OPAMPS – Problems

(1)

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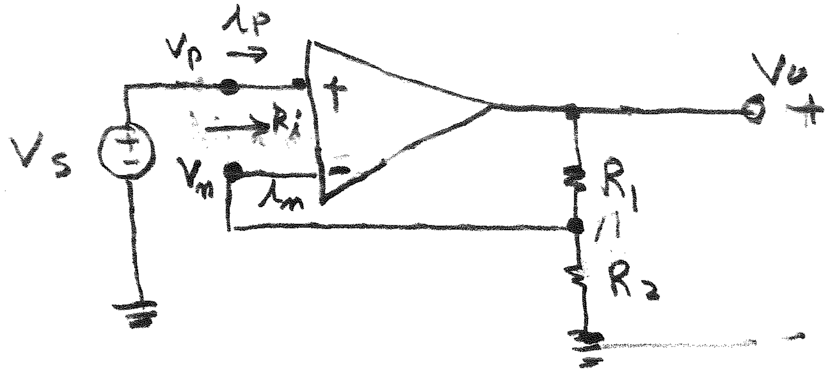
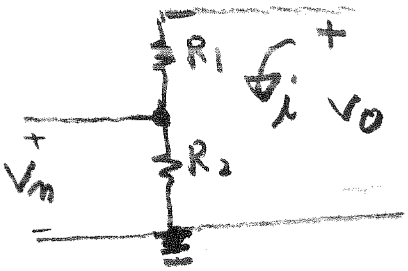
problem 1. Consider the noninverting op amp with $R_1 = 3\,K\Omega$, $R_2 = 2\,K\Omega$ given below:



Find V_o if $V_s = 10$ V

$R_i \to \infty$ (open ckt) $\Rightarrow i_p = 0$, $i_n = 0$

Eq. ckt.



$V_o = i(R_1 + R_2)$

$i = \dfrac{V_o}{R_1 + R_2}$

$$V_n = i R_2 = \frac{R_2}{R_1 + R_2} V_o$$

since $R_i \Rightarrow \infty \Rightarrow$ open ckt $V_p = V_s$ (noninverting condition) and since we have an ideal op amp, we have $V_p = V_n$

$$\therefore \quad V_s = \frac{R_2}{R_1 + R_2} V_o \quad \Longrightarrow$$

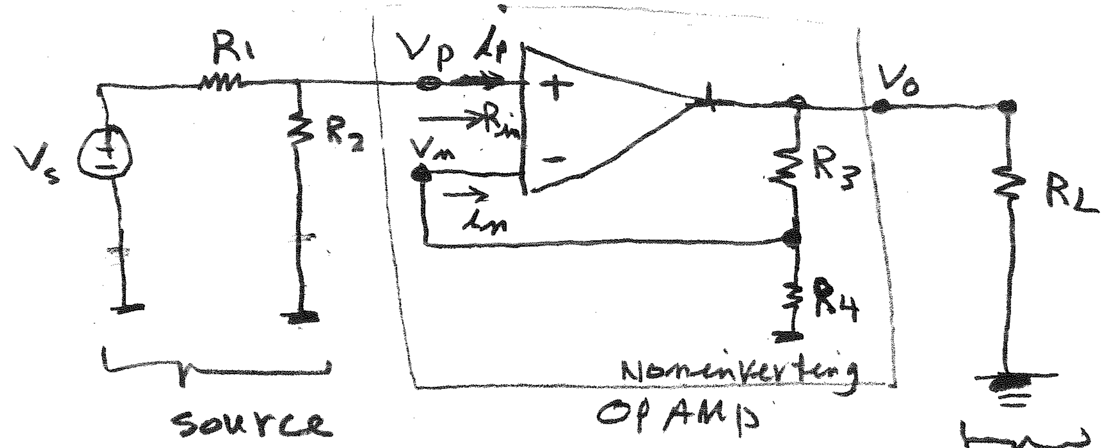
$$V_o = \frac{R_1 + R_2}{R_2} V_s \qquad V_o = 2.5$$

For $R_1 = 3\,K\Omega$, $R_2 = 2\,K\Omega$ $\quad V_s = 10$

$$\therefore \quad V_o = \left(\frac{3+2}{2}\right) V_s = 2.5\, V_s$$

$V_o = 25$ volts

problems. Consider the noninverting op AMP given below.



Assume: source $\begin{cases} V_s = 100 \text{ V}, & R_1 = 2K\Omega \\ R_2 = 3 K\Omega \end{cases}$

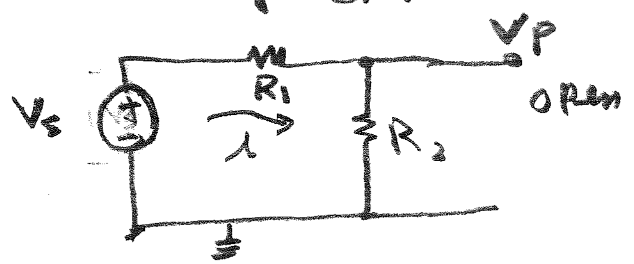
OP AMP $\{ R_3 = 4K\Omega$, $R_4 = 2 K\Omega$

Find Vo if $R_L = 1 \ K\Omega$

solution.

$R_i \Rightarrow \infty \quad i_p = 0, \ i_n = 0$ open ckt

source eq. ckt



$V_s = i(R_1 + R_2) \qquad i = \frac{V_s}{R_1 + R_2}$

$V_p = R_2 i = \underbrace{\left(\frac{R_2}{R_1 + R_2}\right)}_{K_s} V_s \qquad \boxed{V_p = K_s V_s}$

$K_1 = \frac{3}{5} = \frac{6}{10} = 0.6$

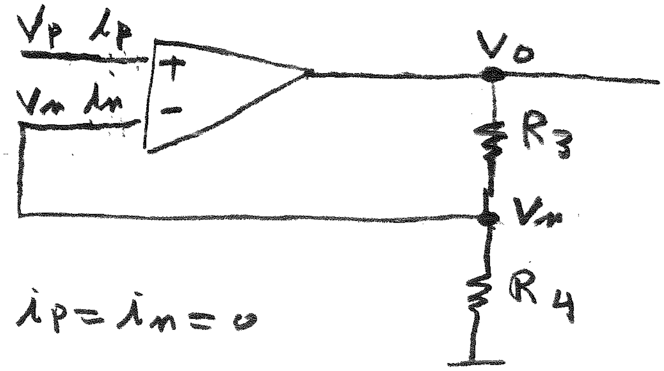
$V_p = 0.6 \ (100)$

$V_p = 60$ Volt

Attenuation.

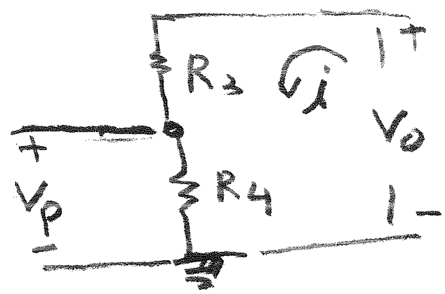
since the noninverting op AMP has <u>Zero</u> output resistance $R_o \Longrightarrow$ very small $R_o \Longrightarrow 0$

The op AMP equivalent ckt is



Recall $i_p = i_n = 0$

$V_p = V_n$



$V_o = i\,(R_3 + R_4)$

$i = \dfrac{V_o}{R_3 + R_4}$

$V_p = i\,R_4 \quad \Longrightarrow \quad V_p = \left(\dfrac{R_4}{R_3 + R_4}\right) V_o$

or.

$$V_o = \underbrace{\left(\frac{R_3 + R_4}{R_4}\right)}_{K_{AMP}} V_p \qquad V_o = K_{AMP} V_p$$

for $R_3 = 4$, $R_4 = 2$

$$K_{AMP} = \frac{R_3 + R_4}{R_4} = \frac{4+2}{2} = \frac{6}{2} = 3$$

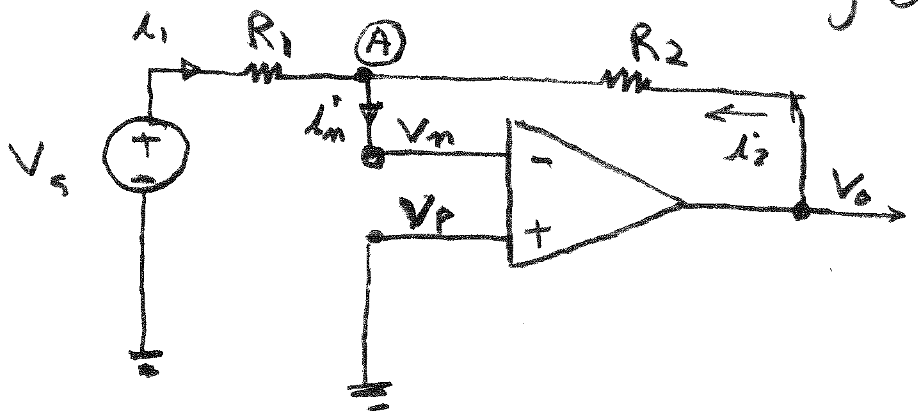
$$V_o = 3\,(60) = 180\ V$$

The overal ckt gain is:

$$K_{ckt} = \frac{V_o}{V_s} = K_s \times K_{AMP} = 0.6\,(3) = 1.8$$

$V_o = K_{ckt} V_s \qquad V_o = 1.8 \times 100 = 180\ V$

Problem 3. Given the inverting op Amp below:



Assume $R_1 = 10\ k\Omega$ $R_2 = 20\ k\Omega$

and $V_s = 120V$ find V_o.

Solution.

In this op-Amp V_p is Grounded, therefore

$$V_p = 0$$

Node A is at V_n $V_A = V_n$. write $\sum i_i = 0$ for node A. We will have:

$$\frac{V_s - V_n}{R_1} + \frac{V_o - V_n}{R_2} - i_n = 0 \quad *$$

Recall op Amp constraints:

$$V_p = V_n \text{ and } i_p = i_n = 0$$

However, $V_p = 0 \Rightarrow V_n = 0$ substituting the above in $*$, we will have:

$$\frac{V_s}{R_1} + \frac{V_o}{R_2} = 0$$

Solution problem 3 cont.

$$\frac{V_s}{R_1} = -\frac{V_o}{R_2}$$

$$V_o = -\frac{R_2}{R_1} V_s$$ where $K_v = \frac{R_2}{R_1}$ is

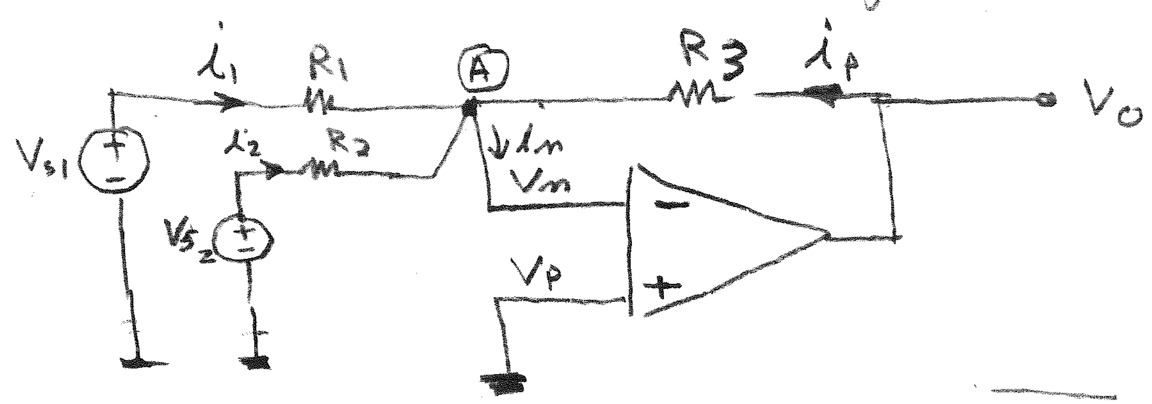
the closed-loop gain. since the voltage gain is negative, it indicates a signal inversion.

For $R_1 = 10\ K\Omega$ $R_2 = 20\ K\Omega$, $V_s = 120$

we will have.

$$V_o = -\left(\frac{20}{10}\right)(120) = -60$$

Problem 4. Given the summing op AMPS below



Given $V_{s1} = 100\ V$, $V_{s2} = 120 V$, $R_1 = 10 K\Omega$

$R_2 = 20 K\Omega$ and $R_3 = 100 K\Omega$

Find V_o

Solution. Write $\sum i_i = 0$ for node A.

$$(*) \quad \frac{V_{s1} - V_A}{R_1} + \frac{V_{s2} - V_A}{R_2} + \frac{V_o - V_A}{R_3} - i_n = 0$$

Node A is connected to V_n. $\Rightarrow$ $V_A = V_n$

Also, the noninverting input is grounded.

$\Rightarrow$ $V_p = 0$ op amp constraints are

$V_p = V_n = 0$ and $i_n = 0$

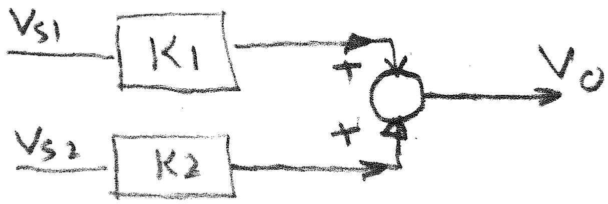
We can rewrite * as

$$\frac{V_{s1}}{R_1} + \frac{V_{s2}}{R_2} + \frac{V_o}{R_3} = 0$$

$$\Rightarrow \quad V_o = \underbrace{\left(-\frac{R_3}{R_1}\right)}_{K_1} V_{s1} + \underbrace{\left(-\frac{R_3}{R_2}\right)}_{K_2} V_{s2}$$

Solution Problem 4. cont.

$$V_o = K_1 v_{s1} + K_2 v_{s2}$$

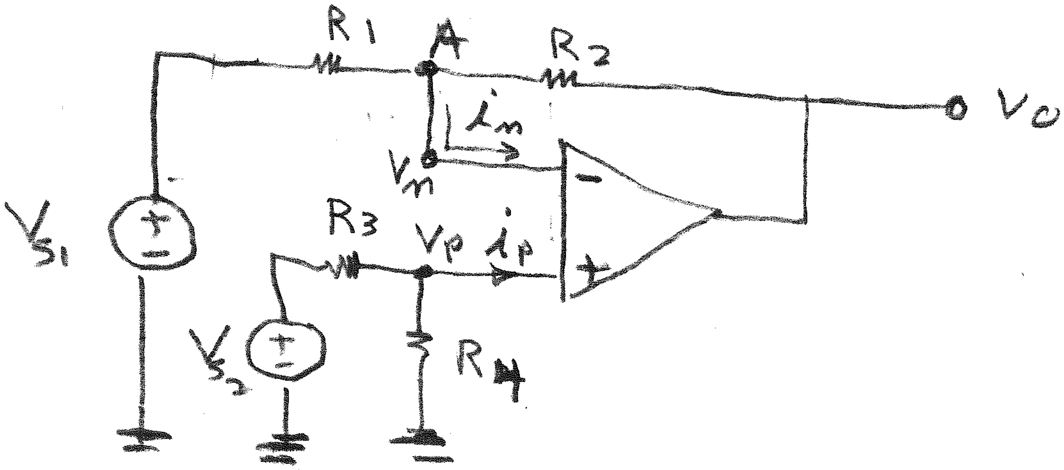


$$K_1 = \frac{100}{10} = 10 \qquad K_2 = \frac{100}{20} = 5$$

$$v_o = -10(100) - 5(120)$$

$$= -1000 - 600 = -1600 \text{ V}$$

Problem 5. Consider the differential OpAmp given below:



Assume $R_1 = 10\,K\Omega$ $R_2 = 20 K\Omega$, $R_3 = 30 k\Omega$ $R_4 = 40\,K\Omega$, $V_{S1} = 10V$ $V_{S2} = 20V$

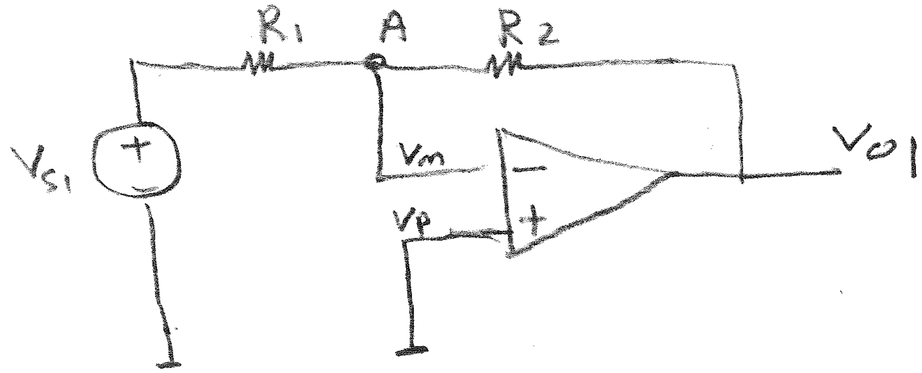
Find Vo.

Solution. The op-amp above is called a differential or subtractor. This op-Amp has two inputs, one applied at the inverting input and one at the noninverting input.

Use superpositon. Apply V_{S1} while V_{S2} is off. This means there is no excitation at noninverting input. $\Rightarrow V_p = 0$

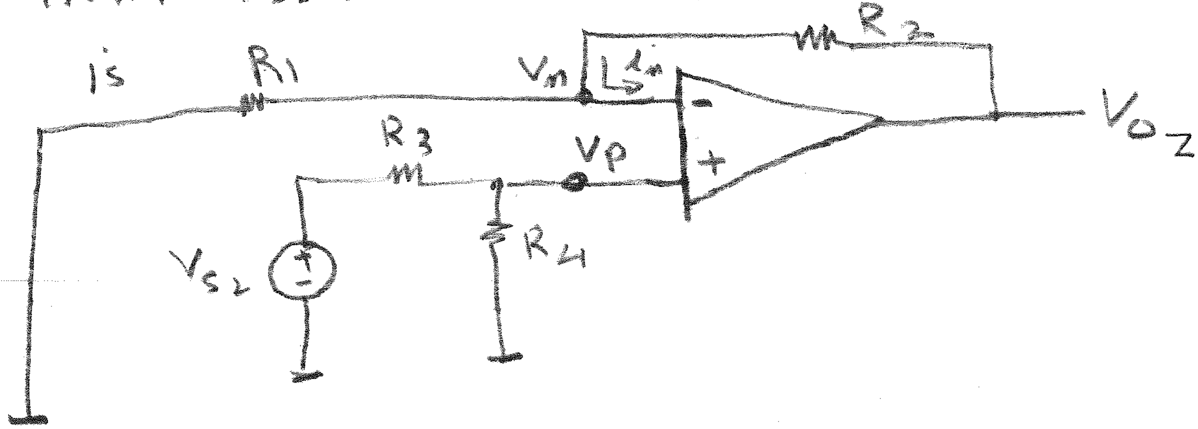
Problem 5 solution continued.

For this case the ckt is



$$V_{o1} = -\frac{R_2}{R_1} V_{s1}$$

Next V_{s2} is on and V_{s1} is off. The eq. ckt is



For this eq. ckt, we have:

$$V_{o2} = \left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right) V_2$$

Addup v_{o1} and v_{o2} using supperposition. principle

$$v_o = v_{o1} + v_{o2}$$

Problem 5 solution cont.

$$v_o = -\underbrace{\frac{R_2}{R_1}}_{K_1} v_{s1} + \underbrace{\left(\frac{R_4}{R_3+R_4}\right)\left(\frac{R_1+R_2}{R_1}\right)}_{K_2} v_{s2}$$

$$v_o = -K_1 v_{s1} + K_2 v_2$$

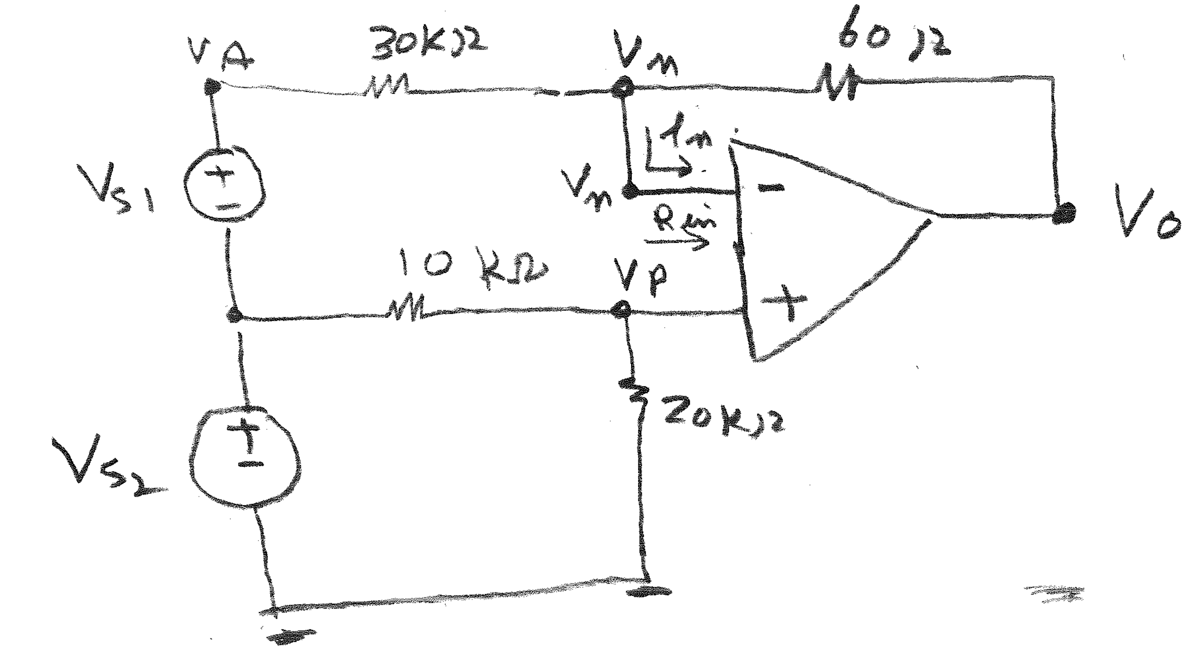
$$K_1 = \frac{20}{10} = 2 \qquad K_2 = \frac{40}{30+40} + \frac{10+20}{10}$$

$$K_2 = \frac{4}{7} + 3 = 3.57$$

$$v_o = -2(10) + 3.57(20)$$

$$= -20 + 71.4 = 51.4 \text{ V}$$

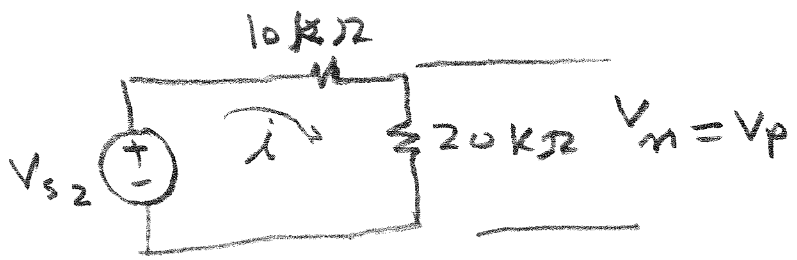
Problem 6. Consider the op amp below:



Find Vo in terms of V_{s1} and V_{s2}.

Solution. $R_{in} \longrightarrow \infty \quad V_p = V_n \quad i_p = i_n = 0$

Eq. ckt is

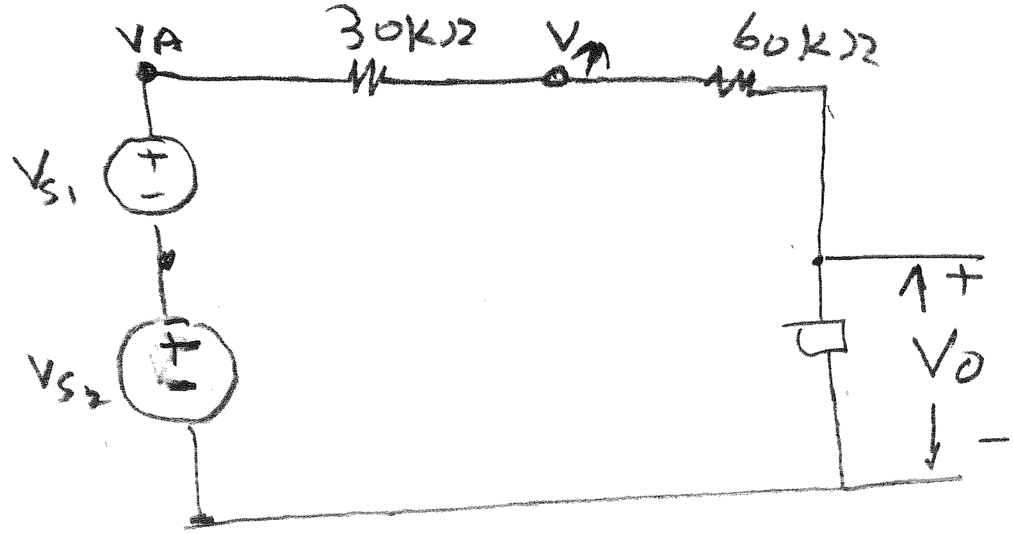


$V_s = i(10+20) = 30\,i \qquad i = \dfrac{V_s}{30}$

$V_p = V_n = i\,(20k\Omega) = \dfrac{20}{30} V_{s_2} = \dfrac{2}{3} V_{s2}$

problem6. solution cont.

Now, we know V_n. To calculate V_o, we have:



$V_A = V_{S1} + V_{S2}$

$\sum i_i = 0$ at node n

$$\frac{V_n - V_A}{30K\Omega} + \frac{V_n - V_o}{60K\Omega} = 0$$

or

$$2(V_n - V_A) + (V_n - V_o) = 0$$

$$2\left[\frac{2}{3}V_{S2} - (V_{S1} + V_{S2})\right] + \left(\frac{2}{3}V_{S2} - V_o\right) = 0$$

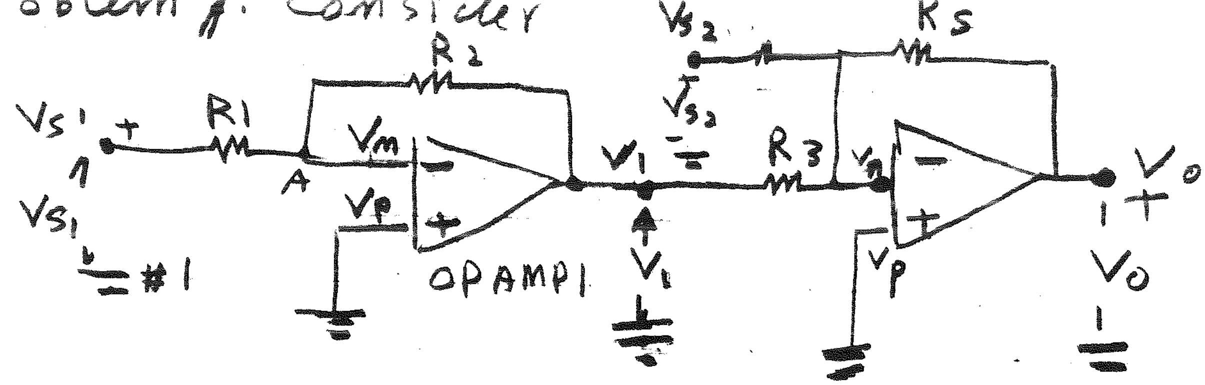
$$\frac{4}{3}V_{S2} - 2V_{S1} - 2V_{S2} + \frac{2}{3}V_{S2} - V_o = 0$$

$$V_o = -2V_{S1}$$

$$\frac{V_o}{V_{S1}} = -2$$

OP-AMP Problems.

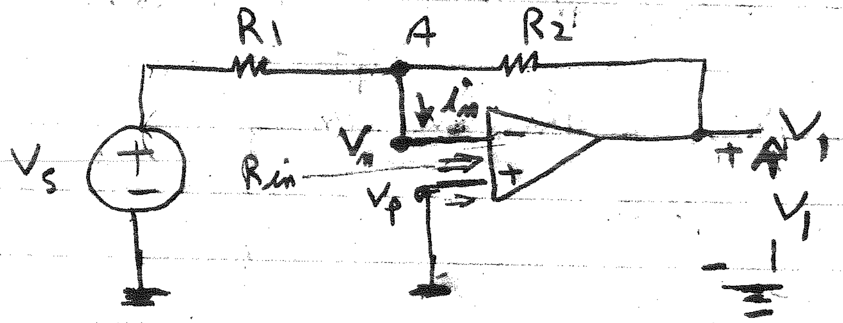
Problem 7. Consider



Find vo in terms of the input V_{s1} and V_{s2}

Solution: Here, we have two Op-Amps. First, we calculate the voltage V_1 in term of V_{s1}.

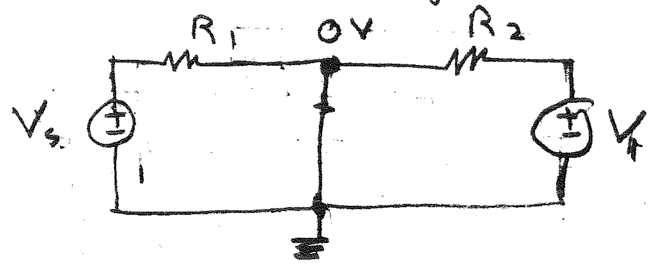
~~For~~ op-amp 1) operating in the inverting mode, we will have



Σi at node A. Note node A is at V_n voltage

$$\frac{V_s - V_n}{R_1} + \frac{V_1 - V_n}{R_2} - i_n = 0$$

However, R_{in} is very high $i_p = i_n \approx 0$ and $V_n = V_p = 0$

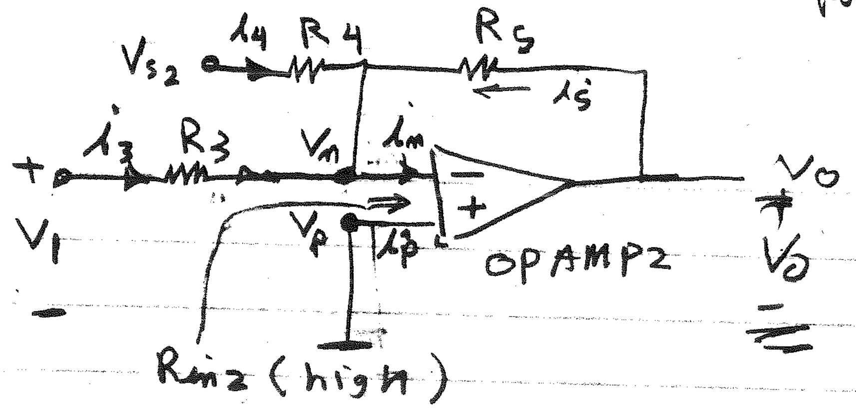


$$\frac{V_s}{R_1} + \frac{V_1}{R_2} = 0$$

Problem 7 solution cont.

$$V_1 = -\frac{R_1}{R_2} V_s$$

For opAMP2, we will have the following:



Again opAMP2 has a very high ~~input~~ resistance,

$i_p = i_n = 0 \qquad V_p = V_n = 0$

$$\frac{V_1 - \cancel{V_n}^{\,0}}{R_3} + \frac{V_{s2} - \cancel{V_n}^{\,0}}{R_4} + \frac{V_o - \cancel{V_n}^{\,0}}{R_5} = 0$$

note that $V_n = 0$, therefore, we have:

$$\frac{V_1}{R_3} + \frac{V_{s2}}{R_4} + \frac{V_o}{R_5} = 0$$

multeply the above by R_5 and then rewrite the above as

$$V_o = -\frac{R_5}{R_3} V_1 - \frac{R_5}{R_4} V_{s2}$$

Recall that $V_1 = -\frac{R_1}{R_2} V_s$, therefore, we will have the following:

Problem 7 solution cont.

$$V_o = -\frac{R_5}{R_3}\left(-\frac{R_2}{R_1} V_{s1}\right) - \frac{R_5}{R_4} V_{s2}$$

$$\boxed{V_o = \frac{R_2 R_5}{R_1 R_3} V_{s1} - \frac{R_5}{R_4} V_{s2}}$$

$$V_o = K_1 V_{s1} - K_2 V_{s2}$$

where $K_1 = \frac{R_2 R_5}{R_1 R_3}$ and

$$K_2 = \frac{R_5}{R_4}.$$

By selecting R_1, R_2, R_3, R_5 and R_5, you can obtain the desired output voltage.