The Ohio State University
Department of Electrical and Computer Engineering

# ECE 205
# Spring 2009

## Quiz #3

May 8, 2009

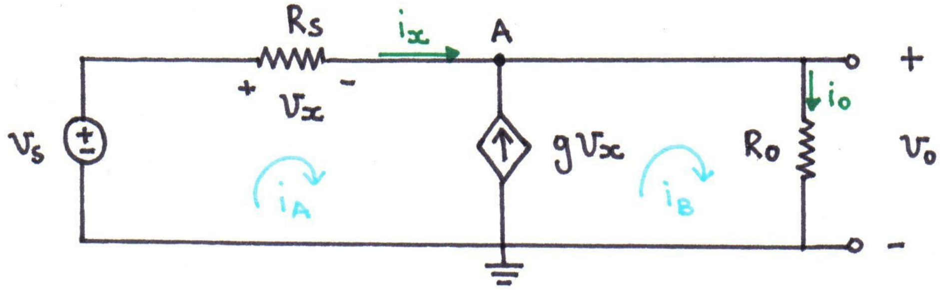
Name (print) ________________________________________

Grade____________

**- Note -**

- **There is 1 problem on the following pages.**
- **Please read the questions carefully.**

**"No aid is given, received or observed"**  **Signature____________**




Find an expression for the voltage gain $v_o / v_s$

## SOLUTION 1 NODE VOLTAGE ANALYSIS

$$\begin{cases} \dfrac{V_A - V_s}{R_s} + \dfrac{V_A}{R_o} = g v_x & \text{NODE EQUATION} \\ v_x = v_s - v_o & \text{CONTROLLING VARIABLE IN TERMS OF THE OTHER UNKNOWNS} \\ V_A = V_o & \end{cases}$$

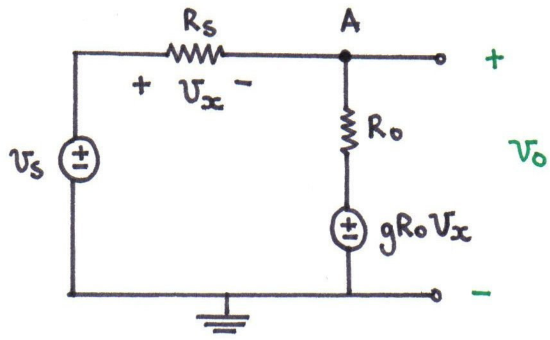
$$\Rightarrow \quad \frac{V_o}{R_s} + \frac{V_o}{R_o} = \frac{V_s}{R_s} + g(v_s - v_o)$$

$$\left(\frac{1}{R_s} + \frac{1}{R_o} + g\right) v_o = \left(\frac{1}{R_s} + g\right) v_s$$

$$\left(\frac{R_o + R_s + gR_sR_o}{\cancel{R_s}R_o}\right) v_o = \left(\frac{1 + gR_s}{\cancel{R_s}}\right) v_s$$

$$\boxed{\frac{v_o}{v_s} = \frac{R_o(1 + gR_s)}{R_o + R_s + gR_sR_o}}$$

## SOLUTION 2    SOURCE TRANSFORMATION




$$V_A = V_o \qquad (2)$$

$$\frac{V_A - V_s}{R_s} + \frac{V_A - gR_o V_x}{R_o} = 0 \qquad (1)$$

$$V_x = V_s - V_o \qquad (3)$$

Combining (1), (2) and (3) ⇨ $(V_o - V_s) R_o + R_s \left[ V_o - gR_o(V_s - V_o) \right] = 0$

$$(R_o + gR_sR_o) V_s = (R_o + R_s + gR_sR_o) V_o$$

$$\boxed{\frac{V_o}{V_s} = \frac{R_o + gR_sR_o}{R_o + R_s + gR_sR_o}}$$

## SOLUTION 3    MESH CURRENT ANALYSIS

$$\begin{cases} gV_x = i_B - i_A & (1) \text{ CONSTRAINT IMPOSED BY THE CURRENT SOURCE} \\ R_s i_A + R_o i_B = V_s & (2) \text{ SUPERMESH EQUATION} \\ V_x = R_s i_A & (3) \text{ CONTROLLING VARIABLE IN TERMS OF THE OTHER UNKNOWNS} \end{cases}$$

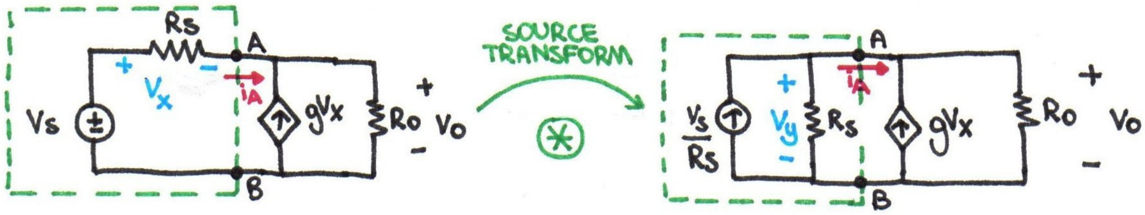
(1) and (3) ⇨ $gR_s i_A + i_A = i_B$ ⇨ $i_A = \frac{1}{1 + gR_s} i_B$

substitute in (2) ⇨ $\frac{R_s}{1 + gR_s} i_B + R_o i_B = V_s$

$$\frac{R_s + R_o(1 + gR_s)}{1 + gR_s} i_B = V_s \quad ⇨ \quad i_B = \frac{1 + gR_s}{R_s + R_o(1 + gR_s)} V_s$$

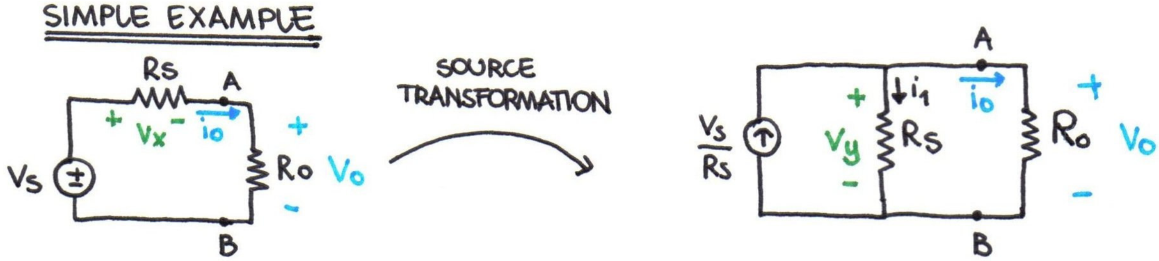
$$\boxed{V_o = R_o i_B} \quad ⇨ \quad \boxed{\frac{V_o}{V_s}} = \frac{R_o i_B}{V_s} = \boxed{\frac{R_o(1 + gR_s)}{R_s + R_o(1 + gR_s)}}$$

# SOURCE TRANSFORMATION




if you transform the independent voltage source into a current source you "loose" the info about $V_x$ . NOTE THAT $V_x \neq V_y$ (in general)

## SIMPLE EXAMPLE




$i_o = \frac{V_s}{R_s + R_o}$

$V_o = \frac{R_o}{R_s + R_o} V_s$

$V_x = \frac{R_s}{R_s + R_o} V_s$

using current division rule $\quad i_1 = \frac{R_o}{R_s + R_o} \cdot \frac{V_s}{R_s}$

$i_o = \frac{\cancel{R_s}}{R_s + R_o} \cdot \frac{V_s}{\cancel{R_s}} = \frac{V_s}{R_s + R_o}$

$V_o = R_o i_o = \frac{R_o}{R_s + R_o} V_s$

$V_y = V_o \neq V_x$

$i_o$ and $V_o$ do not change but the potential difference across $R_s$ changes (if $R_s \neq R_o$)

**WHEN YOU USE SOURCE TRANSFORMATION THE TWO CIRCUITS WILL HAVE THE SAME i-v CONSTRAINTS AT THE TERMINALS A-B THAT IS, EITHER MODEL DELIVERS THE SAME VOLTAGE AND CURRENT TO THE REST OF THE CIRCUIT, HOWEVER**

- **EQUIVALENT SOURCES DO NOT HAVE THE SAME INTERNAL POWER LOSS**
- **IN THE TRANSFORMATION YOU "LOOSE THE INFO" ABOUT THE CURRENT AND POTENTIAL DIFFERENCE ACROSS $R_s$.** So if you want to apply source transformation (*), you have to derive first an expression for $V_x$ : $\boxed{V_x = R_s \cdot i_A}$ then you can change the source and consider the "equivalent current $i_A$" (= current through the terminal A) But <u>you CAN'T SAY $V_x = V_y$</u>