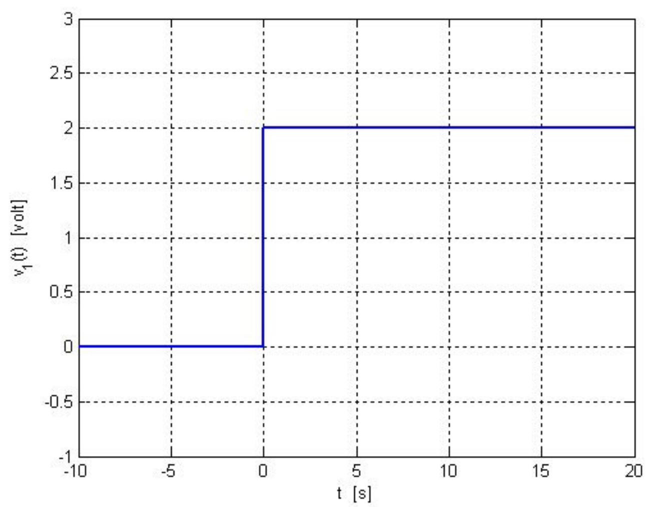
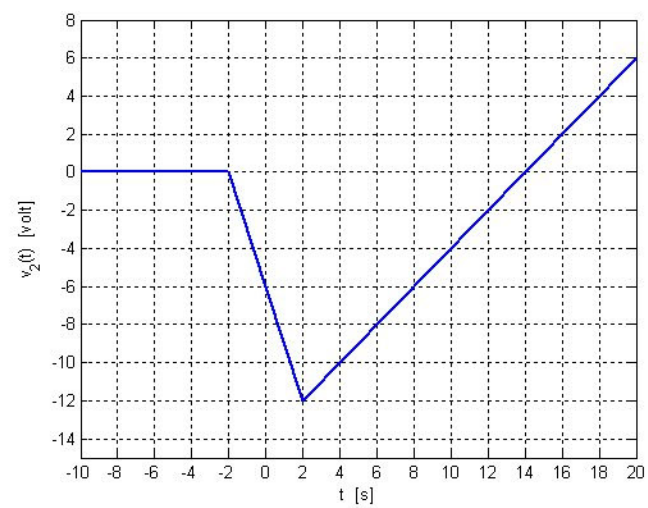
Problem#1: MATLAB Problem

Sketch the following waveforms in MATLAB.

a) $v_1(t) = 2u(t)$

b) $v_2(t) = -3r(t+2) + 4r(t-2)$





Problem#2: Problem 5-4 textbook

Express the following signals as the sum of singularity functions. (step or ramp functions)

a) $v_1(t) = \begin{cases} 2 & t < 1 \\ -5 & 1 \le t < 2 \\ 0 & 2 < t \end{cases}$

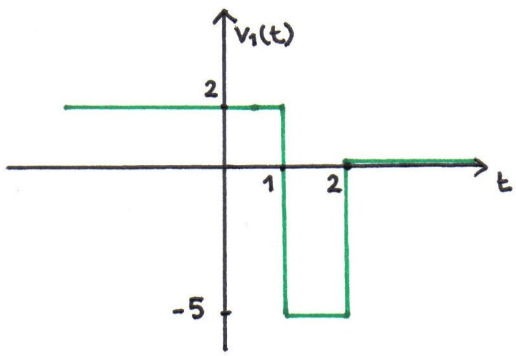
b) $v_2(t) = \begin{cases} 0 & t < 0 \\ -4t & 0 \le t < 2 \\ -12 + 2t & 2 \le t < 6 \\ 0 & 6 \le t \end{cases}$

(a) significant instants of time $t=1$, $t=2$

$$\underline{v_1(t) = a + b\,u(t-1) + c\,u(t-2)}$$

$a, b, c \in \mathbb{R}$ = UNKNOWNS

$$\begin{cases} a = 2 \\ a + b = -5 \\ a + b + c = 0 \end{cases} \qquad \begin{aligned} a &= 2 \\ b &= -7 \\ c &= 5 \end{aligned}$$



$$\boxed{v_1(t) = 2 - 7u(t-1) + 5\,u(t-2)}$$

NOTE: $2 - 7u(t-1) = 2u(1-t) - 5u(t-1)$

(b) significant instants of time $t=0$, $t=2$, $t=6$

SOLUTION 1 (analytical)

$$v_2(t) = x_0(t) + x_1(t)\,u(t) + x_2(t)\,u(t-2) + x_3(t)\,u(t-6)$$

where $x_0(t), x_1(t), x_2(t)$ and $x_3(t)$ are functions that need to be determined

$$\begin{cases} 0 & t < 0 & \rightsquigarrow & x_0(t) = 0 \\ -4t & 0 \le t < 2 & \rightsquigarrow & x_0(t) + x_1(t) = -4t \\ -12 + 2t & 2 \le t < 6 & \rightsquigarrow & x_0(t) + x_1(t) + x_2(t) = -12 + 2t \\ 0 & t \ge 6 & \rightsquigarrow & x_0(t) + x_1(t) + x_2(t) + x_3(t) = 0 \end{cases}$$

therefore

$$x_0(t) = 0$$
$$x_1(t) = -4t$$
$$x_2(t) = -12 + 2t - x_1(t) = -12 + 6t$$
$$x_3(t) = -\left(x_0(t) + x_1(t) + x_2(t)\right) = -(-12+2t) = 12 - 2t$$

$$v_2(t) = -4t\,u(t) + (6t-12)\,u(t-2) + (12-2t)\,u(t-6)$$
$$= -4t\,u(t) + 6(t-2)\,u(t-2) - 2(t-6)\,u(t-6)$$

recall that $\boxed{r(t-a) = (t-a)\,u(t-a)}$ $a \in \mathbb{R}$

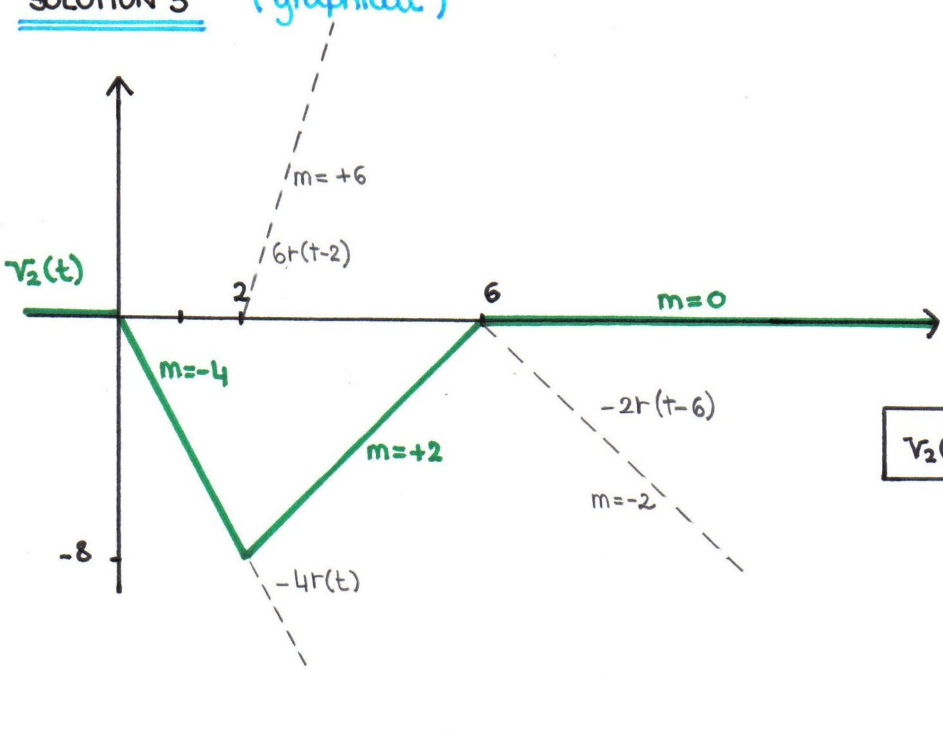
$$\boxed{v_2(t) = -4\,r(t) + 6\,r(t-2) - 2\,r(t-6)}$$

SOLUTION 2 (analytical)

$$v_2(t) = -4t\left[u(t) - u(t-2)\right] + (-12+2t)\left[u(t-2) - u(t-6)\right]$$
$$= -4t\,u(t) + \left[4t - 12 + 2t\right]u(t-2) - (-12+2t)\,u(t-6)$$
$$= -4t\,u(t) + 6(t-2)\,u(t-2) - 2(t-6)\,u(t-6)$$

$$\boxed{v_2(t) = -4\,r(t) + 6\,r(t-2) - 2\,r(t-6)}$$

SOLUTION 3 (graphical)



m = slope

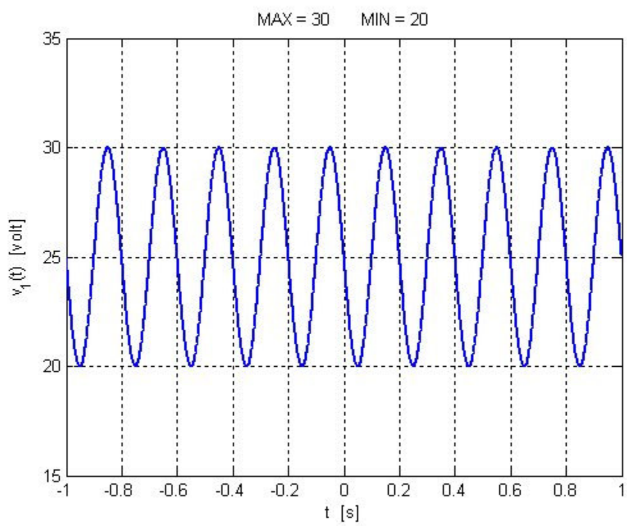
$$\boxed{v_2(t) = -4\,r(t) + 6\,r(t-2) - 2\,r(t-6)}$$

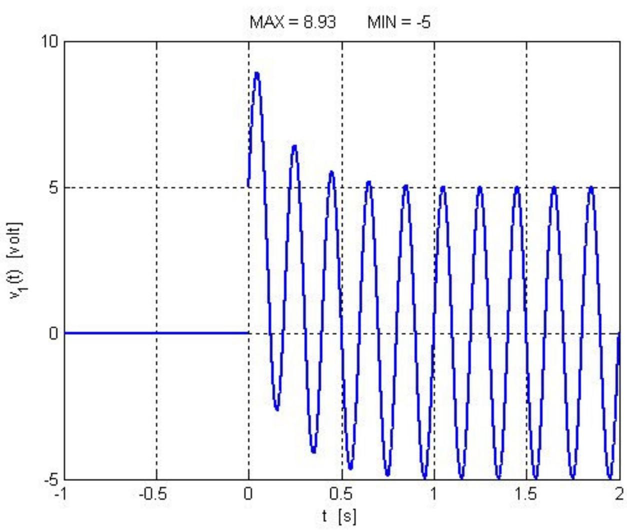
Problem#3:

Sketch the following waveforms. Find the maximum and minimum values of each waveform.

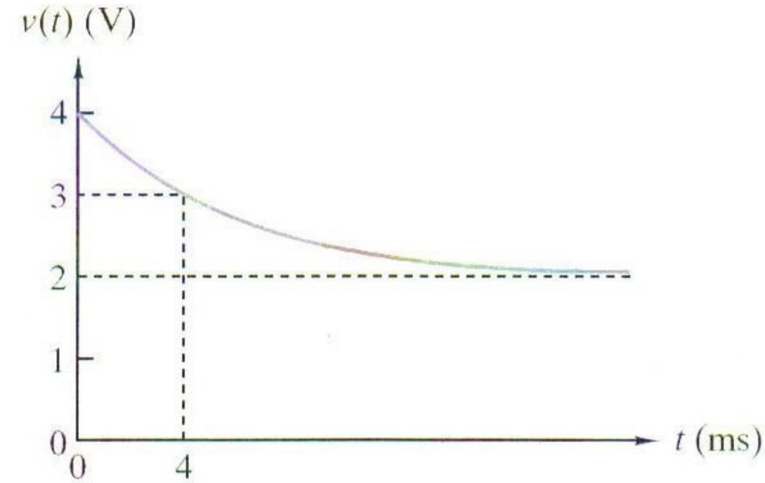
a) $v_1(t) = 25 - 5\sin(10\pi t)$

b) $v_2(t) = 5\left[e^{-5t} + \sin(10\pi t)\right]u(t)$





Problem#4: Problem 5-27 textbook
Write an expression for the waveform in Figure.



$$v(t) = \left(a + b\,e^{-\alpha\cdot t}\right) u(t)$$

(1) $v(0) = a + b = 4$

(2) $v(4\cdot 10^{-3}) = a + b\,e^{-4\alpha\cdot 10^{-3}} = 3$

(3) $\lim_{t\to\infty} v(t) = a = 2$

$a = 2$

$b = 2$

$2 + 2e^{-4\alpha 10^{-3}} = 3 \quad \Rightarrow \quad e^{-4\alpha\cdot 10^{-3}} = \frac{1}{2}$

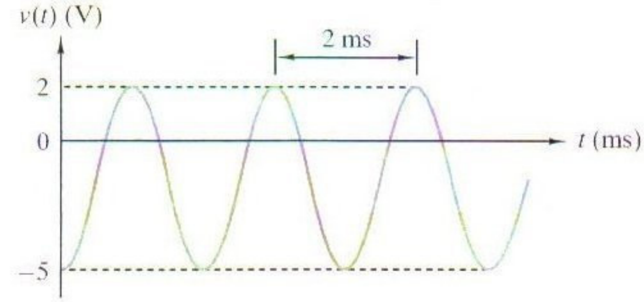
$e^{4\alpha\cdot 10^{-3}} = 2$

$4\alpha \cdot 10^{-3} = \ln(2)$

$\alpha = \frac{\ln(2)}{4\cdot 10^{-3}} = 173.287 \text{ rad/s}$

$$\boxed{v(t) = \left[2 + 2\exp\left(-173.287\, t\right)\right] u(t)}$$

Problem#5: Problem 5-26 textbook
Find the expression for the sinusoidal waveform in Figure.



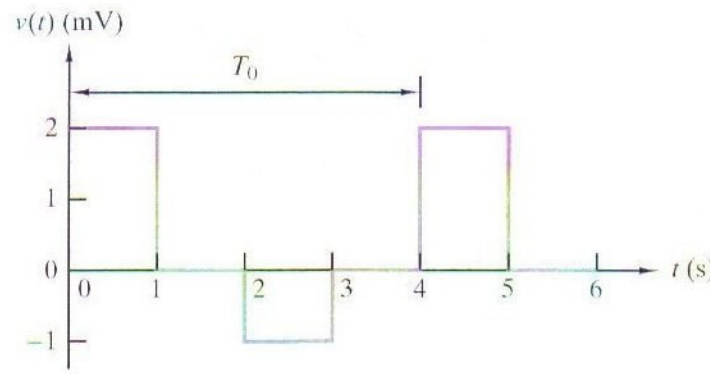
$V_{pp} = 2-(-5) = 7 \quad \Rightarrow \quad$ amplitude sinusoidal fnc $= 3.5$

$v(0) = -5 \quad \Rightarrow \quad$ shift $= -1.5$

$$v(t) = -1.5 - 3.5\cos\left(\frac{2\pi t}{T_0}\right)$$

$T_0 = 2\text{ms} = 0.002 \quad \Rightarrow \quad \boxed{v(t) = -1.5 - 3.5\cos(1000\pi t)}$

Problem#6: Problem 5-34 textbook
Find the V_p, V_{pp}, V_{avg}, and V_{rms} for the periodic waveform in given Figure.



$V_{max} = 2\text{ mV} \qquad V_{min} = -1\text{ mV}$

$$\boxed{V_p} = \max\{|V_{max}|, |V_{min}|\} = \max\{2, 1\} = \boxed{2\text{ mV}}$$

$$\boxed{V_{pp}} = V_{max} - V_{min} = 2-(-1) = \boxed{3\text{ mV}}$$

$$\boxed{V_{avg}} = \frac{1}{T}\int_t^{t+T} v(x)\,dx = \frac{1}{4}\left[\int_0^1 2\,dx + \int_2^3 (-1)\,dx\right] = \frac{1}{4}(2-1) = \boxed{0.25\text{ mV}}$$

$$\boxed{V_{rms}} = \sqrt{\frac{1}{T}\int_t^{t+T}[v(x)]^2\,dx} = \sqrt{\frac{1}{4}\left[\int_0^1 (2^2)\,dx + \int_2^3 (-1)^2\,dx\right]} = \sqrt{\frac{1}{4}(4+1)} = \frac{\sqrt{5}}{2} = \boxed{1.118\text{ mV}}$$

Problem#7: Problem 5-31 textbook

Find V_p, V_{pp}, V_{avg}, and V_{rms} for each of the following sinusoids.

a) $v_1(t) = 10\cos(2000\pi t) + 10\sin(2000\pi t)$

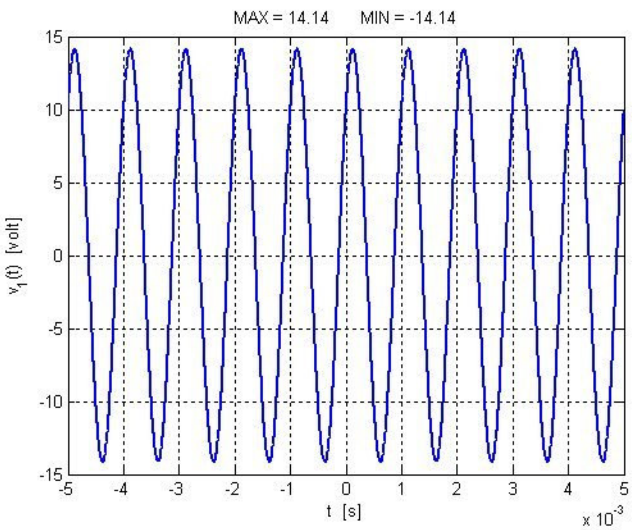
b) $v_2(t) = -30\cos(2000\pi t) - 20\sin(2000\pi t)$

(a) $\boxed{V_p} = V_A = \sqrt{10^2 + 10^2} = \boxed{14.1421}$

$\boxed{V_{pp}} = 2V_A = \boxed{28.1421}$

$\boxed{V_{avg} = 0}$ (symmetric wrt x-axis)

$\boxed{V_{rms}} = \sqrt{\frac{1}{0.001}\int_0^{0.001} [v(x)]^2 dx} = \boxed{10} = \frac{V_A}{\sqrt{2}}$

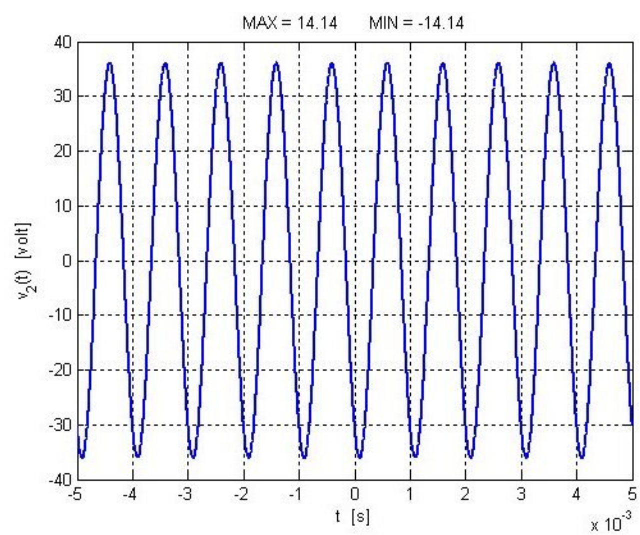


(b) $\boxed{V_p} = V_A = \sqrt{20^2 + 30^2} = \boxed{36.056}$

$\boxed{V_{pp}} = 2V_A = \boxed{72.111}$

$\boxed{V_{avg} = 0}$

$\boxed{V_{rms} = \frac{V_A}{\sqrt{2}} = 25.495}$



Problem#8: MATLAB Problem

The value of the waveform $v(t) = \left(V_A - V_B e^{-\alpha t}\right)u(t)$ is 5V at t=0, 8V at t=5 ms, and approaches 12 V when t→∞. Find α, V_A, and V_B then sketch the waveform in MATLAB.

$$v(t) = \left(V_A - V_B e^{-\alpha t}\right) u(t)$$

$$v(0) = V_A - V_B = 5V$$

$$v(5\cdot 10^{-3}) = V_A - V_B e^{-\alpha 5\cdot 10^{-3}} = 8V$$

$$\lim_{t\to\infty} v(t) = V_A = 12V$$

$$V_A = 12V$$

$$V_B = 7V$$

$$4 = 7e^{-5\alpha\cdot 10^{-3}} \quad \Rightarrow \quad e^{5\alpha 10^{-3}} = \frac{7}{4}$$

$$5\alpha\cdot 10^{-3} = \ln\left(\frac{7}{4}\right)$$

$$\alpha = 200 \ln\left(\frac{7}{4}\right) = 111.923$$

$$\boxed{v(t) = \left[12 - 7\exp(-111.923t)\right] u(t)}$$