COMPLEX POWER

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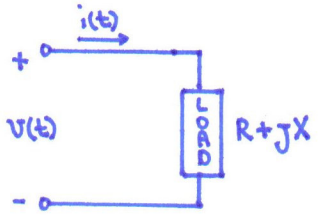
ECE 205 - SP09



$v(t) = V_A \cos(\omega t + \phi_v)$ $\qquad V_{rms} = V_A/\sqrt{2}$

$i(t) = I_A \cos(\omega t + \phi_I)$ $\qquad I_{rms} = I_A/\sqrt{2}$

$$\mathbf{V} = V_{rms} \angle \phi_v$$

$$\mathbf{I} = I_{rms} \angle \phi_I$$

$\theta = \phi_v - \phi_I$ POWER FACTOR ANGLE

$\boxed{P} = \frac{V_A I_A}{2} \cos\theta = \boxed{V_{rms} I_{rms} \cos\theta}$ [W] AVERAGE POWER

$\boxed{Q} = \frac{V_A I_A}{2} \sin\theta = \boxed{V_{rms} I_{rms} \sin\theta}$ [VARs] REACTIVE POWER

$p(t) = v(t) \cdot i(t) = \underbrace{P(1+\cos 2\omega t)}_{\text{NET ENERGY} \neq 0} - \underbrace{Q \sin(2\omega t)}_{\text{TRANSFERS NO ENERGY}}$ INSTANTANEOUS POWER

$W > 0$ if $P > 0$ $\Rightarrow$ $-90° < \theta < 90°$ ENERGY FLOW SOURCE → LOAD

$W < 0$ if $P < 0$ $\Rightarrow$ $90° < \theta < 270°$ ENERGY FLOW LOAD → SOURCE

$\boxed{S} = VI^* = \boxed{P + jQ}$ COMPLEX POWER

$= Z|I|^2 = (R+jX) I_{rms}^2$

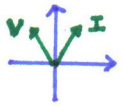
$R = \frac{P}{I_{rms}^2}$ $\qquad X = \frac{Q}{I_{rms}^2}$

$|S| = V_{rms} I_{rms}$ [VA] APPARENT POWER

$pf = \cos\theta$ POWER FACTOR

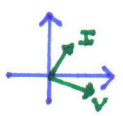
$\theta > 0$ $\Rightarrow$ $\phi_v > \phi_I$ VOLTAGE LEADS $\Rightarrow$ lagging power factor

$Q > 0$, $X > 0$ INDUCTIVE LOAD

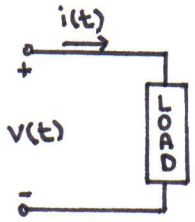


$\theta < 0$ $\Rightarrow$ $\phi_I > \phi_v$ CURRENT LEADS $\Rightarrow$ leading power factor

$Q < 0$, $X < 0$ CAPACITIVE LOAD



EXAMPLE 1



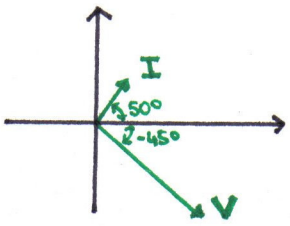
Given $i(t)$ and $v(t)$ calculate the <u>average power</u> and the <u>reactive power</u>. Is the load absorbing or delivering net energy? net energy?

(a) $v(t) = 1500 \cos(\omega t - 45°)$ V

$i(t) = 2 \cos(\omega t + 50°)$ A

$V_A = 1500 \quad \phi_v = -45° \quad V_{rms} = 1500/\sqrt{2}$

$I_A = 2 \quad \phi_I = 50° \quad I_{rms} = 2/\sqrt{2}$



CURRENT LEADS → CAPACITIVE LOAD → $Q < 0$

$\theta = \phi_v - \phi_I = -45° - 50° = -95°$

$\boxed{P} = V_{rms} I_{rms} \cos\theta = \frac{1500}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \cos(-95°) = 1500(-0.087) = \boxed{-130.7 \text{ W}}$

$\boxed{Q} = V_{rms} I_{rms} \sin\theta = \frac{1500}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \sin(-95°) = 1500(-0.996) = \boxed{-1494.3 \text{ VARs}}$

Since $P < 0 \Rightarrow W < 0 \Rightarrow$ the load is <u>delivering power</u>

(b) $v(t) = 90 \cos(\omega t + 60°)$

$i(t) = 10.5 \cos(\omega t - 20°)$

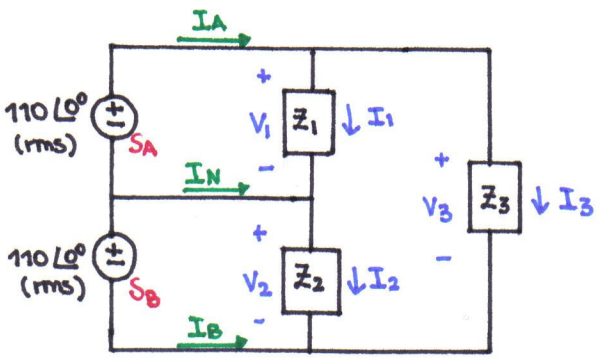
$\theta = \phi_v - \phi_I = 60° + 20° = 80°$

$P = \frac{90}{\sqrt{2}} \cdot \frac{10.5}{\sqrt{2}} \cos 80° = 82.05 \text{ W}$

$Q = \frac{90}{\sqrt{2}} \cdot \frac{10.5}{\sqrt{2}} \sin 80° = 465.322 \text{ VARs}$

Since $P > 0 \Rightarrow W > 0 \Rightarrow$ the load is <u>absorbing power</u>

EXAMPLE 2



The three loads draw complex powers

$S_1 = 1250 + j\,500$ VA

$S_2 = 800 + j\,0$ VA

$S_3 = 2000 + j\,400$ VA

(a) Find I_A, I_N and I_B

(b) Find the complex power produced by each source

(a) $V_1 = 110\angle 0^\circ = 110$

$S_1 = V_1 I_1^* = 1250 + j\,500 \quad \Rightarrow \quad I_1^* = \frac{S_1}{V_1} = 11.364 + j\,4.545$

$\boxed{I_1 = 11.364 - j\,4.545}$ A

$V_2 = 110\angle 0^\circ = 110$

$S_2 = V_2 I_2^* = 800 + j0 \quad \Rightarrow \quad I_2^* = \frac{S_2}{V_2} = 7.273 + j0$

$\boxed{I_2 = 7.273}$ A

$V_3 = V_1 + V_2 = 220$

$S_3 = V_3 I_3^* = 2000 + j\,400 \quad \Rightarrow \quad I_3^* = \frac{S_3}{V_3} = 9.09 + j\,1.818$

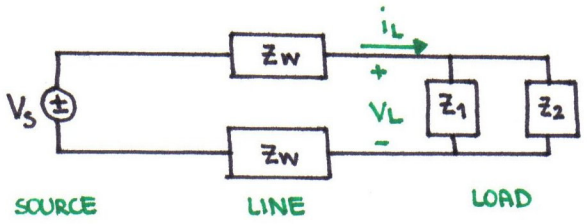
$\boxed{I_3 = 9.09 - j\,1.818}$ A

$\boxed{\begin{aligned} I_A &= I_1 + I_3 = 20.455 - j\,6.363 \text{ A} \\ I_B &= -(I_2 + I_3) = -16.363 + j\,1.818 \text{ A} \\ I_N &= I_2 - I_1 = -4.092 + j\,4.545 \text{ A} \end{aligned}}$

(b) $\boxed{S_A} = -(110)\,I_A^* = -110\,(20.455 + j\,6.363) = \boxed{-2250 - j\,699.9}$ VA

$\boxed{S_B} = (110)\,I_B^* = 110\,(-16.363 - j\,1.818) = \boxed{-1800 - j\,200}$ VA

EXAMPLE 3



The two loads absorb complex power

- $S_1 = 12 + j6$ kVA
- $|S_2| = 15$ kVA at 0.75 lagging power factor

Then $|V_L| = 4.4$ kV (rms)

$Z_W = 3 + j8\ \Omega$

(a) Find the line current and source voltage

Choose $\underline{\phi_v = 0}$ for the sake of simplicity $\Rightarrow$ $V_L = 4400 \angle 0°$

$$\begin{cases} pf = \cos\theta = 0.75 \\ pf \text{ lagging} \Rightarrow \theta > 0 \end{cases} \qquad \begin{cases} \theta = \cos^{-1}(0.75) \\ \theta > 0 \end{cases} \Rightarrow \boxed{\theta = 41.4°}$$

$$\phi_{I_2} = \phi_v - \theta = -41.4°$$

$$S_2 = |S|(\cos\theta + j\sin\theta) = 15000\,(0.75 + j\,0.661) = 11250 + j\,9920$$

$$S_1 = 12000 + j\,6000$$

$$\boxed{S_{LOAD}} = S_1 + S_2 = 23250 + j\,15920 = \boxed{V_L I_L^*}$$

$$I_L^* = \frac{23250 + j15920}{4400} = 5.284 + j\,3.618$$

$$\boxed{I_L} = 5.284 - j\,3.618 = \boxed{6.404 \angle -34.4°}$$

$$\boxed{V_S} = 2Z_W \cdot I_L + V_L = 2(3+j8)(6.404 \angle -34.4°) + 4400$$

$$= (10 \angle 53.2°)(6.404 \angle -34.4°) + 4400$$

$$= 4489.6 + j\,62.836 = \boxed{4490.03 \angle 0.8°}$$

(b) Find the complex power produced by the source

$$\boxed{S_S} = -V_S I_L^* = +(4490.03 \angle 0.8°)(6.404 \angle 34.4°)$$

$$= \boxed{23495.7 + j\,16575.4}\ \text{VA}$$