Notes for Lecture #3
Pulse Width Modulation

8
Sequential Method of Simulation
of Power Electronic Systems

8.0 Introduction
8.1 Decoupled and Coupled Power Electronic Systems
8.2 Analysis of Decoupled Systems
   8.2.1 Analysis of Chopper-Fed DC Motor
   8.2.2 Analysis of Inverter-Fed Induction Machine
8.3 Analysis of Coupled Systems
   8.3.1 Synchronous Machine Fed from a Naturally
        Commutated Inverter
   8.3.2 Induction Machine Fed from a Forced-Commutated
        Current Source Inverter
   8.3.3 Computer-Aided Analysis of AC Machine-
        Converter Group
8.4 Summary
8.5 References

8.0 INTRODUCTION

Our earlier analysis of power electronic converter systems composed of a static
power converter, an electrical machine, and associated control strategy was done
using global simulation programs such as ATORECS, BACKO, and SACMA [1-3].
In these computer-aided analysis simulation programs, no prior knowledge of the
sequence of operation of the converter was assumed. For the study, we had to
know the circuit topology of the converter, the type of electrical machine, and the
control logic for the turn-on instants of the controlled semiconductors. In this
chapter we discuss another method of simulation called the sequential method. In
this method, as the name suggests, the sequence of operation of the power converter...
is assumed to be known; further, the time instants at which the change of sequence occurs is also assumed to be known. This implies that in one class of problems, the output voltages or currents of the power converter are known to be independent of the operating modes of the electrical machine. We refer to this class of systems as decoupled systems. In other words, the analysis of the system can be reduced to the analysis of the electrical machine fed from nonideal sources, such as pulsed sources for dc machines and nonsinusoidal voltage or current sources for ac machines. In another class of problems, only the sequence of operation of the converter is known, although the instants at which the change of sequence occurs might have to be determined during the analysis. We refer to this class of systems as coupled systems. We discuss both these methods of analysis in this chapter.

8.1 DECOUPLED AND COUPLED POWER ELECTRONIC SYSTEMS

Consider the power electronic system consisting of a three-phase line-commutated converter and a dc machine shown in Fig. 8.1. The control strategy used is such that the six thyristors of the converter are turned on at a constant phase-angle delay in order to obtain a rectifying mode of converter operation. A simplified analysis of this converter in the rectifying mode is well known [4]. The sequence of firing of the thyristors is shown in Fig. 8.1. The analysis of the dc motor fed from the converter can easily be done if we assume that the commutation overlap angles are neglected. In other words, the output voltage of the converter can be assumed to be as shown in Fig. 8.3 if the commutation is instantaneous. It is thus possible to decouple the converter operation from the electrical machine. The analysis of the system is therefore simplified to one in which a nonideal dc voltage source such as the one shown in Fig. 8.2 is used to feed the dc machine. Note that the voltage drops due to the source resistance and inductance are taken into account indirectly by adding an equivalent series resistor at the output side of the converter.

Let the commutation interval of the converter be considered for the system. It is well known that the commutation overlap angle depends on the value of the instantaneous current to be commutated. To analyze this operating condition, we utilize the coupled system, where only the turn-on instants of the thyristors are known. The sequence of operation of the different thyristors is shown in Fig. 8.3, which indicates an operating sequence 1-5, 1-2-3, 2-3, 2-3-4, 3-4, 3-4-5, 4-5, 4-5-6, 5-6, 5-6-1, 6-1-2, 1-2, and so on. The output voltage waveform is

FIGURE 8.2 Output voltage of a three phase line-commutated converter assuming instantaneous commutation.

FIGURE 8.3 Sequence of operation of thyristors in a three phase converter with commutation overlap angle $\mu$. 
only partially defined because the instants at which the change of sequence from 1-2-3 to 2-3, and so on, occur are unknown and have to be computed during the analysis. The typical output waveform is shown in Fig. 8.4. This class of systems are coupled systems.

EXAMPLE 8.1. Consider the chopper-fed dc machine shown in Fig. 8.5. Discuss how the analysis of the system can be done assuming a decoupled system.

Solution: The operating sequence of the chopper system is well known [5] and can easily be established by reference to Fig. 8.5. The conduction of thyristor TM supplies a source voltage Vd to the armature of the dc machine. Assuming an initial voltage of suitable polarity across the commutating capacitor Cc, turning on the auxiliary thyristor, TA, results in turning off the main thyristor by the application of an inverse voltage across the main thyristor. Assuming that this commutation period is negligible, the output waveform of the chopper circuit can be determined independent of the operating mode of the dc machine. The system can then be analyzed as a decoupled system. That is, the output voltage waveform of the chopper is a pulsed dc voltage, as shown in Fig. 8.6. The ratio between the conduction interval of the main thyristor and the period of the chopper is designated as the duty cycle (D) of the chopper.

8.2 ANALYSIS OF DECOUPLED SYSTEMS

8.2.1 Analysis of Chopper-Fed DC Motor

Various methods are available for obtaining a variable dc voltage from a constant dc source [6]. In thyristor-type dc-to-dc converters, commutating circuits are required which are similar to those used in many inverters. A classical chopper scheme employing an impulse commutation technique is chosen for this study [7]. A fixed frequency modulation is used for the chopper circuit, because in this scheme, harmonics of a particular frequency alone will occur and can easily be filtered. The variable dc voltage available from the chopper is used to feed the armature of the dc motor with separate excitation, thus controlling the speed of the motor. An inductor has been introduced in the armature circuit to reduce the armature current ripples. In what follows, a chopper-fed dc motor group is analyzed and the steady-state performance characteristics of the dc motor are obtained.

Figure 8.7 shows a dc-to-dc converter and the associated control logic signals for firing of the thyristors. The operating sequences of the converter are characterized by four intervals: precharging interval, duty interval, commutation interval, and freewheeling interval.

PRECHARGING INTERVAL. The control logic used here is such that initially, only the auxiliary thyristor, TA, is turned on periodically; the commutating capacitor Cc is allowed to get charged to a voltage equal to the dc voltage source. The polarity of the voltage across the commutating capacitor is such that point A in Fig. 8.7 is more positive than point B. This precharging interval is essential to ensure proper commutation in the chopper circuit. Normally, the operating cycle of the chopper consists of a duty interval, a commutation interval, and a freewheeling interval.
DUTY INTERVAL. With the firing of main thyristor, T_{m}, the duty interval begins and the dc source voltage appears across the dc motor, assuming a negligible forward voltage drop across the main thyristor. The capacitor voltage reverses its polarity and attains the same magnitude at the end of a half-period of the resonant circuit constituted by \( L_{c} \) and \( C_{c} \), and negligible resistance. Any further discharge of the capacitor is prevented by the blocking diode, \( D_{c} \), connected in series with the commutating inductor, \( L_{c} \). As soon as the auxiliary thyristor, \( T_{a} \), is turned on, the commutation interval begins and the duty interval terminates.

COMMUTATION INTERVAL. The firing of the auxiliary thyristor marks the beginning of the commutation interval. The load current flowing through the main thyristor is transferred to the commutating capacitor and the auxiliary thyristor. The capacitor voltage is applied across the anode-cathode terminals of the main thyristor as an inverse voltage. It is necessary that this inverse voltage be applied across the main thyristor for at least the turn-off time of the main thyristor. The capacitor gets charged from the dc source through the load and changes its polarity. The commutation interval can last until the capacitor gets charged to a voltage equal to the source voltage or can terminate when the main thyristor is refired. In the former case, the capacitor voltage can have a tendency to get charged to a voltage greater than the source voltage in inductive and emf types of loads. Such a tendency initiates conduction of the freewheeling diode \( D_{f} \), which marks the beginning of the freewheeling interval. In the latter case, where the commutation

interval is terminated by refiring of the main thyristor even before the capacitor voltage has attained the supply voltage value, there results a reduced voltage available for commutation of the main thyristor in the next cycles of operation. It can therefore be noted that commutation failure is bound to occur as the duty interval approaches the period of the chopper. Commutation failure results in permanent conduction of the main thyristor, and the control over duty interval can be regained only after opening of the load circuit followed by a precharging interval.

FREEWHEELING INTERVAL. During this interval, the load current is transferred through the freewheeling diode and the dc motor armature circuit. In the case of emf loads, this interval terminates as soon as the load current falls below a value leading to blocking of the freewheeling diode or as soon as the main thyristor is turned on. The former case occurs when the motor is lightly loaded, as the duty cycle is low. This results in a discontinuous armature current; otherwise, the armature current is continuous.

STEADY-STATE ANALYSIS OF THE CHOPPER-FED DC MOTOR WITH SEPARATE EXCITATION. For purposes of analysis, the following assumptions are made:

1. The armature circuit resistance \( R_{a} \) is constant.
2. The inductance \( L_{a} \) of the armature circuit is constant.
3. The speed \( N \) of the motor is constant.
4. The forward voltage drops of semiconductor elements and the source impedance are neglected.
5. The field of the motor is separately excited and the flux is constant.

Let the armature current at the end of the duty interval be \( I_{d} \) and let its value at the end of the commutation and freewheeling intervals be \( I_{c} \) and \( I_{f} \), respectively. Let \( N \) be equal to the speed of the motor. Since the field is excited from a constant separate dc source, the induced emf is proportional to the speed of the motor. Our objective is to find the average value of the armature current for given values of speed \( N \) and known values for the various intervals.

The differential equations governing the commutation, duty, and freewheeling intervals of the chopper-fed motor group can now be formulated.

COMMITUTION INTERVAL \( 0 < t < k_{T}T \). Let the instantaneous armature current during the commutation interval be designated as \( i_{c} \). The equations governing this interval are

\[
\begin{align*}
V_{d} &= R_{c}i_{c} + L_{a} \frac{di_{c}}{dt} + K_{m} \frac{N}{m} + V_{c} \\
C_{c} \frac{dv}{dt} &= i_{c} \\
\end{align*}
\]

where

\[
K_{m} = \text{back emf constant for the dc motor}
\]

\[
V_{c} = \text{instantaneous voltage across the commutating capacitor}
\]
Sequential Method of Simulation

The initial conditions for the capacitor voltage and the armature currents are defined by

\[ v_c |_{t=0} = -v_{cf} \]  \hspace{1cm} \text{(8.3)}

\[ i_c |_{t=0} = i_d \]  \hspace{1cm} \text{(8.4)}

Let the final values of \( v_c \) and \( i_c \) at the end of the commutation interval be defined as given in

\[ v_c |_{t=kT} = v_{cf} \]  \hspace{1cm} \text{(8.5)}

\[ i_c |_{t=kT} = i_c \]  \hspace{1cm} \text{(8.6)}

Solutions for \( i_c \) and \( v_c \) can be written

\[ i_c = i_{cl} \exp(-\beta t) \sin \omega t - I_d \frac{w_0}{\omega} \exp(-\beta t) \sin(\omega t + \phi) \]  \hspace{1cm} \text{(8.7)}

\[ v_c = v_d - K_m N + v_{cl} \frac{w_0}{\omega} \exp(-\beta t) \sin(\omega t + \phi) + I_d \frac{d}{\omega C_c} \exp(-\beta t) \sin \omega t \]  \hspace{1cm} \text{(8.8)}

where

\[ i_{cl} = (V_d - K_m N + v_d)/\omega L_a \]

\[ v_{cl} = v_d - K_m N \]

\[ \omega_0^2 = 1/L_a C_c \]

\[ \beta = \sqrt{R/2L_a} \]

\[ \omega^2 = \omega_0^2 - \beta^2 \]

\[ \phi = \tan^{-1}(\omega/\beta) \]

\[ \exp = \text{exponential operation} \]

If the losses in the circuit are relatively small, then \( \omega_0 \approx \beta \) and the simplifications given below are valid:

\[ \omega_0 \approx \omega \]

\[ X = \sqrt{\frac{C_c}{C_a}} \omega L_a = \frac{1}{\omega C_c} \]

Analysis of Decoupled Systems

\[ Q = \frac{X}{R_a} \]

\[ \phi = \frac{\pi}{2} \]

Equations (8.7) and (8.8) can be simplified as follows:

\[ i_c = (i_{cl} \sin \omega t + I_d \cos \omega t) \exp \left(-\frac{\omega t}{2Q}\right) \]  \hspace{1cm} \text{(8.11)}

\[ v_c = (v_d - K_m N + (X_d \sin \omega t - v_{cl} \cos \omega t) \exp \left(-\frac{\omega t}{2Q}\right)) \]  \hspace{1cm} \text{(8.12)}

Equations (8.11) and (8.12) describe the performance of the converter and the dc motor during the commutation interval. We derive next the equations governing the performance of the system during the freewheeling interval.

FREEWHEELING INTERVAL \( k_c T < t < (k_c + k_f)T \). The condition for the existence of this interval is derived from the final value of the capacitor voltage at the end of the commutation interval. This is given by

\[ V_{cf} = v_d \]  \hspace{1cm} \text{(8.13)}

That is, this interval exists only if the commutating capacitor tends to get charged to a value greater than that of the dc source voltage. The load current circulates only through the armature and the freewheeling diode; the equation governing the operation of the system is

\[ R_a \frac{di_f}{dt} + K_m N = 0 \]  \hspace{1cm} \text{(8.14)}

The initial and final conditions for the freewheeling load current \( i_f \) are defined by

\[ i_f |_{t=k_c T} = 0 \]

\[ i_f |_{t=(k_c + k_f)T} = i_f \]  \hspace{1cm} \text{(8.15)}

The solution for the freewheeling current \( i_f \) is given by

\[ i_f = \frac{-K_m N}{R_a} \left(1 - \exp\left[\frac{R_a}{L_a} (t + k_f T)\right]\right) + I_c \exp\left[\frac{R_a}{L_a} (t - k_c T)\right] \]  \hspace{1cm} \text{(8.16)}

The freewheeling interval is terminated when the load current becomes zero or when the main thyristor is turned on.
DUTY INTERVAL \((k_c + k_d)T < t \leq T\). The load current \(i_d\) during the duty interval is described by

\[
R_a \frac{di_d}{dt} + L_a \frac{di_d}{dt} + \frac{N_i}{K_m} = V_d
\]

(8.17)

The initial and final conditions of current are given by

\[
\begin{align*}
\frac{di_d}{dt} & = I_f \\
\frac{di_d}{dt} & = I_d
\end{align*}
\]

(8.18, 8.19)

The following equation implies that the steady-state solution is reached:

\[
\frac{di_d}{dt} = 0
\]

(8.20)

Using the relation given by Eq. (8.20), the solution for the load current during the duty interval can be obtained:

\[
i_d = \frac{V_d}{R_a} \left[1 - \exp \left(\frac{-T_d}{T_e}\right)\right] i_f \exp \left(\frac{T_d}{T_e}\right)
\]

(8.21)

where

\[
T_d = \frac{R_a}{L_a} \left[-t + (k_c + k_d)T\right]
\]

Let us analyze the operation of the chopper-motor group assuming that the load current is continuous. Further, for low values of the duty cycle \(k_d\), the magnitude of the initial capacitor voltage is equal to the dc source voltage. The initial current \(i_d\) is to be calculated by an iterative procedure satisfying the differential equations for the three intervals with the various boundary conditions. Four different methods are discussed in the literature [6] for the calculation of \(i_d\), \(i_c\), and \(i_f\) without resorting to an iterative procedure. The average value of the armature current can then be computed as a function of speed. These methods are based on the following assumptions:

Method 1: negligible commutation interval
Method 2: negligible ripple in the armature current
Method 3: constant current during commutation
Method 4: relatively small losses in the commutating circuit

The assumption of a negligible commutation interval enables the study of the system as a decoupled system.

Analysis of Decoupled Systems

METHOD 1: STUDY OF THE DECOUPLED SYSTEM. The assumption of a negligible commutation interval leads to a decoupled system. The following equations hold good under this condition:

\[
k_c = 0 \quad I_c = I_d
\]

(8.22)

Using Eqs. (8.16) and (8.21), we can derive

\[
I_f = \frac{V_d}{R_a} \left[1 - \exp \left(\frac{-T}{T_e}\right)\right] + I_f \exp \left(\frac{T}{T_e}\right)
\]

(8.23)

\[
I_d = \frac{V_d}{R_a} \frac{K_m}{N_i} \left[1 - \exp \left(\frac{-T}{T_e}\right)\right] + I_f \exp \left(\frac{T}{T_e} + \frac{k_d T}{T_e}\right)
\]

(8.24)

\[
I_f = \frac{V_d}{R_a} \exp \left(\frac{k_d T}{T_e}\right) \left[1 - \frac{K_N}{R_a} \exp \left(\frac{T}{T_e}\right) - 1\right] - \frac{K_N}{R_a}
\]

(8.25)

\[
I_d = \frac{V_d}{R_a} \exp \left(-\frac{k_d T}{T_e}\right) \left[1 - \frac{K_N}{R_a} \exp \left(-\frac{T}{T_e}\right) - 1\right] + \frac{K_N}{R_a}
\]

(8.26)

\[
I_{av} = \frac{V_d}{R_a} \left[\frac{k_c}{K_m} - \frac{K_N}{R_a}\right]
\]

(8.27)

The average value of the torque developed by the motor is given by

\[
T_{em} = \frac{60K_f}{m_{av}} \frac{T}{2\pi}
\]

(8.28)

The waveforms predicted for the armature current and the chopper output voltage are shown in Fig. 8.8. Analysis of the chopper-fed dc motor based on the other simplified assumptions (methods 2, 3, and 4) described above is left as a series of exercises to the reader. The torque-speed characteristics of the chopper-fed dc motor obtained by the four methods are shown in Fig. 8.9. In Fig. 8.9, different values of the duty cycle \(k_d\) have been assumed for the computation. The experimental measurement – of the test points are also indicated on Fig. 8.9.

For experimental verification of the four methods of analysis, tests were conducted on a 1.5-kW 220-V dc motor with separate excitation. The field current of the motor was maintained constant at its rated value of 3.7 A. A 200-μH choke coil was included in the armature circuit for smoothing the armature current waveform. Three sets of experiments were conducted for values of \(k_d\) = 0.11, 0.32, and 0.45, with the input voltage of the chopper maintained constant at 200 V. The relevant details of the chopper-motor group are as follows:
Sequential Method of Simulation

\[ V_{d} = 200 \text{ V} \quad T = 6.67 \text{ ms} \quad L_a = 200 \text{ mH} \quad C = 20 \mu \text{F} \quad R_a = 5 \Omega \]

\[ T_e = 40 \text{ ms} \quad K_m = 0.1435 \text{ V/rpm} \quad \text{nominal speed} \, N = 1392 \text{ rpm} \]

nominal torque \( T_e = 10 \text{ N-m} \)

In each experiment, the load was varied and the average torque and speed were measured. Figure 8.9 shows these characteristics. Waveforms of the armature current observed during the test conditions showed that there was a continuous conduction in the armature current.

From the characteristics it is seen that the assumption of negligible commutation (method 1) leads to a very simple expression for the torque-speed relationship, but the prediction is very much in error; this method underestimates the torque-developing capability of the dc motor. Further, this assumption is not justified, because the commutation interval varies over a wide range depending on the loading conditions of the motor. The commutation interval is, in general, comparable to the duty interval even under the heavily loaded conditions of the motor. Except for loads where the speed factor, \( N/N_e \), approaches unity, the test points lie more or less midway between the curves that have been predicted using method 3 and method 2. In general, method 4 predicts the characteristics that are in reasonable agreement with experimental values. However, a relatively large deviation is noted in the values corresponding to heavily loaded conditions of the motor. Better agreement between experimental and computed results from method 4 could be expected if the increased value of the apparent resistance of the armature due to the high-frequency pulsating current is used in place of the dc value in the equations. A method of measuring the apparent resistance of the

![Vd t](image)

**FIGURE 8.8** Chopper output voltage and armature current waveforms—Method 1.

![N/N](image)

**FIGURE 8.9** Speed-torque characteristics of chopper-fed dc motor.

Analysis of Decoupled Systems

![N/N](image)

**FIGURE 8.10** Speed-torque characteristics of chopper-fed dc motor.

armature circuit is suggested in the literature [5]. The speed-torque characteristics of the dc drive system, taking into account the apparent resistance, are shown in Fig. 8.10. For this computation, method 4, wherein relatively small losses in the commutating circuit are assumed, was used.
Analysis of Decoupled Systems

such that the induction motor sees the waveforms as they would be when implemented in a practical system [8].

DYNAMIC EQUATIONS OF INDUCTION MACHINE. For the purpose of analysis, a symmetrical induction machine in a two-axis model is considered. Although any arbitrary reference frame could be used for simulation of an induction machine, let us choose a stationary reference frame for its representation. Recall that the transformation between the d-q variables and the real stator variables is time invariant. The ac drive system is assumed to be a decoupled system; that is, the induction machine is fed from an inverter whose output voltages are independent of the operating modes of the induction machine. Note that the output voltages of the inverter are usually nonsinusoidal.

The machine equations in the stationary reference frame in terms of flux linkages is per unit values are given by

\[
\frac{d\psi_{sd}}{dt} = (r_{sd} - r s_{sd})\omega_N
\]

\[
\frac{d\psi_{rd}}{dt} = (r_{rd} - \omega \psi_{rd})\omega_N
\]

\[
\frac{d\psi_{dq}}{dt} = (r_{dq} + \omega \psi_{dq})\omega_N
\]

\[
\frac{d\psi_{sq}}{dt} = (r_{sq} - r s_{sq})\omega_N
\]

Note that the transformed stator and rotor flux linkages are given by

\[
\psi_{sd} = L_s i_{sd} + L_m i_{rd}
\]

\[
\psi_{rd} = L_r i_{rd} + L_m i_{sd}
\]

\[
\psi_{dq} = L_r i_{dq} + L_m i_{sq}
\]

\[
\psi_{sq} = L_s i_{sq} + L_m i_{rq}
\]

where

\[
L_s \quad \text{and} \quad L_r = \text{stator and rotor self-inductances}
\]

\[
L_m = \text{mutual inductance between stator and rotor windings}
\]

\[
\omega_N = \text{nominal synchronous speed of the rotating magnetic field}
\]

The mechanical equation for the drive is given by

\[
\frac{d\omega}{dt} = \frac{T_s - T_L}{J_M}
\]
Sequential Method of Simulation

where

\[ T_e = \text{electromagnetic torque developed; this is given by} \]

\[ T_e = \frac{L_m}{\sigma L_r} (\phi_a \phi_d - \phi_d \phi_a) \]  \hspace{2cm} (8.32)

\[ T_L = \text{load torque} \]

\[ \omega_r = \text{rotor speed} \]

\[ \sigma = (L_d L_r - i_m^2)/L_r \]

Note that the flux linkages in coupled circuits tend to change more slowly than the currents; therefore, the use of flux linkages provides more computational stability.

The expressions for the transformed currents can be derived using Eq. (8.30); these are given by

\[ i_{sd} = \frac{1}{L_s} \left( \phi_{sd} - \frac{L_m}{L_r} \phi_{rd} \right) \]

\[ i_{rd} = \frac{1}{L_r} \left( \phi_{rd} - \frac{L_m}{L_s} \phi_{sd} \right) \]  \hspace{2cm} (8.33)

\[ i_{rq} = \frac{1}{L_s} \left( \phi_{rq} - \frac{L_m}{L_r} \phi_{sq} \right) \]

\[ i_{sq} = \frac{1}{L_s} \left( \phi_{sq} - \frac{L_m}{L_r} \phi_{rq} \right) \]

The actual line currents, \( i_a, i_b, \) and \( i_c \) of the motor are related to the transformed stator currents as given in

\[ i_a = i_{sd} \]

\[ i_b = -\frac{i_{rd}}{2} + \frac{\sqrt{3}}{2} i_{sq} \]  \hspace{2cm} (8.34)

\[ i_c = -\frac{i_{rd}}{2} - \frac{\sqrt{3}}{2} i_{sq} \]

For the analysis of the induction machine, all the equations derived in this section are used.

SIMULATION OF THREE-PHASE INVERTER OUTPUT VOLTAGE WAVEFORMS.

The inverter configuration and the motor connections are shown in Fig. 8.11. The
three-phase bridge inverter is operated in such a way that at any instant of time, the poles have defined voltages; that is, at any instant, one of the devices will be conducting in each leg. The motor phase voltages can, therefore, easily be derived; these equations are

\[
\begin{align*}
V_{a0} &= \frac{2V_{an} - V_{bn} - V_{cn}}{3} \\
V_{b0} &= \frac{2V_{bn} - V_{an} - V_{cn}}{3} \\
V_{c0} &= \frac{2V_{cn} - V_{an} - V_{bn}}{3}
\end{align*}
\]  \hspace{1cm} (8.35)

For a balanced three-phase supply, the transformed voltages \(V_{ad}\) and \(V_{aq}\) can be computed using

\[
\begin{align*}
V_{ad} &= V_{a0} \\
V_{aq} &= \frac{V_{bn} - V_{cn}}{\sqrt{3}}
\end{align*}
\]  \hspace{1cm} (8.36)

Note that Eqs. (8.35) and (8.36) are derived assuming negligible voltage drop in the devices and neglecting the effect of snubber circuits. We now consider three different control strategies for the inverter and derive the corresponding inverter output voltage waveforms:

1. 180° square-wave inverter control
2. Sinusoidal pulse-width-modulated (PWM) inverter control
3. Symmetrical sinusoidal PWM (SSPWM) inverter control

EQUATIONS FOR 180° SQUARE-WAVE INVERTER CONTROL STRATEGY. The output voltage waveforms that are applied to the three-phase induction motor are shown in Fig. 8.12. Referring to Fig. 8.12, we see the following characteristics of inverter operation for this method of control:

1. Semiconductor devices switch sequentially every 60°.
2. At any instant, three semiconductor devices are conducting.
3. Each semiconductor device conducts for a duration of 180°.

The frequency of the inverter is varied by varying the frequency of switching of the devices; the output voltage of the inverter can be varied by varying the input voltage to the inverter.

Referring to Fig. 8.11, we note that the pole voltage \(V_{an}\) is positive when either the thyristor T1 or the diode D1 is conducting. When T1 conducts, the current \(i_a\) is positive and when D1 conducts, \(i_a\) is negative. This can be stated in the form

FIGURE 8.12 Output voltage waveforms for 180° mode of operation.
Sequential Method of Simulation

\[ v_{an} = \begin{cases} v_d & \text{if } T1 \text{ or } DI \text{ is ON (i.e., } TDI \text{ is .TRUE.)} \\ 0 & \text{if } T1 \text{ and } DI \text{ are OFF (i.e., } TDI \text{ is .FALSE.)} \end{cases} \quad (8.37) \]

In a similar way, this logic holds good for the other two phases. The DC supply current due to phase \( a \) is \( i_a \) when \( TDI \) is .TRUE. and is zero when \( TDI \) is .FALSE. Thus the DC source current can be computed using the logical statement

\[ i_{dc} = -\frac{1}{3} TDI + \frac{1}{3} TD3 + \frac{1}{3} TD5 \quad (8.38) \]

For implementing the 180° square-wave inverter on the digital computer, the following six states are switched after every 60° at the required frequency of operation of the inverter:

STATE 1: \( TDI = .TRUE.; \quad TD3 = .FALSE.; \quad TD5 = .TRUE. \)
STATE 2: \( TDI = .TRUE.; \quad TD3 = .FALSE.; \quad TD5 = .FALSE. \)
STATE 3: \( TDI = .TRUE.; \quad TD3 = .TRUE.; \quad TD5 = .FALSE. \)
STATE 4: \( TDI = .FALSE.; \quad TD3 = .TRUE.; \quad TD5 = .FALSE. \)
STATE 5: \( TDI = .FALSE.; \quad TD3 = .TRUE.; \quad TD5 = .TRUE. \)
STATE 6: \( TDI = .FALSE.; \quad TD3 = .FALSE.; \quad TD5 = .TRUE. \)

EQUATIONS FOR SINUSOIDAL PWM INVERTER CONTROL. The power circuit configuration for this inverter is the same as that shown in Fig. 8.11, in which either thyristors or transistors can be used. The PWM control strategy is realized by using a high-frequency triangular waveform as the carrier wave and three-phase sinusoidal waveforms as the modulating waves; the intersection points of the triangular waveform with the sinusoidal waveforms define the turn-on and turn-off instants for the six transistors. The inverter output voltage waveforms, \( v_a, v_b, \) and \( v_c \), are shown in Fig. 8.13. For example, if \( TDI \) is ON when the phase \( a \) sine-wave amplitude is greater than the triangular wave (i.e., \( v_a > v_d \)), \( TDI \) is OFF when the amplitude of the phase \( a \) sine wave is less than the triangular wave (i.e., \( v_a < v_d \)). This logic also holds true for the other two phases.

Let the equations for the three-phase sinusoidal waves be

\[ v_a = M \sin \omega t \]
\[ v_b = M \sin \left( \omega t - \frac{\pi}{3} \right) \]
\[ v_c = M \sin \left( \omega t - \frac{2\pi}{3} \right) \quad (8.39) \]

where \( M \) represents the modulation index:

\[ M = \frac{\text{amplitude of sine wave}}{\text{amplitude of carrier (triangular) wave}} \]

FIGURE 8.13 Output voltage waveforms for sinusoidal PWM inverter.

Further, let the equations for the triangular carrier wave be

\[ x_1 = -1.0 + 2N \frac{\omega t}{\pi} \quad \text{for } 0 < \omega t < \frac{\pi}{N} \]
\[ x_2 = 3.0 - 2N \frac{\omega t}{\pi} \quad \text{for } \frac{\pi}{N} < \omega t < \frac{2\pi}{N} \quad (8.40) \]
Analysis of Decoupled Systems

For numerical computation of the intersection points, the proper slopes of the triangular waveform are chosen after every interval of $\pi/N$. The inverter output voltages are therefore defined as follows:

$$
\begin{align*}
\text{If } v_a & \geq x_1 \text{ or } x_2, \text{ then } v_{an} = V_d \\
\text{If } v_b & \geq x_1 \text{ or } x_2, \text{ then } v_{bn} = V_d \\
\text{If } v_c & \geq x_1 \text{ or } x_2, \text{ then } v_{cn} = V_d \\
\text{Otherwise, } v_{an} = 0; v_{bn} = 0; v_{cn} = 0
\end{align*}
$$

(8.41)

The expression for the dc source current is similar to Eq. (8.38).

EQUATIONS FOR SFPWM CONTROL STRATEGY. The limitation of the three-phase bridge inverter shown in Fig. 8.11 is that any type of modulation technique cannot be implemented using it, because of the pole switching constraint [8]. Any desired waveform could be applied to the induction machine by feeding each phase of the machine separately by a single-phase bridge inverter. This power circuit configuration is shown in Fig. 8.14. The apparent limitations of the power circuit are:

1. The requirement of a large number of semiconductor devices, resulting in higher cost than for a conventional three-phase bridge configuration
2. The presence of zero-sequence currents in the stator windings

The advantages of the configuration are:

1. Possible implementation of any type of modulation technique without a pole switching constraint
2. Availability of higher output voltage than with a conventional three-phase bridge configuration for the same dc link voltage
3. Possible operation of the devices in order to reduce the switching losses

The induction machine fed from separate inverters as shown in Fig. 8.14 can be modeled by considering it to be similar to a star connection with neutral connected. Equations (8.29) to (8.33) are valid in this case. In addition, an expression for zero-sequence current is required:

$$
\frac{d}{dt} [s0] = (v_s - r [s0])/\omega_N
$$

(8.42)

Referring to Fig. 8.14, we can derive the relations between the real and the transformed voltages, $v_{s0}$, $v_{sQ}$, and $v_{s0}$ and $v_{s1}$, $v_{s2}$, and $v_{s3}$:

$$
\begin{align*}
v_{sd} &= \frac{1}{3} \left[ v_{s1a2} + (v_{s1b2} + v_{s1c2})/2 \right]
\end{align*}
$$

\text{where} 
\begin{align*}
v_{s1a2} &= v_{s1a} + v_{s1b} \\
v_{s1b2} &= v_{s1b} + v_{s1c} \\
v_{s1c2} &= v_{s1c} + v_{s1a}
\end{align*}
Sequential Method of Simulation

\[ v_{sq} = \frac{v_{bcb2} - v_{ccl2}}{\sqrt{3}} \]
\[ v_{s0} = \frac{v_{a1a2} + v_{bcb2} + v_{ccl2}}{3} \]  \hspace{1cm} (8.43)

The relations between the transformed and the real line currents are given by

\[ i_s = i_{sd} + i_{s0} \]
\[ i_b = -\frac{i_{sd}}{2} + \frac{\sqrt{3}}{2} i_{sq} + i_{s0} \]
\[ i_c = -\frac{i_{sd}}{2} - \frac{\sqrt{3}}{2} i_{sq} + i_{s0} \]  \hspace{1cm} (8.44)

In the SSPWM control strategy, each pulse width is proportional to the sine of the center point and the width index; further, the pulses are placed symmetrically with respect to the center point, as shown in Fig. 8.15. In simulating these waveforms on a digital computer, the starting and ending instants of each pulse in a half cycle are calculated \([8]\). Let \( P_j \) be the width of the \( j \)th pulse, for \( j = 1 \) to \( N \):

\[ P_j = \frac{W}{N} \sin C_j \]  \hspace{1cm} (8.45)

where

\[ C_j = \left[ \frac{j}{N} (2j - 1) / 2 \right] \text{ center point of the } j \text{th pulse} \]
\[ N \text{ number of pulses per half cycle} \]
\[ S_j = C_j - P_j / 2 \text{ starting point of the } j \text{th pulse} \]
\[ E_j = C_j + P_j / 2 \text{ end point of the } j \text{th pulse} \]

In one half-cycle, for phase \( a \), the following relations hold good:

- \( S_j \) to \( E_j \): \( v_{a1a2} = V_d \); \( T_a \) is .TRUE. and \( T_a \) is .TRUE.
- \( E_j \) to \( S_{j+1} \) for \((j+1) < N \): \( v_{a1a2} = 0 \); \( T_a \) is .TRUE. and \( T_{a} \) is .TRUE.
- \( 0 \) to \( S_j \) and \( E_N \) to \( \pi \): \( v_{a1a2} = 0 \); \( T_a \) is .TRUE. and \( T_{a} \) is .TRUE.

In another half-cycle,

- \( S_j \) to \( E_j \): \( v_{a1a2} = -V_d \); \( T_{a} \) is .TRUE. and \( T_{a} \) is .TRUE.
- \( E_j \) to \( S_{j+1} \) for \((j+1) < N \): \( v_{a1a2} = 0 \); \( T_{a} \) is .TRUE. and \( T_{a} \) is .TRUE.
- \( 0 \) to \( S_j \) and \( E_N \) to \( \pi \); \( v_{a1a2} = 0 \); \( T_{a} \) is .TRUE. and \( T_{a} \) is .TRUE.

This logic can be extended similarly for phases \( b \) and \( c \). The dc supply current is given by

\[ i_{dc} = i_{sd} + i_{bd} + i_{cd} \]  \hspace{1cm} (8.46)

Analysis of Decoupled Systems

![Single Phase Bridge Inverter Diagram](image)

**NO. OF DIVISIONS/HALF CYCLE** = \( N = 6 \)

**WIDTH OF EACH INTERVAL** = 30°

**WHEN** \( W = 1 \),

\[ P_1 = (30) \sin (15°) = 7.77° \] (THE SINEWAVES ARE DRAWN)

\[ P_1 = (30) \sin (45°) = 21.2° \] (ONLY FOR REFERENCE)

\[ P_1 = (30) \sin (75°) = 28° \]

\[ P_2 = (30) \sin (105°) = 28° \]

\[ P_3 = (30) \sin (135°) = 21.2° \]

\[ P_4 = (30) \sin (165°) = 7.77° \]

**FIGURE 8.15** SSPWM waveforms for a single phase bridge inverter \((N = 6, W = 1)\).**

**where,**

\[ i_a \text{ if } T_a \text{ is .TRUE. and } T_{a} \text{ is .TRUE}. \]

\[ i_{ad} = -i_a \text{ if } T_{a} \text{ is .TRUE. and } T_a \text{ is .TRUE.} \]

\[ 0 \text{ if both } T_a \text{ and } T_{a} \text{ are .TRUE. or } T_{a} \text{ and } T_a \text{ are .TRUE.} \]
SEQUENTIAL METHOD OF SIMULATION

\[ \begin{align*}
  u_1 &= u_2 = u_3 = u_c \\
  w_t &= \text{constant}
\end{align*} \]

FIGURE 8.16 Output voltage waveforms for SSPWM inverter (N = 6; W = 1).

Note that similar relations hold between \( i_d \) and \( i_q \), and between \( i_0 \) and \( i_c \). Recall that the expression "Tail is TRUE" signifies that either the transistor Tail or Dal in phase a is conducting.

Figure 8.16 shows the output voltage waveforms for the SSPWM inverter.

DIGITAL COMPUTER SIMULATION OF INVERTER-FED INDUCTION MACHINE. It is possible to develop a simulation program which would be able to generate the required output voltage waveforms; further, it would compute the corresponding transformed voltages with a view to calculating the performance characteristics of the induction machines. The generalized flowchart applicable to all three types of inverter control strategies is discussed previously and shown in Fig. 8.17. The flowchart indicates the required steps for computation of the free-acceleration characteristics of the induction motor when fed from different non-sinusoidal voltage sources during acceleration and steady-state operation. The input data required are the equivalent-circuit parameters of the induction machine in per unit quantities: frequency, supply voltage, load torque, and number of pulses per half cycle for PWM inverters. The transformed voltages \( v_{ad} \) and \( v_{aq} \) are computed after every step length and the

ANALYSIS OF DECOUPLED SYSTEMS

START

SET MOTOR PARAMETERS AND INITIAL CONDITIONS

READ DATA AND SET TIME = 0

CALCULATE \( v_{a1}, v_{a2}, v_{c1} \)

\( \left( v_{a1a2}, v_{a1b2}, v_{c1c2} \right) \)

COMPUTE TRANSFORMED QUANTITIES: \( v_{sd}, v_{sq}, v_{so} \)

COMPUTE NUMERICALLY THE MACHINE PERFORMANCE: FLUX LINKAGES, SPEED

RUNGE-KUTTA METHOD

COMPUTE \( i_{sd}, i_{sq}, i_{so}, i_{r0}, i_{g} \)

COMPUTE MACHINE LINE CURRENTS, TORQUE AND D.C. LINE CURRENT

PRINT AND STORE RESULTS FOR PLOTTING

INCREMENT STEP LENGTH: \( T = T + DT \)

NO

YES

CALL SUBROUTINE FOR PLOTTING

END

FIGURE 8.17 Flowchart for computation of induction machine performance fed from three phase inverter.
Sequential Method of Simulation

FIGURE 8.18 Computed waveforms for induction machine fed from square-wave inverter—180° conduction.

machine equations are solved numerically. The step length used for a square-wave inverter is

$$DT = \frac{1.0}{f(60)}$$

so that there are 50 points in every one-sixth of a period. For PWM and SPPWM inverters, the step length is 60 μs, which is much less than the minimum pulse width.

The typical computed waveforms under different operating conditions are shown in Figs. 8.18, 8.19, and 8.20; these figures show motor line current, torque, inverter input current, and speed as functions of time and torque versus speed. In all cases the ratio of voltage to frequency is kept constant. The circles in the torque-speed characteristics shown in Fig. 8.19 and in Fig. 8.20

Analysis of Decoupled Systems

FIGURE 8.19 Computed waveforms for induction machine fed from sinusoidal PWM inverter.
are due to the speed oscillations. In these cases the load torque has been kept constant at a value of 0.65 pu. The base quantities and induction machine parameters are given in Table 8.1.

The simulation technique presented here can also be used to study other dynamic characteristics, such as a sudden increase in load, behavior during abnormal operating conditions such as switching off, single phasing, and so on.

### Analysis of Coupled Systems

#### TABLE 8.1 Base Quantities and Motor Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine:</td>
<td>Three phase induction machine: 2.2 kW, 50 Hz, 400 V, star-connected</td>
</tr>
<tr>
<td>Base Quantities:</td>
<td></td>
</tr>
<tr>
<td>Phase voltages:</td>
<td>$V_N = 2 V_{rms} = 325 V$</td>
</tr>
<tr>
<td>Phase current:</td>
<td>$I_N = 2 I_{rms} = 6.36 A$</td>
</tr>
<tr>
<td>Synchronous frequency:</td>
<td>$\omega_N = 314.15 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Flux linkage:</td>
<td>$\phi_N = V_N/\omega_N = 1.034 \text{ Volt sec}$</td>
</tr>
<tr>
<td>Power:</td>
<td>$P_N = 1.5 V_N I_N = 3.1 \text{ kW}$</td>
</tr>
<tr>
<td>Mechanical angular speed:</td>
<td>$\Omega_N = \omega_N/\rho = 314.15 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Torque:</td>
<td>$M_N = P_N/\Omega_N = 9.869 \text{ Nm}$</td>
</tr>
<tr>
<td>Impedance:</td>
<td>$Z_N = V_N/\Omega_N = 51.1 \text{ Ohms}$</td>
</tr>
<tr>
<td>Inductances:</td>
<td>$L_N = Q_N/\Omega_N = 0.1625 \text{ H}$</td>
</tr>
<tr>
<td>Motor parameters:</td>
<td></td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.0684 pu</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.02485 pu</td>
</tr>
<tr>
<td>$L_s$</td>
<td>2.67 pu</td>
</tr>
<tr>
<td>$L_f$</td>
<td>2.67 pu</td>
</tr>
<tr>
<td>$L_m$</td>
<td>2.5846 pu</td>
</tr>
<tr>
<td>$T_s$</td>
<td>0.124 sec</td>
</tr>
<tr>
<td>$T_f$</td>
<td>0.341 sec</td>
</tr>
<tr>
<td>$T_m$</td>
<td>0.636 sec</td>
</tr>
</tbody>
</table>

$$\sigma = (L_s I_s^2 - L_f I_f^2)/L_m I_f$$

#### 8.3 ANALYSIS OF COUPLED SYSTEMS

For our study we consider the most commonly used ac drive systems using three-phase synchronous and induction machines. In order to provide variable-speed operation of these machines, a variable-frequency power supply is required. This power conversion is usually obtained from a constant-frequency supply mains by the use of a group of converters; for example, a three-phase rectifier supplies a...
variable dc voltage or variable dc current output, and a three-phase voltage source or current source inverter provides a variable-frequency power supply.

Earlier we considered a voltage source inverter-fed ac machine and analyzed its performance as a decoupled system. In this section we discuss the case of coupled systems that arise when a current source inverter feeds an ac machine. For the analysis, we consider two cases:

1. A synchronous machine supplied from a naturally commutated inverter
2. An induction machine supplied from a forced-commutated current source inverter

We first establish the equations describing the operating conditions of the two systems. Then we discuss a generalization procedure for developing a computer-aided analysis program that would enable us to study the various types of ac drive systems using synchronous or induction machines.

8.3.1 Synchronous Machine Fed from a Naturally Commutated Inverter

We consider a balanced three-phase synchronous machine with the stator windings connected in delta; the synchronous machine is fed from a dc current source through a three-phase naturally commutated inverter (NCI). The firing signals for the thyristors are obtained from a rotor position sensor in such a way that operation of the converter is in the naturally commutated inverter mode. Figure 8.21 shows the schematic diagram of the synchronous machine-converter system.

**EQUATIONS OF THE SYNCHRONOUS MACHINE-CONVERTER GROUP.** For the analysis, the real machine quantities are transformed into d-q quantities. Derivations of these transformed equations are available in Chapter 2. For the

\[
\begin{bmatrix}
V_{sd} \\
V_{sq} \\
V_{rd} \\
V_{rq}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{s} r + L_{eq} \omega_r & 0 & M_{ar} & 0 \\
0 & \frac{1}{s} r + L_{eq} \omega_r & 0 & M_{ar} \\
M_{sr} & M_{sr} \omega_r & r_r + L_{eq} \omega_r & L_{eq} \\
-M_{sr} \omega_r & -M_{sr} & -L_{eq} \omega_r & r_r + L_{eq} \omega_r
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
i_{rd} \\
i_{rq}
\end{bmatrix}
\]

(8.47)

where

- \( r_s, r_r \) = stator and rotor resistances
- \( L_s, L_r \) = stator and rotor self-inductances
- \( M_{sr} \) = mutual inductance between stator and rotor windings
- \( \omega_r \) = rotor speed of an equivalent two-pole machine
- \( p \) = differential operator \( d/dt \)

The torque equation for the system is given by

\[
J \frac{d\omega}{dt} + f\omega + PT_L = P^2 M_{ar} (i_{rd} - i_{rq}) (d i_{sd} - d i_{sq})
\]

(8.48)

where

- \( P \) = number of pole pairs
- \( T_L \) = load torque
- \( J \) = total inertia of the system
- \( f \) = frictional coefficient of the load
- \( \omega_r \) = rotor speed of an equivalent two-pole machine

Note that Eq. (8.47) can be obtained from the general equation (2.50); to obtain Eq. (8.47), \( \omega_d \) is made zero in Eq. (2.50), since the d-q axes are stationary. The synchronous machine fed by the NCI is a coupled system since we take into account the commutation interval of the thyristors. Further, let us take into consideration the dc link inductance \( L_d \) and the resistance \( r_d \). In the naturally commutated mode of operation, we can associate 12 sequences of operation in a period, as shown in Fig. 8.3. This would therefore require 12 sets of equations if no further assumptions are made. However, we can note that the three-phase bridge converter exhibits cyclic operation, in which there are only two sequences of operation for a given set of thyristors. For example, in the first sequence of operation, called the "sequence between commutations," thyristors 1 and 6 conduct initially. When thyristor 2 is turned on, the second sequence of operation begins; this is characterized by simultaneous conduction of thyristors 5, 1, and 2. This sequence
is called the "sequence during commutation." Figure 8.22 shows the operating conditions of the circuit during the two operating sequences. The same two operating sequences occur in a cyclic manner for the other sets of thyristors, with a phase difference of 60°. If we take into account this cyclic operation, it is sufficient to write only two sets of equations describing the dynamics of the coupled system which is applicable for one-sixth of a period.

The dynamic equations for the system during the commutation interval can be calculated:

\[
\begin{align*}
\mathbf{p}_1 &= \begin{bmatrix} L_r U / \sqrt{\delta} U & L_r U / \sqrt{\delta} U & 0 & M_{sr} U / \sqrt{2} U \end{bmatrix} \\
\mathbf{p}_l &= \begin{bmatrix} 0 & L_s R & -M_{sr} S / S & 0 \end{bmatrix} \\
\mathbf{p}_{ld} &= \begin{bmatrix} 0 & -M_{sr} L_s S & 0 \end{bmatrix} \\
\mathbf{p}_{lr} &= \begin{bmatrix} M_{sr} & M_{sr} S & 0 \end{bmatrix} \left( \frac{\sqrt{2}}{2} U \right) \left( \frac{\sqrt{2} U}{2} \right) \end{align*}
\]

where

\[
\begin{align*}
U &= L_s L_r - M_{sr}^2 / 2 \\
S &= L_s L_r - M_{sr}^2 \sin \phi \\
L_s &= L_d + L_a / 2 \\
E_1 &= V_d - (r_d + r_s / 2) i_d - r_s i_d \sqrt{\delta} \\
E_2 &= -r_s i_d \\
E_3 &= V_d + M_{sr} \omega L_s \left( \sqrt{2} - r_d \right) - L_s \omega l_q \\
E_4 &= V_d + M_{sr} \omega L_a \left( \sqrt{2} - r_d \right) - L_a \omega l_q
\end{align*}
\]

The dynamic equations for the interval between commutations can be calculated; these equations are the same as those given by Eq. (8.49) with the following differences. The differential equation for \( i_{3d} \) in Eq. (8.49) is replaced by

\[
\frac{di_{3d}}{dt} = \frac{1}{6} \frac{di_{3d}}{dt}
\]

Further, the value of \( E_2 \) is given by

\[
E_2 = \frac{E_21}{E_22}
\]
Derivation of Eqs. (8.49) to (8.51) is left to the reader as exercises. The torque equation for both the operating sequences is

\[ J_p \omega = -f_w - P T_L - P^2 M_{st} \left( \frac{1}{v^2} + \frac{i_{sd}}{v_{dq}} \right) \]  

(8.52)

8.3.2 Induction Machine Fed from a Forced-Commutated Current Source Inverter

We consider a balanced three-phase induction machine with the stator windings connected in delta; the induction machine is fed from a dc current source through a forced-commutated inverter (FCI). A current source inverter (CSI) configuration is used for the FCI. This configuration uses six main thyristors and six auxiliary thyristors; further, there are three capacitors which are properly charged and discharged during turn-on of the auxiliary and main thyristors; thus they provide the necessary reverse voltage across the main thyristors for their turn-off. Figure 8.23 shows the schematic diagram of the induction machine-converter group.

EQUATIONS OF THE INDUCTION MACHINE-CONVERTER GROUP. For the analysis, once again we use the d-q transformation equations and transform the machine equations into d-q quantities. Let us assume, initially, that the two main thyristors, TM1 and TM6, are conducting. Then thyristors TA6 and TM2 are turned on simultaneously. This will result in instantaneous commutation of main thyristor TM6. Main thyristor TM2 is not turned on, however, because it is still reverse biased. The first sequence of operation of the inverter is characterized by charging of the commutating capacitor at constant current as shown in Fig. 8.24. The equations for this sequence are

\[ i_{sq} = \frac{1}{2} \]  

(8.53)

\[ i_{sd} = \frac{1}{6} \]

\[ V_d = (r_d + L_d i) + V_f - \frac{v_{sd}}{6} + \frac{v_{sq}}{2} \]  

(8.54)

The second sequence of operation begins when main thyristor TM2 starts conducting. Figure 8.24 shows this operating condition. The corresponding equations are

\[ i_{sq} = \frac{1}{2} \]  

(8.55)

\[ i_{sd} = \frac{1}{6} \]

\[ V_d = (r_d + L_d i) + V_f - \frac{v_{sd}}{6} - \frac{v_{sq}}{2} \]  

The third sequence is characterized by the conduction of only two main thyristors, TM1 and TM2. Figure 8.24 shows this operating condition. The equations describing this sequence are

\[ i_{sq} = \frac{1}{2} \]  

(8.56)

\[ i_{sd} = \frac{1}{6} \]

\[ V_d = (r_d + L_d i) - \frac{v_{sd}}{6} + \frac{v_{sq}}{2} \]

It is possible to establish a set of equations that would describe all the three sequences of operation:
Sequential Method of Simulation

Analysis of Coupled Systems

Equation (8.56) is applicable to all three sequences, with the following particular conditions:

First sequence:

\[ \epsilon = 1 \]

\[ p_{sd} = -\epsilon \frac{di}{dt} \]

this replaces the corresponding differential equation in Eq. (8.56)

\[ E_1 = U - \left( r_d - \frac{r_s}{2} \right) i + \epsilon \frac{i_{sd}}{\sqrt{s}} \]

\[ U = V_d - V_f \]

\[ E_2 = \frac{E_{23}}{2} \]

\[ E_{24} = 6U_s L_{sr} E_3 - \epsilon \sqrt{s} \frac{1}{s} \]

\[ E_{25} = E_{24} \]

The expressions for \( E_3 \) and \( E_4 \) are the same as those defined for the NCI-fed synchronous machine.

Second sequence:

\[ \epsilon = 1 \]

\[ U = V_d - V_f \]

\[ v_{sd} = \frac{\sqrt{3}}{2} V_f \]

\[ E_2 = \frac{\sqrt{3}}{2} V_f - r_d i_{sd} \]

All other expressions are identical to those described for the first sequence.

Third sequence:

\[ \epsilon = -1 \]

\[ U = V_d \]

All other expressions are identical to those described for the first sequence. Further, for all the sequences, the torque equation is given by Eq. (8.48). We can note that the three sequences have been defined only for one-sixth of the period. Once again, cyclic operation of the various thyristors in the power circuit can be taken into consideration and the analysis need be carried out only for one-sixth of the period.

FIGURE 8.24 Operating sequences for CSI-fed induction machine.
8.3.3 Computer-Aided Analysis of AC Machine-Converter Group

It is possible to establish a simulation program for the analysis of a power electronic system comprising an ac machine-converter group. Such a general-purpose program has been developed and it is called SECMA [9]. The flowchart for the SECMA simulation program is shown in Fig. 8.25.

The SECMA program permits analysis of various ac drive schemes using either current source inverter (CSI)-fed or voltage source inverter (VSI)-fed ac machines. Table 8.2 lists the subsystems pertaining to the study of VSI-fed ac machines. Table 8.3 shows the subsystems relating to the study of CSI-fed ac machines.

The interesting feature of the SECMA program is that it is possible to study various types of control strategies for the converter-machine group. One example is the use of a speed-regulation loop which does not require a speed sensor [9].

As an ac drive scheme using a CSI-fed induction machine is shown in Fig. 8.26. The input current to the CSI is obtained from a three-phase bridge rectifier operated in the current-regulated mode. A proportional-integral (PI) controller has been used for the current control loop. The speed reference values for the input current $i_f$ and the speed of the induction motor $\omega_s$ are computed from

$$\omega_s = \frac{k_v}{s}$$

$$i_f = i_{sn} \left[ 1 + \frac{1}{3\sqrt{2}} \frac{r}{2 \tau} \right]$$  \hspace{1cm} (8.57)

where $i_{sn}$ represents the stator current corresponding to the nominal value of flux; all other quantities have been defined earlier for the ac machine. The current-source inverter uses only one commutating capacitor but requires six auxiliary thyristors and six main thyristors. The parameters of the machine are shown in Table 8.4. Figure 8.27 shows the starting transient for the induction motor line current, voltage, speed, commutating capacitor voltage, rectifier output current, and motor torque as functions of time. These waveforms have been obtained using the SECMA simulation program.

An examination of the motor line current and voltage in Fig. 8.27 shows a progressive increase in the stator frequency. The rectifier output current is nearly constant during starting. This is ensured by the PI current controller incorporated in the control circuit of the three-phase converter. Figure 8.28 shows the waveforms for a step increase in the stator current reference.

We consider a VSI-fed induction machine in Fig. 8.28. A variable voltage at the input of the VSI is obtained from a dc chopper. Further, the scheme shown in Fig. 8.28 uses an input LC filter at the output of the dc chopper. The speed regulation of the induction machine is done without a speed sensor. The details of a state variable type of controller are shown in Fig. 8.29. Table 8.5 shows the parameters of the ac drive scheme.

Figure 8.29 shows the starting transients of the VSI-fed induction motor. The different waveforms that are plotted are the motor line current, line voltage, inverter input voltage, chopper output current, and motor speed. Note that a
### TABLE 8.2 SECMA Simulation Subsystems for VSI-fed AC Machines

<table>
<thead>
<tr>
<th>Types of voltage sources</th>
<th>Operating inverter modes</th>
<th>Control strategies</th>
<th>Analysis modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. d.c. voltage source</td>
<td>1. 180° conduction</td>
<td>1. self-regulated speed control &amp; speed sensor</td>
<td>1. starting transient</td>
</tr>
<tr>
<td>2. d.c. voltage source-chopper group</td>
<td>2. PWM with constant duty cycle</td>
<td>2. speed regulation without speed sensor</td>
<td>2. step change in load torque</td>
</tr>
<tr>
<td></td>
<td>3. PWM with variable duty cycle</td>
<td>3. step change in speed reference</td>
<td>3. step variation of load torque</td>
</tr>
<tr>
<td></td>
<td>4. two-level with hysteresis current control</td>
<td>4. step variation of speed reference</td>
<td>4. step variation of flux</td>
</tr>
</tbody>
</table>

### TABLE 8.3 SECMA Simulation Subsystems for CSI-fed AC Machines

<table>
<thead>
<tr>
<th>Types of current sources</th>
<th>Control strategies</th>
<th>Analysis modes</th>
<th>Operating inverter modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. current-regulated three phase rectifier with PI regulator</td>
<td>1. speed regulation loop with speed sensor</td>
<td>1. starting transient</td>
<td>1. 180° conduction</td>
</tr>
<tr>
<td>2. current-regulated three phase rectifier with sampled-data regulator</td>
<td>2. speed regulation without speed sensor</td>
<td>2. step change in load torque</td>
<td>2. PWM with constant duty cycle</td>
</tr>
</tbody>
</table>

### TABLE 8.4 CSI-fed Induction Machine System Parameters

<table>
<thead>
<tr>
<th>Type of machine</th>
<th>Slip-ring induction machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power</td>
<td>4.7 kW</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
</tr>
<tr>
<td>Nominal voltages</td>
<td>220 V / 127 V</td>
</tr>
<tr>
<td>Nominal line current</td>
<td>20 A / 35 A</td>
</tr>
<tr>
<td>Stator resistance, r_s</td>
<td>0.0713 Ohm</td>
</tr>
<tr>
<td>Rotor resistance, r_r</td>
<td>0.159 Ohm</td>
</tr>
<tr>
<td>Stator self inductance, L_s</td>
<td>0.0378 H</td>
</tr>
<tr>
<td>Rotor self inductance, L_r</td>
<td>0.0220 H</td>
</tr>
<tr>
<td>Mutual inductance, M_m</td>
<td>0.0278 H</td>
</tr>
<tr>
<td>Moment of inertia, J_i</td>
<td>0.08 kg m²</td>
</tr>
<tr>
<td>Frictional coefficient, f_r</td>
<td>0.04 Nm/rad/sec</td>
</tr>
<tr>
<td>D.C. link resistance, r_d</td>
<td>1 Ohm</td>
</tr>
<tr>
<td>D.C. link inductance, L_d</td>
<td>0.030 H</td>
</tr>
<tr>
<td>Commutating capacitance, C_d</td>
<td>30 µF</td>
</tr>
</tbody>
</table>
FIGURE 8.26 CSI-fed induction machine ac drive system.
FIGURE 8.28 Step change in $i_s$ for CSI-fed induction motor.

FIGURE 8.29 VSI-fed induction machine ac drive system.

TABLE 8.5 VSI-fed Induction Machine System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of machine</td>
<td>squirrel-cage induction machine</td>
</tr>
<tr>
<td>Nominal power rating</td>
<td>11 kW</td>
</tr>
<tr>
<td>Nominal frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Nominal stator winding voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>Stator resistance, $r_s$</td>
<td>0.5 Ohm</td>
</tr>
<tr>
<td>Rotor resistance, $r_f$</td>
<td>0.5 Ohm</td>
</tr>
<tr>
<td>Stator self inductance, $L_s$</td>
<td>0.069 H</td>
</tr>
<tr>
<td>Rotor self inductance, $L_f$</td>
<td>0.069 H</td>
</tr>
<tr>
<td>Mutual inductance, $M_{sr}$</td>
<td>0.067 H</td>
</tr>
<tr>
<td>Moment of inertia, $J_s$</td>
<td>0.08 kg m$^2$</td>
</tr>
<tr>
<td>Frictional coefficient, $f_s$</td>
<td>0.1 Nm/rad/sec</td>
</tr>
<tr>
<td>D.C. link filter resistance, $r_d$</td>
<td>0.5 Ohm</td>
</tr>
<tr>
<td>D.C. link filter inductance, $L_d$</td>
<td>0.040 H</td>
</tr>
<tr>
<td>D.C. link filter capacitor, $C_d$</td>
<td>2000 μF</td>
</tr>
</tbody>
</table>
Figure 8.30 Starting transients for VSI-fed induction motor.

Figure 8.31 Step change in reference speed—VSI-fed induction motor.
8.4 SUMMARY

Analysis of power electronic converter-electrical machine group has been discussed. The analysis is based on equations established for known set of operating sequences.

In Section 8.1 we discussed two types of systems, decoupled and coupled. In the case of decoupled systems, it is possible to predict the output voltage waveform independent of the operating conditions of the electrical machine. Usually, decoupled systems arise when the commutation intervals in the operation of the converter are neglected.

Analysis of two examples of decoupled systems was discussed in Section 8.2. The first example covered the chopper-fed dc motor. The analysis of this system was based on the assumption of negligible commutation interval. In the second example we discussed an induction motor fed from a voltage-source inverter. This analysis was based on the dq transformation equations for the induction machine. Three different control strategies—180° conduction, PWM, and SSFWM—were considered for the inverter control.

In Section 8.3 we considered the case of coupled systems. The first example discussed the case of an ac drive using a synchronous machine and a naturally commutated inverter. In the second example an ac drive using an induction machine and a forced-commutated current-source inverter was discussed. Finally, in Section 8.3.3 we discussed a generalized method of analysis of coupled or decoupled ac drive systems with their associated control strategies.

8.5 REFERENCES

1. V. Rajagopalan, ATOSEC Users' Manual, Département d'Ingénierie, Université du Québec à Trois-Rivières, Quebec, Canada (August 1985).


3. S. Davat, Etude—mise au point et applications d'une méthode de simulation globale de convertisseurs statiques connectés à des charges électriques complexes (Study and development of a global simulation program for the study of converter-fed complex electric loads), Docteur-Ingénieur Thesis, Laboratoire d'électrotechnique et d'électronique industrielle, Toulouse, France (June 27, 1979).


References


9. German Cruz Jovanne, Etude et mise au point d'un programme de simulation numérique par séquences (SECMA) d'ensembles constitués de convertisseurs statiques et de machines à courant alternatif. Application à la simulation de variateurs électrotechnique et d'électronique industrielle, Toulouse, France (February 22, 1982).