Problem#1: Problem 6-2 textbook

The voltage across a 3mF capacitor is $v_C(t) = 20\sin(2\pi 10t)\,\mathrm{V}$. Derive expressions for $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing or delivering power?

$$i_c(t) = C\,\frac{dv_c}{dt} = (3\cdot 10^{-3})\cdot 20\cdot(2\pi 10)\cos(2\pi 10t)$$

$$\boxed{i_c(t) = 1.2\pi\cos(2\pi 10t)}\ \mathrm{A}$$

$$p_c(t) = v_c(t)\cdot i_c(t) = 20\sin(2\pi 10t)\,.\,1.2\pi\cos(2\pi 10t)$$

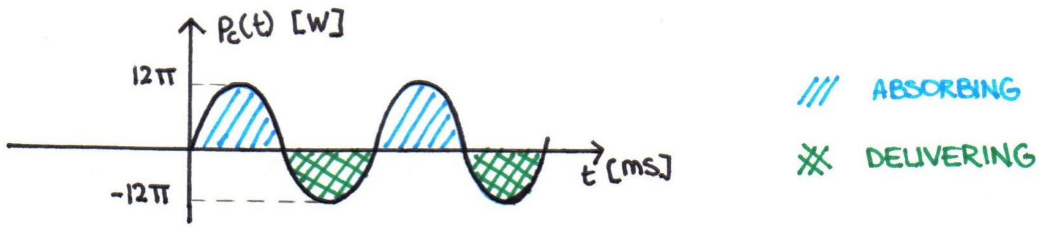
$$= 24\pi\sin(2\pi 10t)\,.\,\cos(2\pi 10t)$$

$$\sin\alpha\cos\alpha = \frac{\sin 2\alpha}{2}$$

since $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$$\boxed{p_c(t) = 12\pi\sin(2\pi 20t)}\ \mathrm{W}$$

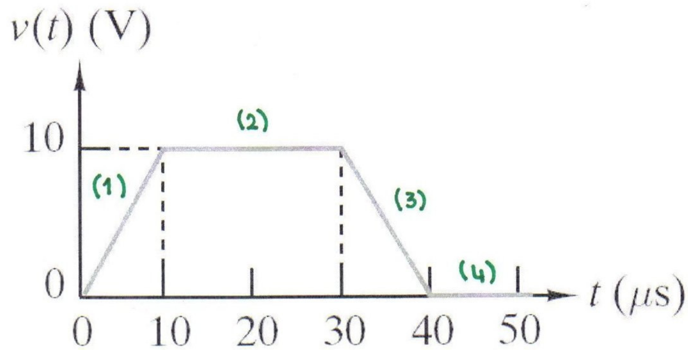
$$T_0 = \frac{1}{20} = 50\,\mathrm{mS}$$



The capacitor absorbs and delivers power

Problem#2: Problem 6-6 textbook

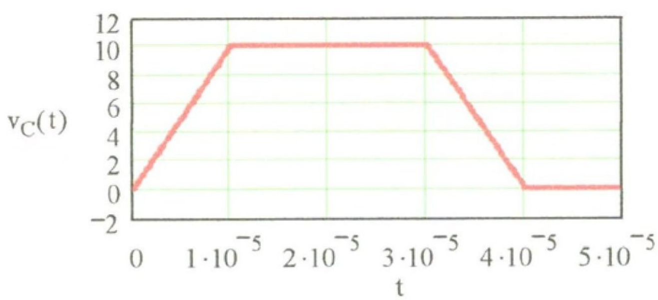
The voltage across a 0.5 μF capacitor is shown in the Figure. Sketch $i_C(t)$ and $p_C(t)$. Is the capacitor absorbing or delivering power?

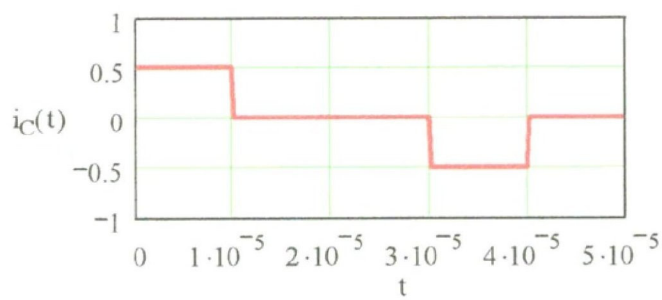


(1) slope in $[0, 10]\,\mu s = \dfrac{10}{10^{-5}} = 10^6 \;\Rightarrow\; i_c(t) = C\dfrac{dv_c}{dt} = 0.5\cdot 10^{-6}\cdot 10^{6} = 0.5\text{ A} \quad t \in [0,10]\,\mu s$

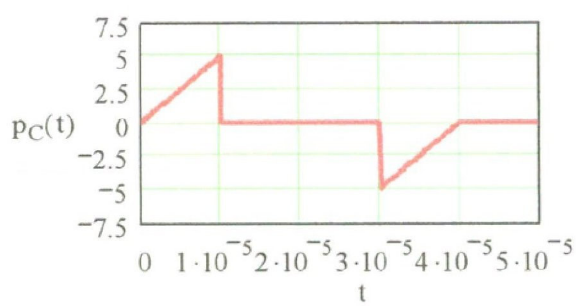
(2)-(4) slope in $[10,30]\,\mu s$ and $[40,50]\,\mu s = 0 \;\Rightarrow\; i_c(t) = C\cdot 0 = 0\text{ A} \quad t \in [10,30]\,\mu s$, $t \in [40,50]\,\mu s$

(3) slope in $[30,40]\,\mu s = \dfrac{-10}{10^{-5}} = -10^6 \;\Rightarrow\; i_c(t) = 0.5\cdot 10^{6}(-10^{-6}) = -0.5\text{ A} \quad t \in [30,40]\,\mu s$





$P_c(t) = v_c(t)\cdot i_c(t)$



The capacitor absorbs power from $0 < t < 10\,\mu s$ and delivers power in $30\,\mu s < t < 40\,\mu s$.

Problem#3: MATLAB Problem

A voltage of $v_L(t) = 5\cos(1000t) - 2\sin(3000t)V$ appears across a 50mH inductor. Derive an expression for $i_L(t)$ assuming $i_L(0)=0$. Discuss the effect of frequency on the relative amplitudes of the sinusoidal components of $i_L(t)$ and $v_L(t)$. Sketch theses waveforms in MATLAB.

$$L = 0.05 \text{ H}$$

$$v_L(t) = 5\cos(1000t) - 2\sin(3000t) \text{ V}$$

$$v_L(t) = L\frac{d\,i_L(t)}{dt} \quad \Rightarrow \quad i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(x)\,dx$$

$$\boxed{i_L(t)} = 0 + \frac{1}{0.05}\int_0^t \left[5\cos(1000x) - 2\sin(3000x)\right]dx$$

$$= 20\left[\frac{5}{1000}\sin(1000x) + \frac{2}{3000}\cos(3000x)\right]_0^t$$

$$= \boxed{\frac{1}{10}\sin(1000t) + \frac{1}{75}\cos(3000t) - \frac{1}{75} \quad \text{A}}$$

NOTE if $v_L(t) = a_1\cos(\omega_1 t) - a_2\sin(\omega_2 t)$

then $$i_L(t) = \frac{1}{L}\left[\frac{a_1}{\omega_1}\sin(\omega_1 t) + \frac{a_2}{\omega_2}\cos(\omega_2 t) - \frac{a_2}{\omega_2}\right]$$

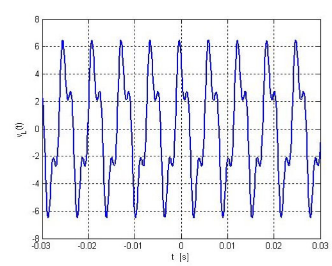
$$= \underbrace{-\frac{a_2}{L\omega_2}}_{DC} + \underbrace{\frac{a_1}{L\omega_1}\sin(\omega_1 t) + \frac{a_2}{L\omega_2}\cos(\omega_2 t)}_{AC}$$

in $v_L(t)$ the ratio of the two amplitudes of the 1KHz and 3KHz AC components is

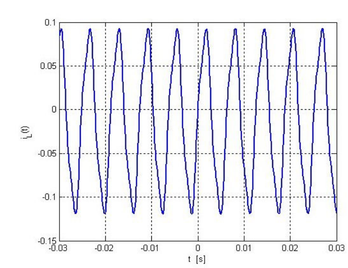
$$\frac{a_1}{a_2} = \frac{5}{2} = 2.5$$

in $i_L(t)$ is

$$\frac{\frac{a_1}{L\omega_1}}{\frac{a_2}{L\omega_2}} = \frac{a_1}{a_2}\cdot\frac{\cancel{L}\omega_2}{\cancel{L}\omega_1} = \frac{5}{2}\cdot\frac{3000}{1000} = 7.5$$

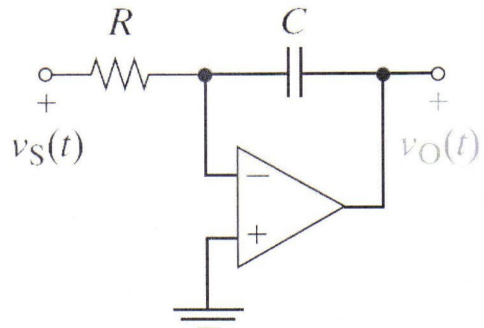


Therefore the integration produces a DC component and reduces the amplitude of the higher frequency (3KHz) component relative to the lower frequency component (1KHz)



Problem#4: Problem 6-21 textbook

The OP AMP integrator in the Figure has R=40 kΩ, C=50 nF, and $v_O(0)$=10 V. The input is $v_S(t) = 10e^{-500t}u(t)\ V$. Find $v_O(t)$ for t>0.



$$\boxed{\begin{array}{l} V_n = V_p \\ i_n = i_p = 0 \end{array}}$$

$V_p = 0 \quad \Rightarrow \quad V_p = 0$

$i_n = 0 \quad \Rightarrow \quad \dfrac{V_s - \overset{=0}{\cancel{V_n}}}{R} + C\dfrac{d\,(V_o - \overset{=0}{\cancel{V_n}})}{dt} = 0$

$$\frac{dV_o}{dt} = -\frac{1}{RC}V_s$$

$$v_o(t) = v_o(0) - \frac{1}{RC}\int_0^t v_s(x)\,dx = 10 - \frac{1}{RC}\int_0^t 10\,e^{-500x}\,dx$$

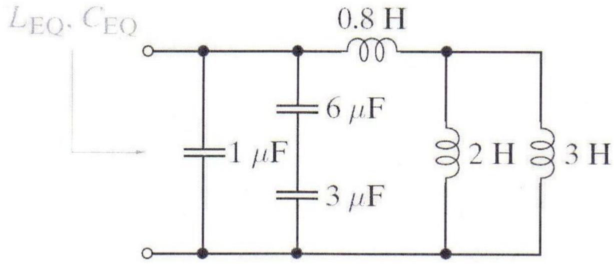
$$= 10 - \frac{1}{40\cdot10^3\cdot50\cdot10^{-9}}\left[-\frac{e^{-500x}}{50}\right]_0^t$$

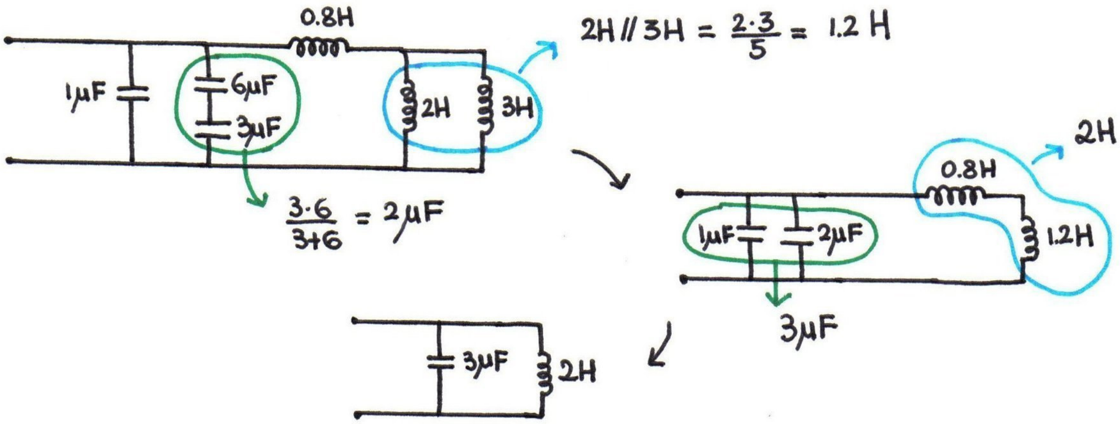
$$= 10 - \overset{10}{\cancel{500}}\left(-\frac{e^{-500t}}{\cancel{50}} + \frac{1}{\cancel{50}}\right)$$

$$\boxed{v_o(t) = 10\,e^{-500t}\ \ V} \qquad t \geq 0$$

Problem#5: Problem 6-40 textbook

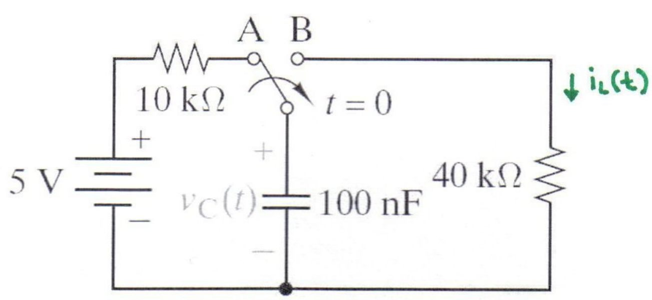
For the circuit given find an equivalent circuit consisting of one inductor and one capacitor.



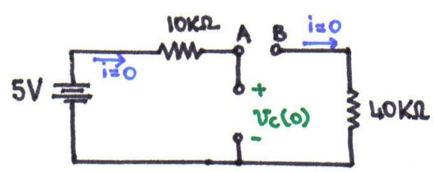


Problem#6: Problem 7-7 textbook

The switch has been in position A for a long time and is moved to position B at t=0. Find $v_c(t)$ for $t \geq 0$.

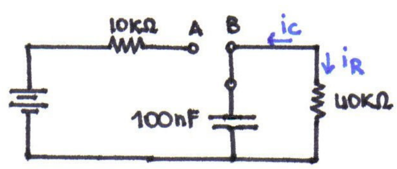


$\boxed{t<0}$



$$v_c(0) = \underline{5V}$$

$\boxed{t \geq 0}$



KCL $\quad i_c + i_R = 0 \quad \Rightarrow \quad C\frac{dV_c}{dt} + \frac{V_c}{R} = 0$

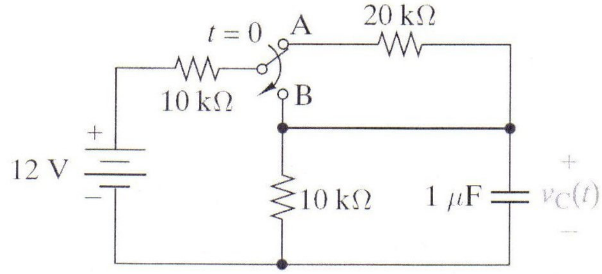
$$\frac{dV_c}{dt} = -250\ V_c$$

$$V_c(t) = V_c(0)\ e^{-250t}$$

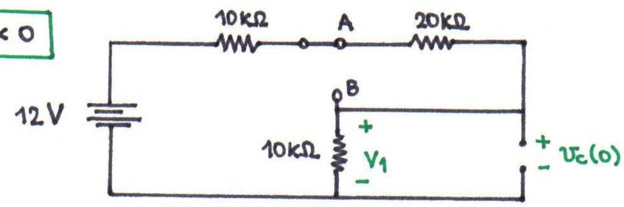
$$\boxed{V_c(t) = 5\ e^{-250t}\ \text{V}} \qquad t \geq 0$$

Problem#7: Problem 7-13 textbook

The switch has been in position A for a long time and is moved to position B at t=0. Find $v_C(t)$ for $t \geq 0$. Identify te forced and natural components in the response.



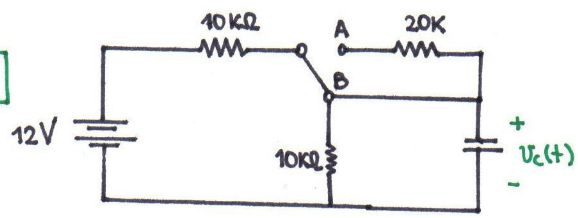
$\boxed{t < 0}$

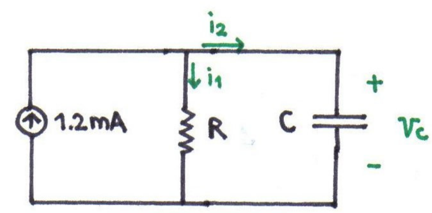


VOLTAGE DIVISION RULE

$$v_c(0) = \frac{10K}{10K + 10K + 20K} \cdot 12V = 3V.$$

$\boxed{t \geq 0}$





$$i_s = \frac{V_s}{10K} = \frac{12}{10 \cdot 10^3} \quad 1.2 \text{ mA}$$

$$R = 10K \,//\, 10K = 5K\Omega$$

$$i_1 + i_2 = 1.2 \text{ mA}$$

$$\frac{V_c}{R} + C\frac{dV_c}{dt} = 1.2 \cdot 10^{-3}$$

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = \frac{1.2 \cdot 10^{-3}}{C}$$

$$\begin{cases} \dfrac{dv_c}{dt} + a v_c = b \\ v_c(0) = v_0 \end{cases}$$

$$a = \frac{1}{RC} = \frac{1}{5 \cdot 10^3 \cdot 10^{-6}} = 200$$

$$v_c(t) = \left(v_0 - \frac{b}{a}\right)e^{-at} + \frac{b}{a}$$

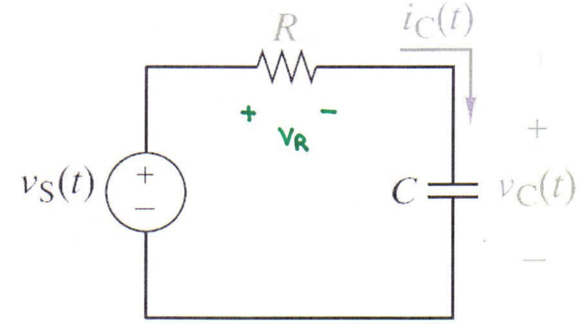
$$b = \frac{1.2 \cdot 10^{-3}}{10^{-6}} = 1200$$

$$b/a = 1200/200 = 6$$

$$\boxed{v_c(t)} = (3-6)e^{-200t} + 6 = \boxed{\underbrace{6}_{\text{NATURAL RESPONSE}} - \underbrace{3e^{-200t}}_{\text{FORCED RESPONSE}} \quad V} \qquad t \geq 0$$

Problem#8: Problem 7-26 textbook

For t≥0 the step response of the current through the capacitor in Figure is $i_C(t) = 20e^{-2000t}\,mA$. Find $v_C(t)$ for t≥0 when C=1μF and $v_C(0)$=5 V.



$$\boxed{v_c(t)} = v_c(0) + \frac{1}{C}\int_0^t i_c(x)\,dx$$

$$= 5 + \frac{1}{\substack{\cancel{10^{-6}} \\ 10^{-3}}} \int_0^t 20\cdot\cancel{10^{-3}}\, e^{-2000x}\,dx$$

$$= 5 + 10^3 \left[-\frac{e^{-2000x}}{100}\right]_0^t \quad = \quad 5 + \overset{10}{\cancel{10^3}}\left[-\frac{e^{-2000t}}{\cancel{100}} + \frac{1}{\cancel{100}}\right]$$

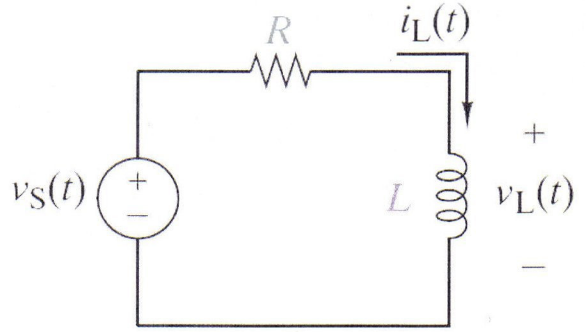
$$= \boxed{15 - 10\,e^{-2000t}\ \text{V}} \qquad t \geq 0$$

Problem#9: Problem 7-27 textbook

For $t \geq 0$ the step response of the current through and voltage across the inductor are:

$i_L(t) = 5 - 10e^{-2000t}\,mA$ and $v_L(t) = e^{-2000t}\,V$

(a) Find v_s, R and L.
(b) Find the energy stored in the inductor at $t = \ln(2)/2$ ms.



(a) $$\boxed{L} = \frac{V_L(t)}{\frac{d\,i_L(t)}{dt}} = \frac{e^{-2000t}}{(-10)(-2000)\,e^{-2000t}\cdot 10^{-3}} = \frac{1}{20} = \boxed{0.05\,H}$$

$$V_R + V_L = V_S \;\Rightarrow\; R\,i_L(t) + L\frac{di_L(t)}{dt} = V_S$$

$$\frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = \frac{V_S}{L} \;\Rightarrow\; i_L(t) = a\,e^{-\frac{R}{L}t} + b \;\Rightarrow\; \frac{R}{L} = 2000$$

$$\boxed{R} = 2000\,L = \boxed{100\,\Omega}$$

since at $t = \infty$ the inductor acts like a short circuit

$$\boxed{V_S} = R\,i_L(\infty) = 100 \cdot 5 \cdot 10^{-3} = \boxed{0.5\,V}$$

(b) $$W_L(t) = \frac{1}{2} L\, i_L^2(t)$$

$$\boxed{W_L\left(\frac{\ln(2)}{2000}\right)} = \frac{1}{2}L\left[5 - 10\exp\left(-\cancel{2000}\frac{\ln(2)}{\cancel{2000}}\right)\right]^2 \cdot (10^{-3})^2$$

$$= \frac{1}{2}L\left[\cancel{5} - 10\,\underbrace{\cancel{\exp(-\ln(2))}}_{\exp(-\ln(2)) = \exp(\ln(\frac{1}{2})) = \frac{1}{2}}\right]^2 \cdot 10^{-6} = \boxed{0}$$