

SET	$\bar{V}_1$	$\bar{V}_2'$	$\bar{I}_1$	$\bar{I}_2'$	$\bar{I}_\varphi$
1	--	479.3∠0.0026°	0.166∠-73.2°	0	0.166∠-73.2°
2	--	474.3∠-2.14°	2.43∠-5.85°	2.37∠-2.14°	0.165∠-74.3°
3	--	1∠0°	0	0	0
4	--	0.7∠-44.4°	0.7∠-44.4°	0.7∠-44.4°	0
5	1.85∠21.9°	--	1∠-45°	1∠-45°	0
6	--	0.989∠-2.27°	1∠-3.76°	0.989∠-2.27°	0.0283∠-69.3°
7	--	0.983∠-0.6°	0.517∠-46.9°	0.491∠-45.6°	0.0283∠-69.3°
8	--	115.1∠-3.87°	18.29∠-33.4°	15.65∠-3.87°	9.03∠-92.3°
9	5400∠-2.1°	--	20∠-37°	20∠-37°	0
10	--	2330∠-0.83°	20.8∠-37.7°	20.8∠-37.7°	0
11	--	230∠-47.1°	259.4∠-44.6°	23∠-47.1°	236.4∠-44.4°
12	--	0	496∠-75.1°	0	496∠-75.1°

## HOMEWORK #1

### SOLUTIONS

- CASE ① -

- NO LOAD -

$$\bar{Z}_1 = R_1 + j\omega L_1 = 1 + j3.77 = 3.9 \angle 75.14^\circ \Omega$$

$$\bar{Z}_\beta = R_\beta // j\omega L_m = 10000 // j3016$$

$$\bar{Z}_\phi = 2,887.5 \angle 73.22^\circ \Omega$$

$$\bar{V}_2 = 480 \times \frac{2887.5 \angle 73.22^\circ}{(1 + j3.77) + (833.61 + j2764.5)}$$

$$\bar{V}_2 = \underline{479.3 \angle -0.0026^\circ \text{ V}}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_{eq}} = \frac{480}{2891.37 \angle 73.22^\circ} = \underline{0.166 \angle -73.2^\circ \text{ A}}$$

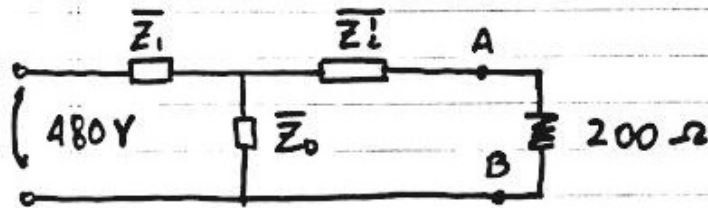
$$\bar{Z}_{eq} = \bar{Z}_1 + \bar{Z}_\phi$$

$$\bar{I}_1 = \bar{I}_\beta \quad \text{since } \bar{I}_2 = \phi$$

CASE (2)

$$R_L' = 200 \Omega \quad X_L' = \emptyset$$

$$\bar{Z}_1 = 3.9 \angle 75.14^\circ = \bar{Z}_2$$



$$\bar{Z}_{eq} = \bar{Z}_1 + \bar{Z}_0 \parallel \bar{Z}_2 \quad (\text{UP TO AB})$$

$$\bar{V}_2' = 480 \times \frac{200}{200 + \bar{Z}_{eq}} = \underline{474.9 \angle -2.13^\circ} \text{ V}$$

$$\bar{I}_2' = \frac{\bar{V}_2'}{R_2} = \frac{474.9 \angle -2.13^\circ}{200} = \underline{2.37 \angle -2.13^\circ} \text{ A}$$

$$\bar{I}_1 = \frac{480}{197.77 \angle 5.85^\circ} = \underline{2.427 \angle -5.85^\circ} \text{ A}$$

$$\bar{Z}_{eq} = \bar{Z}_1 + \bar{Z}_0 \parallel \bar{Z}_2 = 197.77 \angle 5.85^\circ$$

$$\bar{I}_0 = \bar{I}_1 - \bar{I}_2 = \underline{0.165 \angle -74^\circ} \text{ A}$$

CASE (4)

$\bar{Z}_0 \rightarrow$  OPEN CIRCUIT

$$R_1 = 0.02 + j0.99 \Omega$$

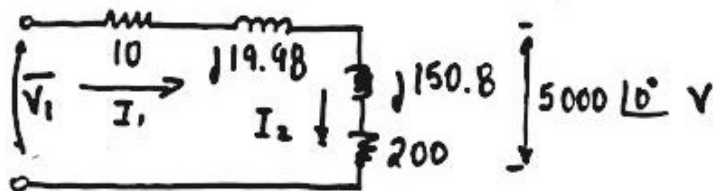
$$R'_2 = 1 \Omega$$

$$\bar{V}_2 = \frac{1 \angle 0^\circ \times 1}{1.427 \angle 44.4^\circ} = \underline{0.7 \angle 44.4^\circ} \text{ V}$$

$$\bar{I}_1 = \frac{1}{1.427 \angle 44.4^\circ} = \underline{0.7 \angle 44.4^\circ} \text{ A}$$

$$\bar{I}_1 = \bar{I}_2$$

CASE (9)



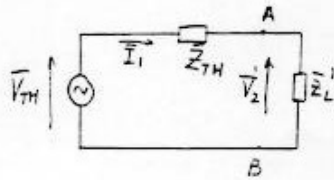
$$\bar{I}_2 = \frac{5000 \angle 0^\circ}{Z_L} = \underline{20 \angle -37.01^\circ} \text{ A} = \bar{I}_1$$

$$\bar{V}_1 = \bar{I} (Z_1 + Z_L) = \underline{5403 \angle 2.1^\circ} \text{ V}$$

(2)

(21)

2. THE THEVENIN EQUIVALENT CIRCUIT IS:



$$\bar{Z}_L' = R_L' + j\omega L_L'$$

$$V_{TH} = I_{\phi}(Z_{\phi})$$

$$\bar{V}_{TH} = \frac{R_f // j\omega L_m}{R_f // j\omega L_m + R_i + j\omega L_i} \bar{V}_i$$

$$Z_{\phi} = R_f // j\omega L$$

$$\bar{Z}_{TH} = R_L' + j\omega L_L' + (R_f // j\omega L_m) // (R_i + j\omega L_i)$$

SET	$\bar{V}_{TH}$	$\bar{Z}_{TH}$	$\bar{V}_2'$	$R_{L \text{ MAX. P}}$
1 1	$479.4 \angle 0^\circ$	$7.79 \angle 75.14^\circ$	$479.3 \angle 0.0026^\circ$	$7.79 \Omega$
2 2	$479.4 \angle 0^\circ$	$7.79 \angle 75.14^\circ$	$474.3 \angle -2.14^\circ$	$7.79 \Omega$
3 4	$1.0 \angle 0^\circ$	$1 \angle 88.9^\circ$	$0.7 \angle -44.4^\circ$	$1 \Omega$
4 4	$5400 \angle 2.1^\circ$	$22.34 \angle 63.41^\circ$	$5000 \angle 0^\circ$	$22.34 \Omega$

$$\text{SET 1: } R_i = 1 \Omega, \omega L_i = 3.77 \Omega, R_f = 10 \Omega, \omega L_m = 3016 \Omega$$

$$R_L' = 1 \Omega, \omega L_L' = 3.77 \Omega, R_L' = \infty, \omega L_L' = \infty$$

$$R_f // j\omega L_m = \frac{3.016 \times 10^3 \angle 90^\circ}{1.0445 \times 10^9 \angle 16.78^\circ} = 2887.5 \angle 73.22^\circ \Omega$$

$$= 833.6 + j2762.6 \Omega$$

$$\bar{V}_{TH} = \frac{2887.5 \angle 73.22^\circ}{834.6 + j2768.37} \times 480 \angle 0^\circ$$

$$= \frac{2887.5 \angle 73.22^\circ}{28424 \angle 73.22^\circ} \times 480 \angle 0^\circ \cong 479.4 \angle 0^\circ \text{ V}$$

$$\bar{Z}_{TH} = 1 + j3.77 + \frac{2887.5 \angle 73.22^\circ \times 3.4 \angle 75.14^\circ}{2891.4 \angle 73.22^\circ}$$

$$\cong 1 + j3.77 + 3.89 \angle 75.14^\circ = 1 + j3.77 + 0.998j3.76$$

$$= 1.998 + j7.53 = 7.79 \angle 75.14^\circ \Omega$$

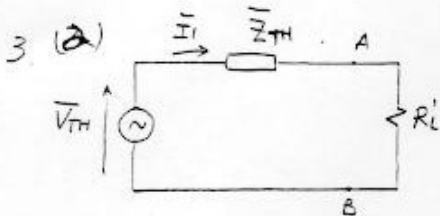
SET 2: SAME AS FOR SET 1, BUT DIFFERENT  $\bar{Z}_L$  (3)

SET 4:  $R_1 = 0.02 \Omega$ ,  $\omega L_1 = 1 \Omega$ ,  $R_f = \infty$ ,  $\omega L_m = \infty$ ,  $R_2' = 0$ ,  
 $\omega L_2' = 0$ ,  $R_L' = 1 \Omega$ ,  $\omega L_L' = 0$

$$\bar{V}_{TH} = \bar{V}_1 = 1.0 \angle 0^\circ \text{ V}; \quad \bar{Z}_{TH} = 0.02 + j1 \approx 1 \angle 88.4^\circ \Omega$$

SET 9:  $R_1 = 10 \Omega$ ,  $\omega L_1 = 19.98 \Omega$ ,  $R_f = \infty$ ,  $\omega L_m = \infty$ ,  $R_2' = 0$ ,  
 $\omega L_2' = 0$ ,  $R_L' = 200 \Omega$ ,  $\omega L_L' = 150.8 \Omega$

$$\bar{V}_{TH} = \bar{V}_1 = 5400 \angle 2.1^\circ; \quad \bar{Z}_{TH} = 10 + j19.98 = 22.34 \angle 63.4^\circ \Omega$$



POWER DELIVERED TO  $R_L'$  IS

$$P_L = I_1^2 R_L'$$

$$\text{AND } \bar{I}_1 = \frac{\bar{V}_{TH}}{\bar{Z}_{TH} + R_L'} = \frac{\bar{V}_{TH}}{(R_{TH} + R_L') + jX_{TH}}$$

$$\therefore P_L = \frac{V_{TH}^2 R_L'}{(R_{TH} + R_L')^2 + X_{TH}^2}; \quad P_L \text{ IS MAX WHEN } \frac{dP_L}{dR_L'} = 0$$

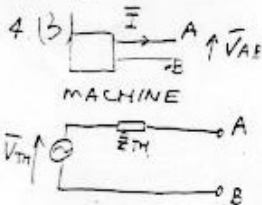
$$\frac{dP_L}{dR_L'} = \frac{V_{TH}^2 [(R_{TH} + R_L')^2 + X_{TH}^2] - V_{TH}^2 R_L' 2(R_{TH} + R_L')}{[(R_{TH} + R_L')^2 + X_{TH}^2]^2} = 0$$

$$\text{OR: } (R_{TH} + R_L')^2 + X_{TH}^2 = 2R_L'(R_{TH} + R_L')$$

$$\text{OR: } R_{TH}^2 + 2R_{TH}R_L' + (R_L')^2 + X_{TH}^2 = 2R_{TH}R_L' + 2(R_L')^2$$

$$\text{OR: } (R_L')^2 = R_{TH}^2 + X_{TH}^2 = Z_{TH}^2; \quad \underline{R_L' = Z_{TH}}$$

FOR EACH SET OF PARAMETERS, POWER DELIVERED TO  $R_L'$  IS MAXIMUM WHEN  $R_L'$  IS EQUAL TO THE MAGNITUDE OF THE THEVENIN EQUIVALENT IMPEDANCE  $\bar{Z}_{TH}$  SEEN BY  $R_L'$



$\bar{V}_{TH} = \bar{V}_{AB}$  WHEN TERMINALS AB ARE OPEN-CIRCUITED

$\bar{Z}_{TH} = \frac{\bar{V}_{TH}}{\bar{I}_{SC}}$  WHERE  $\bar{I}_{SC} = \bar{I}$  WHEN TERMINAL AB ARE SHORTED

$$\therefore \bar{V}_{TH} = 100 \angle 0^\circ \text{ V}; \quad \bar{Z}_{TH} = \frac{100}{\frac{1}{\sqrt{2}}} \angle 90^\circ = 2 \angle 90^\circ \Omega$$

## Matlab Code for case 11

```
clear all
w = 377;
R1 = .3;
L1 = .003;
Rf = 1;
Lm = .0425;
R2 = .2;
L2 = .003;
RL = 10;
LL = 0;
V1 = 440;

Z1 = R1 + i * w * L1;
Z2 = R2 + i * w * L2;
ZL = RL + i * w * LL;
Xm = i * w * Lm;
Zf = Rf * Xm / (Rf + Xm);
Z2L = Z2 + ZL;
Z2Lf = Z2L * Zf / (Z2L + Zf);

I1 = V1 / (Z1 + Z2Lf);
I2 = (V1 - I1 * Z1) / Z2L;
V2 = I2 * ZL;

If = (V1 - I1 * Z1) / Rf;

fprintf('I1 = %6.2f /_ %6.2f\n', abs(I1), angle(I1)*180/pi);
fprintf('I2 = %6.2f /_ %6.2f\n', abs(I2), angle(I2)*180/pi);
fprintf('V2 = %6.2f /_ %6.2f\n', abs(V2), angle(V2)*180/pi);
fprintf('If = %6.2f /_ %6.2f\n', abs(If), angle(If)*180/pi);

% Results:
%-----
% I1 = 259.35 /_ -44.61
% I2 = 22.99 /_ -47.13
% V2 = 229.89 /_ -47.13
% If = 235.93 /_ -40.80
```