Problem#1: Problem 8-1 textbook

Transform the following sinusoids into phasor form and draw a phasor diagram. Use the additive property of phasors to find $v_1(t) + v_2(t)$.

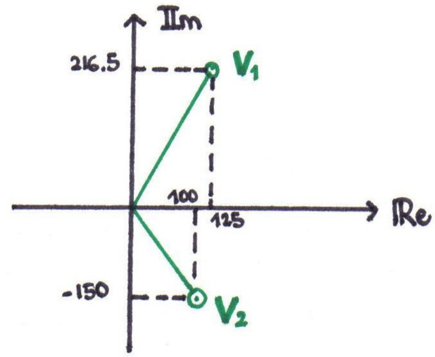
a) $v_1(t) = 250\cos(\omega t + 60°)\text{V}$

b) $v_2(t) = 100\cos(\omega t) + 150\sin(\omega t)\text{V}$

$$\boxed{V_1} = 250\angle 60° = 250\cos(60°) + j\,250\sin(60°) = \boxed{125 + j\,216.5}$$

$$v_2(t) = 100\cos(wt) + 150\cos(wt - 90°)$$

$$\boxed{V_2} = \underbrace{100\angle 0°}_{100 + j0} + \underbrace{150\angle -90°}_{0 - j150} = \boxed{100 - j\,150}$$



$$\boxed{V_3} = V_1 + V_2 = 225 + 66.5\,j$$

$$= \sqrt{225^2 + 66.5^2}\;\angle \tan^{-1}\left(\frac{66.5}{225}\right)\cdot\frac{180}{\pi} = \boxed{234.62\angle 16.465°}$$

↳ $\text{Re}[\cdot] > 0$, $\text{Im}[\cdot] > 0$, thus $0 < \theta < 90°$

$$\boxed{v_1(t) + v_2(t) = 234.62\cos(wt + 16.465°)\ \text{V}}$$

Problem#2: Problem 8-3 textbook

Convert the following phasors into sinusoidal waveforms.

a) $\mathbf{V}_1 = 10e^{-j30°}V, \omega = 10^4\,rad/s$

b) $\mathbf{V}_2 = 60e^{-j220°}V, \omega = 10^4\,rad/s$

c) $\mathbf{I}_1 = 5e^{j90°}A, \omega = 200\,rad/s$

d) $\mathbf{I}_2 = 2e^{j270°}A, \omega = 200\,rad/s$

a) $\boxed{v_1(t)} = \text{Re}\left[10\,e^{-j30°}\cdot e^{j10^4 t}\right] = 10\,\text{Re}\left[e^{j(10^4 t - 30°)}\right] = \boxed{10\cos\left(10^4 t - \frac{\pi}{6}\right)}\ \text{V}$

↓ $\cos\theta = \text{Re}[e^{j\theta}]$

b) $\boxed{v_2(t) = 60\cos\left(10^4 t - 220\frac{\pi}{180}\right)\ \text{V}}$

c) $\boxed{i_1(t) = 5\cos\left(200t + \frac{\pi}{2}\right)\ \text{A}}$ $\quad 90° = \frac{\pi}{2}$

d) $\boxed{i_2(t) = 2\cos\left(200t + \frac{3}{2}\pi\right)\ \text{A}}$ $\quad 270° = \frac{3}{2}\pi$

Problem#3: Problem 8-6 textbook

Convert the following phasors into sinusoids:

a) $\mathbf{V}_1 = 20 + j25V, \omega = 10 rad/s$

b) $\mathbf{V}_2 = 5(8 - j3)V, \omega = 20 rad/s$

c) $\mathbf{I}_1 = 12 - j5 + \frac{4}{j} A, \omega = 300 rad/s$

d) $\mathbf{I}_2 = \frac{3 + j8}{2 - j6} A, \omega = 50 rad/s$

a) $V_1 = 20 + j25 \text{ V}$ $\qquad M = \sqrt{20^2 + 25^2} = 32.016$

$$\theta = \begin{cases} \tan^{-1}\left(\frac{25}{20}\right) \cdot \frac{180}{\pi} = 51.34° \\ 0° < \theta < 90° \end{cases}$$

$V_1 = 32.016 \angle 51.34°$

$v_1(t) = 32.016 \cos(10t + 51.34°) \text{ V}$

b) $V_2 = 40 - j15 \text{ V} = 42.72 \angle -20.556° \text{ V}$

$v_2(t) = 42.72 \cos(20t - 20.556°) \text{ V}$

c) $I_1 = 12 - j5 + \frac{4j}{j^2 = -1} = 12 - j9 = 15 \angle -36.87° \text{ A}$

$i_1(t) = 15 \cos(300t - 36.87°) \text{ A}$

d) $I_2 = \frac{3 + j8}{2 - j6} \cdot \frac{2 + j6}{2 + j6} = \frac{6 + j16 + j18 - 48}{4 + 36} = -\frac{42}{40} + j\frac{34}{40} = 1.35 \angle 141.01°$

$\mathrm{Re}[\cdot] < 0$
$\mathrm{Im}[\cdot] > 0$
$90° < \theta < 180°$

$i_2(t) = 1.35 \cos(50t + 141°) \text{ A}$

Problem#4: Problem 8-10 textbook

Given a sinusoid $v_1(t)$ whose phasor is $\mathbf{V}_1 = -3 + j4$ V, use phasor methods to find the voltage $v_2(t)$ that leads $v_1(t)$ by 90° and has an amplitude of 10 V.

$$V_1 = -3 + j4 = 5 \angle 126.87°$$

$$M_1 = \sqrt{3^2 + 4^2} = 5$$

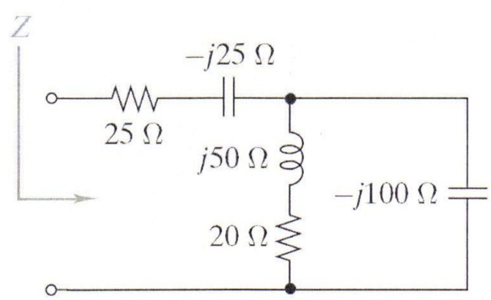
$$\theta_1 = \tan^{-1}\left(\frac{4}{-3}\right) = -53.13 \text{ or } 126.87, \quad 90° < \theta_1 < 180° \Rightarrow \theta_1 = 126.87°$$

$$V_2 = 10 \angle 126.87° + 90° = 10 \angle 216.87°$$

$$\boxed{v_2(t)} = \mathrm{Re}\left[V_2 e^{j\omega t}\right] = 10\cos(\omega t + 216.87°) = \boxed{10\cos(\omega t - 143.13°)} \text{ V}$$

Problem#5: Problem 8-12 textbook

Find the equivalent impedance Z in the circuit. Express the result in both polar and rectangular form.



$$\boxed{Z} = 25 - j25 + \frac{(20 + j50)(-j100)}{(20 + j50) + (-j100)}$$

$$= 25 - j25 + \frac{5000 - j2000}{20 - j50}$$

$$= 25 - j25 + (68.966 + j72.414) = \boxed{93.966 + j47.414}$$

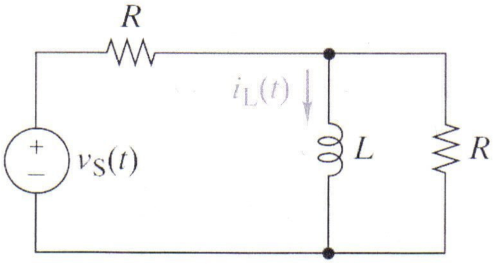
$$|Z| = \sqrt{93.966^2 + 47.414^2} = 105.2506$$

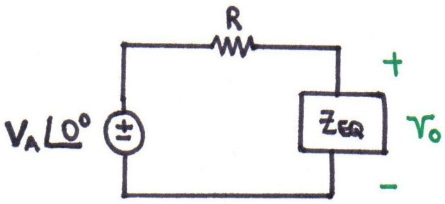
$$\angle Z = \frac{180}{\pi}\tan^{-1}\left(\frac{47.414}{93.966}\right) = 26.775°$$

$$\boxed{Z = 105.2506 \angle 26.775°}$$

Problem#6: Problem 8-22 textbook

The circuit is operating in the sinusoidal steady state with $v_S(t) = V_A \cos(\omega t)$. Derive a general expression for the phasor response I_L.





$L \Rightarrow Z_L = j\omega L$

$R \Rightarrow Z_R = R$

$V_s = V_A \angle 0°$

$$Z_{EQ} = j\omega L \,//\, R = \frac{j\omega RL}{R + j\omega L}$$

$$V_o = \frac{Z_{EQ}}{R + Z_{EQ}} V_s$$

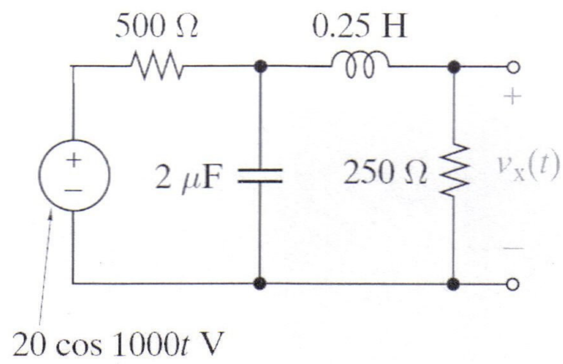
$$I_L = \frac{V_o}{j\omega L}$$

$$V_o = \frac{j\omega \cancel{R} L \; V_s}{\cancel{R}(R + j\omega L) + j\omega \cancel{R} L} = \frac{j\omega L \, V_A}{R + j2\omega L}$$

$$\boxed{I_L} = \frac{V_A}{R + j2\omega L} = \boxed{\frac{R - j2\omega L}{R^2 + (2\omega L)^2} V_A}$$

Problem#7: Problem 8-26 textbook

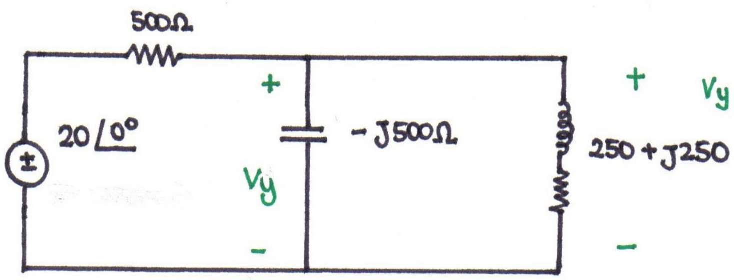
The circuit is operating in sinusoidal steady state. Find the steady state response $v_x(t)$.



$\omega = 1000$ rad/s

$$Z_C = -\frac{j}{\omega C} = -\frac{j}{1000 \cdot 2 \cdot 10^{-6}} = -j\,500\,\Omega$$

$$Z_L = j\omega L = j \cdot 1000 \cdot 0.25 = j\,250\,\Omega$$



$$Z_{EQ} = \frac{(-j500)(250+j250)}{(-j500)+(250+j250)} = 500\,\Omega = \frac{500\,(250-j250)}{(250-j250)}$$

Applying voltage division rule twice :

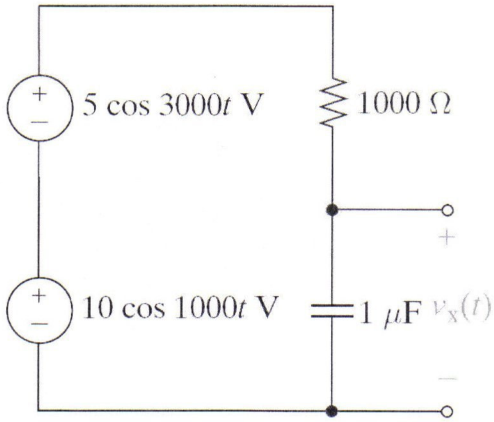
$$V_y = \frac{Z_{EQ}}{500 + Z_{EQ}} \cdot 20 = 10\text{ V}$$

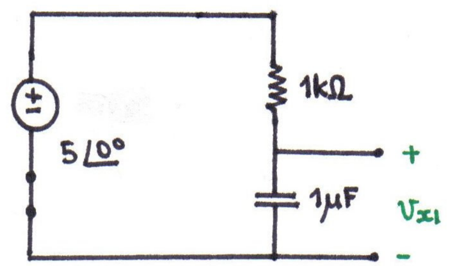
$$V_x = \frac{250}{250 + j250}\, V_y = 5 - j5 \text{ V} = 7.071 \angle{-45^\circ} \text{ V}$$

$$\boxed{v_x(t) = 7.071 \cos(1000t - 45^\circ)} \text{ V}$$

Problem#8: Problem 8-30 textbook

The circuit is operating in sinusoidal steady state. Use superposition to find the response $v_x(t)$. *Note:* the sources do not have the same frequency.



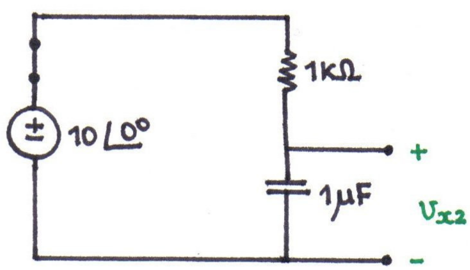


$\omega_1 = 3000$ rad/s

$$Z_{C1} = \frac{-j}{\omega_1 C} = \frac{-j}{3000 \cdot 10^{-6}} = -j\frac{1000}{3}$$

$$V_{x1} = \frac{-j\frac{1000}{3} \cdot 5}{1000 - j\frac{1000}{3}} = 0.5 - j1.5 = 1.581\angle -71.57^\circ$$

$$v_{x1}(t) = 1.581\cos(3000t - 71.57^\circ)$$



$\omega_2 = 1000$ rad/s

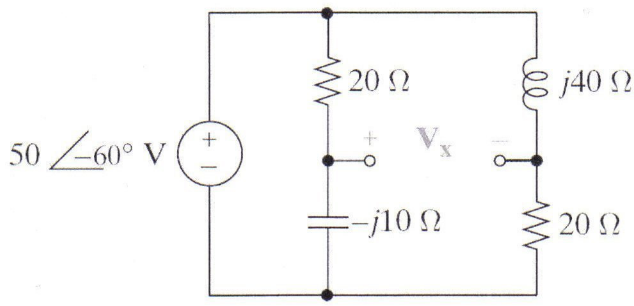
$$Z_{C2} = \frac{-j}{\omega_2 C} = -\frac{j}{10^3 \cdot 10^{-6}} = -j1000$$

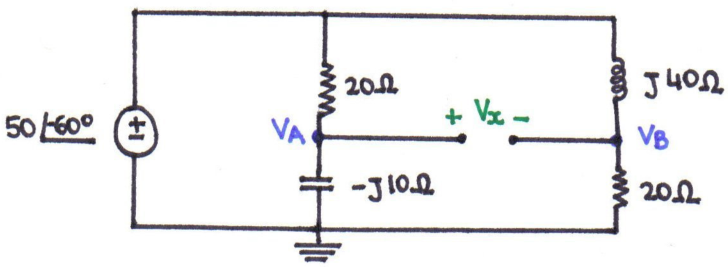
$$V_{x2} = \frac{-j1000 \cdot 10}{1000 - j1000} = 5 - j5 = 7.071\angle -45^\circ$$

$$v_{x2}(t) = 7.071\cos(1000t - 45^\circ)$$

$$\boxed{v_x(t)} = v_{x1}(t) + v_{x2}(t) = \boxed{1.581\cos(3000t - 71.57^\circ) + 7.071\cos(1000t - 45^\circ)}$$

Problem#9: Problem 8-32 textbook
The circuit is operating in the sinusoidal steady state. Find the phasor response $\mathbf{V}_x$.





$$V_A = \left(\frac{-j10}{20 - j10}\right) 50\angle{-60^\circ} = (0.4472\angle{-63.4349^\circ})(50\angle{-60^\circ})$$

$$= 22.36\angle{-123.435^\circ} = -12.3202 - j\,18.66$$

$$V_B = \left(\frac{20}{20 + j40}\right) 50\angle{-60^\circ}$$

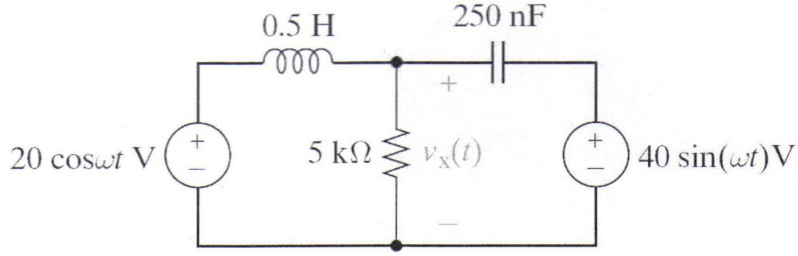
$$= (0.4472\angle{-63.4349^\circ})(50\angle{-60^\circ})$$

$$= 22.36\angle{-123.435^\circ} = -12.3202 - j\,18.66$$

$$\boxed{V_x = V_A - V_B = 0}$$

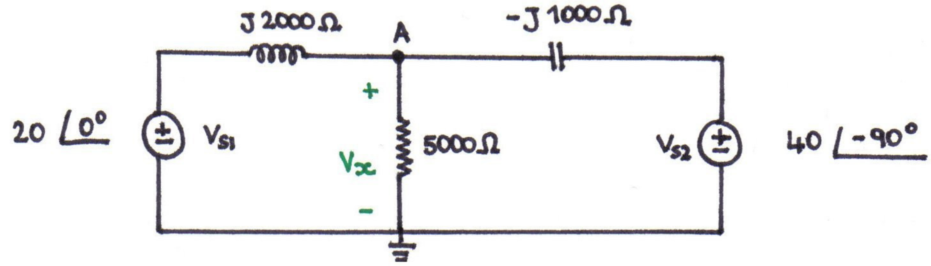
Problem#10: Problem 8-41 textbook

The circuit is operating in the sinusoidal steady state with ω=4 krad/s. Use node-voltage analysis to find the steady-state response $v_x(t)$.



$$Z_C = \frac{-j}{\omega C} = \frac{-j}{4000 \cdot 250 \cdot 10^{-9}} = -j\,1000\ \Omega = 1000\angle{-90}$$

$$Z_L = j\omega L = j\,4000 \cdot 0.5 = j\,2000\ \Omega = 2000\angle{90°}$$



$V_A = V_x$

$$\frac{V_A - V_{S1}}{Z_L} + \frac{V_A - V_{S2}}{Z_C} + \frac{V_A}{5K} = 0$$

$$V_A\left(\frac{1}{Z_L} + \frac{1}{Z_C} + \frac{1}{5K}\right) = \frac{V_{S1}}{Z_L} + \frac{V_{S2}}{Z_C}$$

$$V_A = \frac{Z_L \cdot Z_C \cdot 5K}{Z_L Z_C + Z_L 5K + Z_C 5K} \cdot \left(\frac{V_{S1}}{Z_L} + \frac{V_{S2}}{Z_C}\right)$$

$$= \frac{Z_C V_{S1} + Z_L V_{S2}}{Z_L Z_C + Z_L 5K + Z_C 5K} \cdot 5K$$

$$= \frac{20\not{000}\angle{-90°} + 80\not{000}\angle{0°}}{2 \cdot \not{10^6}\angle{0°} + 5 \cdot \not{10^6}\angle{-90°} + 10 \cdot \not{10^6}\angle{90°}} \cdot 5\not{000}$$

$$= 5\,\frac{20\angle{-90°} + 80\angle{0°}}{2\angle{0°} + 5\angle{-90°} + 10\angle{90°}} = 5\,\frac{80 - j20}{2 - j5 + j10}$$

$$= 5\,\frac{80 - j20}{2 + j5} = 10.3448 - j\,75.8621 = 76.564\angle{-82.235°}$$

$$v_x(t) = V_A(t) = 76.564\cos(4000t - 82.235°)$$