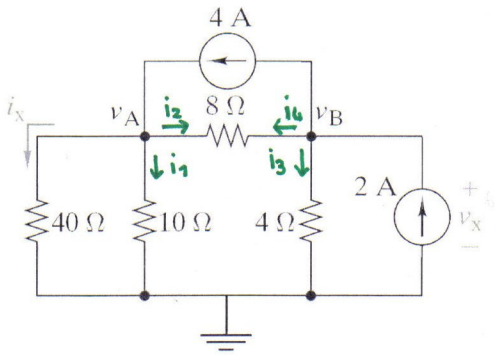
Problem#1: Problem 3-2 textbook

a) Formulate node voltage equations for the circuit.

b) Solve these equations and find v_x and i_x.



(a) $i_x + i_1 + i_2 = 4$ KCL @ NODE A

$$\frac{V_A}{40} + \frac{V_A}{10} + \frac{V_A - V_B}{8} = 4$$

$$V_A + 4V_A + 5(V_A - V_B) = 4$$

$$\boxed{10V_A - 5V_B = 160} \quad (1)$$

$i_4 + i_3 + 4 = 2$ KCL @ NODE B

$i_4 + i_3 = -2$

$$\frac{V_B - V_A}{8} + \frac{V_B}{4} = -2$$

$$V_B - V_A + 2V_B = -16$$

$$\boxed{V_A - 3V_B = 16} \quad (2)$$

(b) Node Voltage equations

$$\begin{cases} 10V_A - 5V_B = 160 & (1) \\ V_A - 3V_B = 16 & (2) \end{cases}$$

(2) → $V_A = 16 + 3V_B$

substituting in (1) → $10(16 + 3V_B) - 5V_B = 160$

$160 + 30V_B - 5V_B = 160$ ⇨ $V_B = 0$

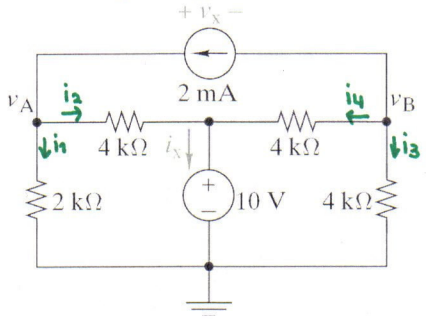
$V_A = 16$

$$\boxed{\begin{aligned} V_x &= V_B = 0\,V \\ i_x &= \frac{4V_A}{40} = \frac{16}{40} = 0.4\,A \end{aligned}}$$

Problem#2: Problem 3-4 textbook

a) Formulate node voltage equations for the circuit.

b) Solve these equations and find v_x and i_x.



(a) $i_1 + i_2 = 2\text{mA}$ KCL @ NODE A

$$\frac{V_A}{2\cdot 10^3} + \frac{V_A - 10}{4\cdot 10^3} = 2\cdot 10^{-3}$$

$$2V_A + V_A - 10 = 8$$

$$3V_A = 18$$

$$\boxed{V_A = 6\text{V}}$$

$i_3 + i_4 = -2\text{mA}$ KCL @ NODE B

$$\frac{V_B}{4\cdot 10^3} + \frac{V_B - 10}{4\cdot 10^3} = -2\cdot 10^{-3}$$

$$V_B + V_B - 10 = -8$$

$$2V_B = 2$$

$$\boxed{V_B = 1\text{V}}$$

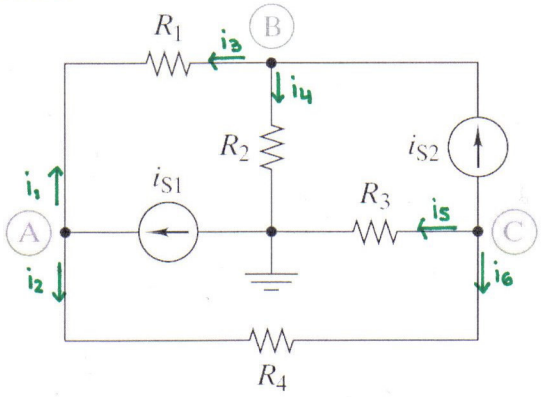
(b) $\boxed{V_x} = V_A - V_B = \boxed{5\text{V}}$

$$\boxed{i_x} = i_2 + i_4 = \frac{V_A - 10}{4\cdot 10^3} + \frac{V_B - 10}{4\cdot 10^3} = \frac{-4}{4\cdot 10^3} + \frac{-9}{4\cdot 10^3} = -\frac{13}{4}\text{ mA} = \boxed{-3.25\text{ mA}}$$

Problem#3: Problem 3-7 textbook

a) Formulate the node voltage equations for the circuit.

b) Use **MATLAB** to solve for v_A, v_B, v_C when R_1=1 kΩ, R_2=2 kΩ, R_3=4 kΩ, R_4=2 kΩ, and i_{s1}=i_{s2}=2 mA.



3 UNKNOWNS

(a) $i_1 + i_2 = i_{s1}$ KCL @ NODE A

$$\boxed{\frac{V_A - V_B}{R_1} + \frac{V_A - V_C}{R_4} = i_{s1}} \quad (1)$$

$i_3 + i_4 = i_{s2}$ KCL @ NODE B

$$\boxed{\frac{V_B - V_A}{R_1} + \frac{V_B}{R_2} = i_{s2}} \quad (2)$$

$i_5 + i_6 + i_{s2} = 0$ KCL @ NODE C

$$\boxed{\frac{V_C}{R_3} + \frac{V_C - V_A}{R_4} = -i_{s2}} \quad (3)$$

(b) Using the numerical values

$$\frac{V_A - V_B}{1} + \frac{V_A - V_C}{2} = 2 \quad \Rightarrow \quad 3V_A - 2V_B - V_C = 4$$

$$\frac{V_B - V_A}{1} + \frac{V_B}{2} = 2 \quad \Rightarrow \quad -2V_A + 3V_B = 4$$

$$\frac{V_C}{4} + \frac{V_C - V_A}{2} = -2 \quad \Rightarrow \quad -2V_A + 3V_C = -8$$

Let $A := \begin{bmatrix} 3 & -2 & -1 \\ -2 & 3 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ $B := \begin{bmatrix} 4 \\ 4 \\ -8 \end{bmatrix}$ $x := \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$

then (1)-(2)-(3) can be written as

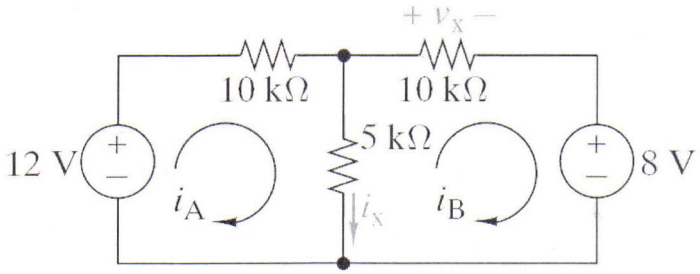
$\boxed{Ax = B}$ $x = A^{-1}B$ using Matlab $x = A \backslash B = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$

$\boxed{V_A = 4V \quad V_B = 4V \quad V_C = 0V}$

Problem#4: Problem 3-9 textbook

a) Formulate the mesh-current equations for the circuit.

b) Use equations to find v_x and i_x.



(a) Resistor unit = kΩ , I drop the 10^3 factor but I Know the current unit = mA

$$10\, i_A + 5(i_A - i_B) = 12$$

$$10\, i_B + 8 + 5(i_B - i_A) = 0$$

$$\begin{cases} 15\, i_A - 5\, i_B = 12 & (1) \\ -5\, i_A + 15\, i_B = -8 & (2) \end{cases}$$

MESH CURRENT EQUATIONS

(2) ⇨ $i_A = 3\, i_B + \frac{8}{5}$ (*)

(1) ⇨ $15\left(3\, i_B + \frac{8}{5}\right) - 5\, i_B = 12$

$45\, i_B + 24 - 5\, i_B = 12$

$40\, i_B = -12$ $\qquad \boxed{i_B = -\frac{12}{40} \cdot 10^3 = -0.3 \text{ mA}}$

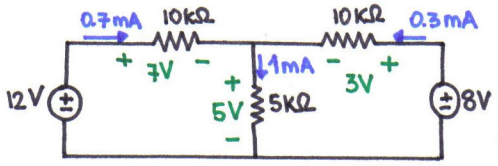
using (*) $\qquad \boxed{i_A = -0.9 + 1.6 = 0.7 \text{ mA}}$

(b)

$$V_x = 10 \cdot 10^3 \cdot (-0.3 \cdot 10^{-3}) = -3\text{V}$$

$$i_x = i_A - i_B = 0.7 - (-0.3) = 1\text{mA}$$

sanity check



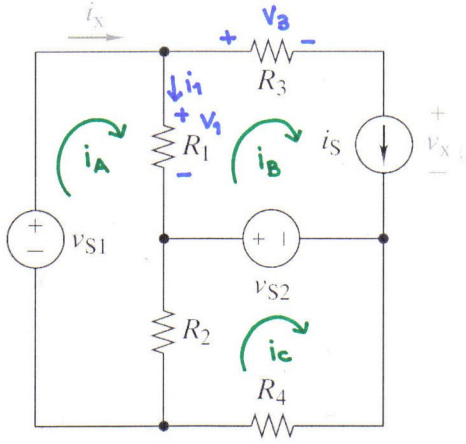
OK !!

Problem#5: Problem 3-13 textbook

a) Formulate mesh-current equations for the given circuit.

b) Solve these equations with **MATLAB** to find v_x and i_x when R_1=, R_2=10 kΩ, R_3=2 kΩ, R_4=1 kΩ, i_s=2.5 mA, v_{s1}=12 V and v_{s2}=0.5 V.

c) Find the power supplied by v_{s1}.



$i_x = i_A$

RESISTOR UNIT = kΩ
I DROP THE 10^3 FACTOR
AND I KNOW THE UNIT
FOR i_A, i_B, i_C IS mA

(a)

A : $R_1 (i_A - i_B) + R_2 (i_A - i_C) = v_{S1}$

B : $i_B = i_S$

C : $R_4 i_C + R_2 (i_C - i_A) = -v_{S2}$

$$\begin{cases} (R_1+R_2)\, i_A - R_2\, i_C = v_{S1} + R_1\, i_S \\ -R_2\, i_A + (R_2+R_4)\, i_C = -v_{S2} \\ i_B = i_S \end{cases}$$

MESH-CURRENT EQUATIONS

(b)

$20\, i_A - 10\, i_C = 12 + 10 \cdot 2.5 = 37$

$-10\, i_A + 11\, i_C = -0.5$

$$\underbrace{\begin{bmatrix} 20 & -10 \\ -10 & +11 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_A \\ i_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 37 \\ -0.5 \end{bmatrix}}_{B}$$

$Ax = B$

using Matlab $x = A \backslash B = \begin{bmatrix} 3.35 \\ 3 \end{bmatrix}$

$i_A = 3.35$ mA
$i_C = 3$ mA

(c)

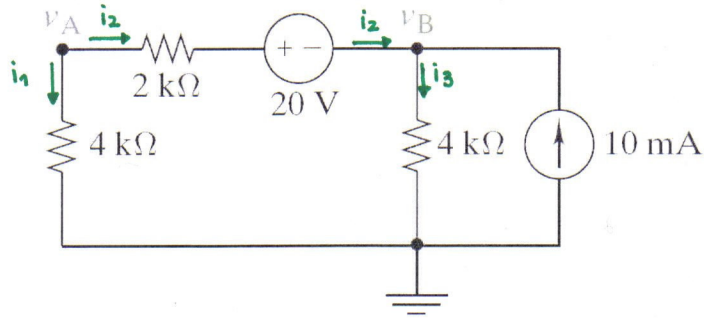
$i_x = i_A = \boxed{3.35 \text{ mA}}$

$v_x = V_1 - V_3 = R_1 i_1 - R_3 i_B = R_1 (i_A - i_B) - R_3 i_B$

$= R_1 i_A - (R_1 + R_3) i_B = 33.5 - 12 \cdot 2.5 = \boxed{3.5 \text{ V}}$

Problem#6: Problem 3-19 textbook

Find the node voltages v_A and v_B in the Figure.



A : $\dfrac{V_A}{4\cdot10^3} + \dfrac{V_A-(20+V_B)}{2\cdot10^3} = 0$

$V_A + 2\,[V_A - 20 - V_B] = 0$

$\boxed{3V_A - 2V_B = 40}$

B : $\dfrac{V_A-20-V_B}{2\cdot10^3} + 10\cdot10^{-3} = \dfrac{V_B}{4\cdot10^3}$

$2\,(V_A - 20 - V_B) + 40 = V_B$

$\boxed{2V_A - 3V_B = 0}$

$$\begin{cases} 3V_A - 2V_B = 40 & (1) \\ 2V_A - 3V_B = 0 & (2) \end{cases}$$

(2) $\Rightarrow$ $V_A = \frac{3}{2}V_B$

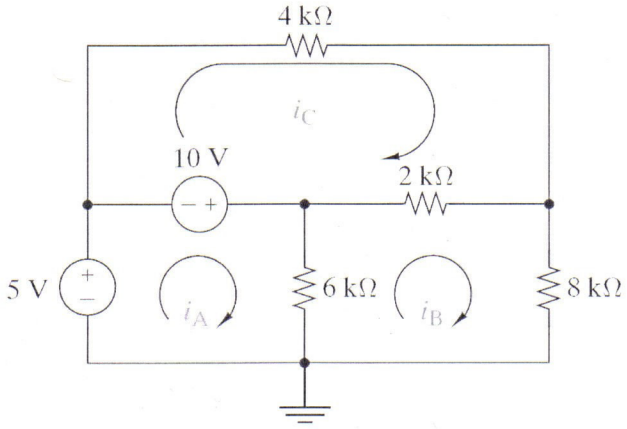
(1) $\Rightarrow$ $\frac{9}{2}V_B - 2V_B = 40$

$\frac{5}{2}V_B = 40 \Rightarrow \boxed{V_B = \frac{80}{5} = 16\,V}$

$\boxed{V_A = \frac{3}{2}.16 = 24\,V}$

Problem#7: Problem 3-20 textbook

Find the mesh currents i_A, i_B and i_C in the circuit.



RESISTOR UNIT = KΩ

I DROP THE 10^3 FACTOR AND I KNOW THAT THE UNIT FOR i_A, i_B, i_C IS mA

A: $5 + 10 = 6(i_A - i_B)$

B: $8 i_B + 6(i_B - i_A) + 2(i_B - i_C) = 0$

C: $4 i_C + 2(i_C - i_B) + 10 = 0$

$$\underbrace{\begin{bmatrix} 6 & -6 & 0 \\ 6 & -16 & 2 \\ 0 & 2 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 15 \\ 0 \\ 10 \end{bmatrix}}_{B}$$

$Ax = B$

using Matlab $\quad x = A \backslash B = \begin{bmatrix} 3.75 \\ 1.25 \\ -1.25 \end{bmatrix}$

$i_A = 3.75$ mA
$i_B = 1.25$ mA
$i_C = -1.25$ mA