3 = 59.86 \text{ kW} + 46.04 \text{ kW} + 0 \text{ kW} = 105.9 \text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 - Q_3 = 34.56 \text{ kVAR} + 34.53 \text{ kVAR} - 46.06 \text{ kVAR} = 23.03 \text{ kVAR}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^2 + Q_{\text{TOT}}^2} = 108.4 \text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{23.03 \text{ kVAR}}{105.9 \text{ kW}}\right] = 0.977 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\, V_T \text{ PF}} = \frac{105.9 \text{ kW}}{\sqrt{3}\,(480 \text{ V})(0.977)} = 130.4 \text{ A}$$