Maximum likelihood estimation of synchronous machine parameters and study of noise effect from DC flux decay data

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Abstract: The paper presents an evaluation of the performance of the maximum likelihood (ML) method when used to estimate the linear parameters of a synchronous machine model from the standstill time-domain flux decay test data. It is shown that a unique set of parameters can be obtained and the noise effects can be dealt with effectively when the ML estimation technique is used. The results also show that accurate machine parameters can be identified when the signal/noise ratio is approximately 200 : 1.

1 Introduction

Many papers have been published [1–8] on synchronous machine modelling using frequency and time-domain parameter estimation techniques. However, the effects of noise on synchronous machine parameter estimation have not been studied extensively [6, 7]. Jaleeli *et al.* [7] used a quasilinearisation-based, least-square algorithm to estimate machine parameters from noise-corrupted data. The percentage error of the estimated parameters was as high as 32% for noise-free data and 259% for noise-corrupted data. Le and Wilson [6] employed the generalised least-square technique, the percentage error in some of their estimated parameters was as high as 100%. These studies confirm that the least-square algorithm is very sensitive to noise-corrupted data [11].

In our previous work [1], we established that, if the transfer functions of a synchronous machine are estimated from the noise-corrupted standstill frequency response (SSFR) data, and the machine parameters are then estimated from the machine transfer functions' time constants, multiple parameter sets will be obtained. This is because the SSFR approach results in a redundant and inconsistent set of nonlinear equations with the number of unknowns being one less than the number of equations. The inconsistency of the equations is due to the noise which is inherently present in any test data.

To overcome the above problems, we proposed a time-domain identification technique [2] to estimate machine parameters using SSFR data. We demonstrated that the multiple solution set problem encountered in the SSFR technique can be eliminated if the maximum likelihood (ML) estimation technique is used for machine parameter estimation [2].

However, in our previous work [1], we used the SSFR data to estimate the machine transfer functions and then the machine parameters from the transfer functions. This two-step identification technique limits the accuracy of estimated parameters and requires a very high signal/noise ratio (i.e. a very low level of noise).

In the present study, a simple DC flux decay test is used to collect the input and output data of the machine for estimation. This offload test method has an advantage over the online test methods because of the expense and difficulty of obtaining transient test data from loaded machines. The DC decay test [3, 4] can be performed at either the factory or the power plant, with the machine stationary. It only requires a supply of direct current.

2 Problem description

The fidelity of synchronous machine models is affected by the proposed model structures, the quality of the experimental data used to identify the model's parameters, and the robustness of the estimation technique. One question needs to be answered, namely if the assumed model structure is correct, can one obtain a unique estimate of the parameters from noise-corrupted frequency response data? The answer to this question cannot be found from measurements, since the measurements are made on a machine with a complex, high-order rotor circuit, with an unknown structure and unknown parameters.

If one assumes a model structure and proceeds with estimating its parameters from actual measurements, the structural error and the effect of noise in the measurements will result in inaccurate parameters. It will not be clear whether the discrepancy between the simulated model response and the measured response is due to the effect of noise on the parameters, the inadequacy of the assumed model structure, or both. Therefore, the structural identification problem and the parameter estimation problems should be studied separately. It is necessary to show that the measurement noise will not corrupt the estimated parameters when the parameters of an assumed structure are estimated from the input/output measurements.

To study this problem, a machine model with a known parameter is simulated and then the data are noise-corrupted using a known noise distribution. The objective is to estimate the parameters of the machine model from the noise-corrupted data and to evaluate the effect of noise on the estimated parameters by comparing them with the known parameters. The problems studied can be stated as follows:

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(i) Using the DC flux decay data, how high can the noise level be so that there are no significant disagreements between the estimated and the actual parameters?

(ii) How close should the initial parameters be selected in the estimation to guarantee the algorithm convergence to the correct estimates?

3 Standstill synchronous machine model for time-domain parameter estimation

Consider the system described by Fig. 1. When the standstill measurements are used for estimation, the mathematical formulation of Fig. 1 can be broken down into two state-space representations for the d- and q-axes, respectively. This is because the coupling factors $\omega\lambda_q$ and $\omega\lambda_d$ are equal to zero under the standstill condition. The state-space representations of the models in Fig. 1 can therefore be written as follows:

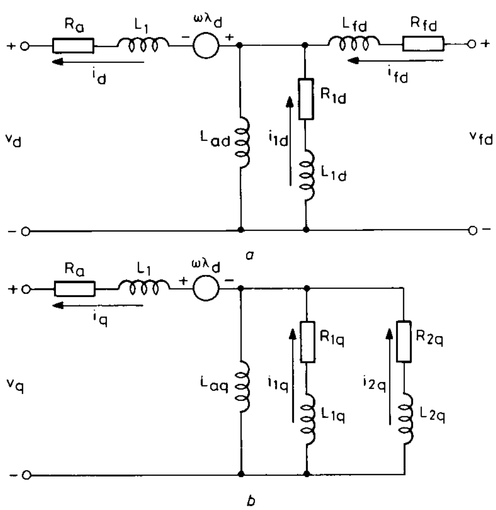

Fig. 1 *Standard synchronous machine model circuit structure*
a d-axis circuit
b q-axis circuit

for the d-axis model,

$$X_d(k+1) = A_d(\theta_d)X_d(k) + B_d(\theta_d)U_d(k) + w(k) \quad (1)$$

$$Y_d(k) = C_d X_d(k) + v(k) \quad (2)$$

for the q-axis model,

$$X_q(k+1) = A_q(\theta_q)X_q(k) + B_q(\theta_q)U_q(k) + w(k) \quad (3)$$

$$Y_q(k) = C_q X_q(k) + v(k) \quad (4)$$

In eqns. 1 and 3, the definitions of the state variables, input and output are

$$X_d = [i_d\, i_{1d}\, i_{fd}]^T \quad X_q = [i_q\, i_{q1}\, i_{q2}]^T$$
$$U_d = [v_d] \quad U_q = [v_q]$$
$$Y_d = [i_d\, i_{fd}]^T \quad Y_q = [i_q]$$

The unknown parameter sets for the d- and q-axes are

$$\theta_d = [R_a\, R_{1d}\, R_{fd}\, L_l\, L_{ad}\, L_{1d}\, L_{fd}]^T$$
$$\theta_q = [R_a\, R_{1q}\, R_{2q}\, L_l\, L_{aq}\, L_{1q}\, L_{2q}]^T \quad (5)$$

In eqn. 1, the input control variable U_d is determined by the d-axis stator voltage alone because, under stationary test conditions, the field circuit is shorted. The quantity w represents the process noise, and the quantity v indicates the measurement noise in eqns. 1 and 3. In the actual system, the process noise w is imbedded in the input control variables U_d and U_q, respectively.

The computation of $A_d(\theta_d)$, $B_d(\theta_d)$, $A_q(\theta_q)$, and $B_q(\theta_q)$ from continuous time-domain representation is described in Reference 8. The explicit parameterization of continuous system representation in terms of θ_d and θ_q is given in Reference 2.

4 Effect of noise on the process and measurement

The model, which mathematically describes the process, is subjected to the deterministic input at each time instant k. Nature also subjects the process to a random input sequence $w(\cdot)$. The sequence $w(\cdot)$ is designated as the process noise sequence. It is assumed to be Gaussian with zero mean and a covariance matrix $Q(\cdot)$. The covariance matrix Q gives a measure of the intensity of the process noise on the model. A high value of the covariance matrix Q corresponds to a noisy process. The measurement noise sequence $v(\cdot)$ is introduced because in physical problems, the measurements are inherently subject to errors. The signal conditioning equipment and sensors introduce random measurement noise. The measurement errors $v(\cdot)$ are assumed to be independent and Gaussian with zero mean value and a known covariance matrix R_0. It is further assumed that the sequences $w(\cdot)$, $v(\cdot)$, and $X(0)$ are independent.

The initial covariance R_0 is constructed from the knowledge of sensor errors, and it represents a measure of the prior confidence in the sensors to produce accurate measurements. Strictly speaking, two experiments performed on the same process will not result in identical measurements. Therefore the covariance of the estimation error is calculated as part of a Kalman filter [9–12] for estimating the machine states and parameters. The covariance of the estimation error is defined as

$$R(k) = COV(e(k), e(k))$$
$$e(k) = Y(k) - \hat{Y}(k) \quad (6)$$

In eqn. 6, $Y(k)$ and $\hat{Y}(k)$ represent the measured and the estimated output, respectively.

5 Study process

The purpose of the study process is to develop a methodology for synchronous machine parameter estimation from the noise-corrupted data due to the DC flux decay test. For this purpose, synthetic time-domain standstill transient response data were generated using standard synchronous machine model parameters. Then, we will assume that the model structure (the number of differential equations representing the machine) and input/output response data are known; however, the model parameters are not known. The objective of the study is to develop a methodology for machine parameter estimation.

For the study process, the synthetic time-domain data are generated using the following procedure. A constant DC voltage signal is first applied to the machine model for a specified period of time so that the initial transient responses no longer exist. The DC supplied voltage is then suddenly removed from the system, and the transient responses are generated for the estimation. In the first stage of the simulation, the noise sequences are not introduced to the signals. Then, in the second stage of the simulation process, the developed time responses are corrupted with Gaussian distributed noise of zero mean and varying degrees of variances depending on the selected signal-to-noise ratios. A low level of signal/noise ratio corresponds to a noisy measurement.

To simulate the real conditions, the noise must only be injected into those variables which are physically accessible during the machine standstill test. For the d-axis test, the machine rotor is turned to the position where the q-axis coincides with phase a. A diagram depicting the test setup for the d-axis measurements is shown in Fig. 2, the adjustable resistor R is used to regulate the inrush current during the startup. A similar test arrangement for the q-axis is shown in Fig. 3, in this case, the rotor d-axis is aligned with phase a.

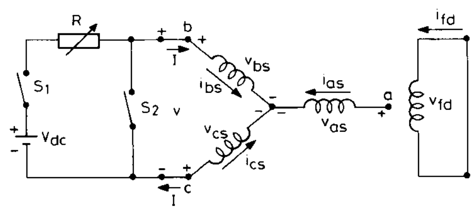

Fig. 2 *Rotor position alignment for d-axis flux decay test*

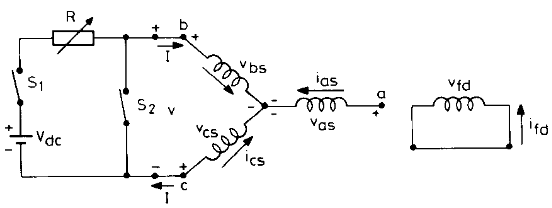

Fig. 3 *Rotor position alignment for q-axis flux decay test*

According to Figs. 2 and 3, the noise corruption process for the simulated data must be performed on the terminal variables such as V, I and i_{fd}, namely

$$\begin{aligned}\bar{V}(k) &= V(k) + \alpha_1(k)v_1(k) \text{ volts}\\ \bar{I}(k) &= I(k) + \alpha_2(k)v_2(k) \text{ amperes}\\ \bar{i}_{fd}(k) &= i_{fd}(k) + \alpha_3(k)v_3(k) \text{ amperes}\end{aligned} \tag{7}$$

In the above equation, $\bar{V}(\cdot)$, $\bar{I}(\cdot)$, and $\bar{i}_{fd}(\cdot)$ represent the noise corrupted terminal quantities, and $V(\cdot)$, $I(\cdot)$, and $i_{fd}(\cdot)$ correspond to the noise-free terminal values. The terms, $\alpha_1(k)$, $\alpha_2(k)$, and $\alpha_3(k)$, are defined as

$$\alpha_i(k) = \left| \frac{\text{signal } (k)}{S/N \text{ ratio}} \right| \tag{8}$$

where signal (k) corresponds to the noise-free signal strength, and the S/N ratio indicates the signal/noise ratio used in the noise corruption process. The S/N ratio is defined as

$$S/N \text{ ratio} = \left[\frac{\sum_{k=0}^{N} \text{signal}^2(k)}{\sum_{k=0}^{N} (\alpha_i(k)v_i(k))^2} \right]^{1/2} \tag{9}$$

The sequence $v_i(\cdot)$ in eqn. 9 is Gaussian distributed with zero mean and unity variance.

In eqn. 7, the noise-free variables $V(k)$, $I(k)$, and $i_{fd}(k)$ can be calculated based on the noise-free simulated stator and field quantities in the dqo reference frame using the Park transformation and the specified rotor positions in Figs. 2 and 3, namely

$$\begin{aligned}V(k) &= -\sqrt{(3)}v_d(k)\\ I(k) &= -\frac{\sqrt{(3)}}{2} i_d(k)\end{aligned} \tag{10}$$

for the d-axis flux decay test, and

$$\begin{aligned}V(k) &= \sqrt{(3)}v_q(k)\\ I(k) &= \frac{\sqrt{(3)}}{2} i_q(k)\end{aligned} \tag{11}$$

for the q-axis flux decay test. Once the data are corrupted with additive noise, the resultant d- and q-axis stator quantities can be calculated using eqns. 10 and 11 again. However, $\{V(\cdot), I(\cdot)\}$ must be replaced by $\{\bar{V}(\cdot), \bar{I}(\cdot)\}$ which represent the noise-corrupted data.

6 Maximum likelihood parameter estimation

To identify the machine parameters, the maximum likelihood estimation algorithm (ML) is used (see Fig. 4). The likelihood function used is defined as [9, 10]

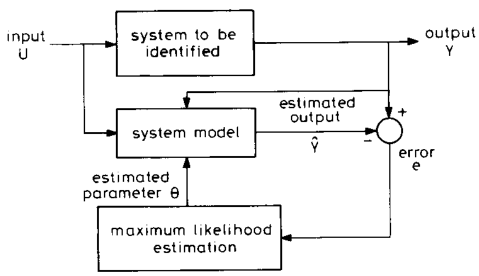

Fig. 4 *Maximum likelihood estimation*

$$L(\theta) = \prod_{k=1}^{N} \left[\frac{1}{\sqrt{\{(2\pi)^m \det (R(k))\}}} \times \exp\left(-\tfrac{1}{2}e^T(k)R^{-1}(k)e(k)\right) \right] \tag{12}$$

where $e(\cdot)$, $R(\cdot)$, N, and m denote the estimation error, the covariance of the output estimation error, the number of data points, and the dimension of Y, respectively. The output estimation error, $e(\cdot)$, and the corresponding covariance, $R(\cdot)$, are defined by eqn. 6. The estimation steps are given in Reference 2, 9 and 10.

7 Estimation procedure

According to the above discussion, the entire estimation effort involves two phases. Phase one is to obtain adequate flux decay test information, and phase two is to estimate the machine parameters using the maximum likelihood method. The estimation procedure for identifying the parameters of the standard synchronous machine model is summarised as follows:

(i) Perform the standstill time-domain flux decay test with the rotor in both positions of the d-axis and the q-axis.

(ii) Transform the available measurement data to *dqo* reference frame using the Park transformation, in particular, using eqn. 10 for the *d*-axis and eqn. 11 for the *q*-axis.

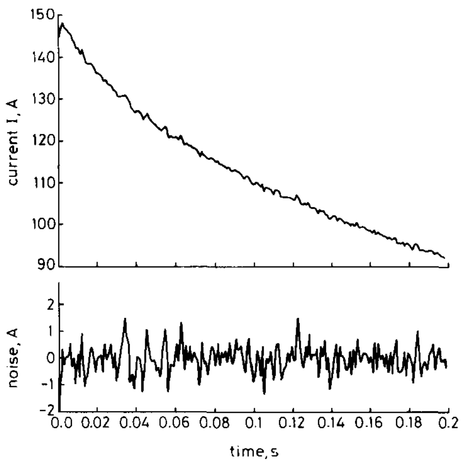

Fig. 5 *Synthetic test data and added noises for q-axis DC flux decay test with initial stator current of 150 A*

(iii) Estimate both the *d*- and *q*-axis standstill model parameters using the available test data calculated in step 2 and the maximum likelihood estimation algorithm.

8 Results

The *q*-axis flux decay test responses are shown in Fig. 5. The initial stator currents prior to the closing of switch S_2 are 150 A for both the *d*-axis and the *q*-axis. The S/N ratio used in Fig. 5 is 200 : 1. Three other noise levels are also used in the estimation study to see the effect of noise on the outcome of the estimation. The sampling interval used in the simulation is 0.001 s, and for each signal 1800 data points are generated. For the flux decay test, the voltage across the machine terminals is zero immediately after the closing of switch S_2. Thereafter, the terminal voltage is not plotted in Fig. 5. However, prior to the flux decay test, the DC terminal voltage and current have already reached a steady-state condition. Consequently, using Ohm's law, the armature winding resistance R_a can easily be computed before the startup of the flux decay test. The computed R_a can then be used for estimating other machine parameters under the flux decay responses.

The results of the estimation for the *d* and *q*-axis parameters are listed in Tables 1 and 2. The percentage errors listed in both tables are calculated according to the formula

$$\% \text{ error} = \frac{(\text{estimated value}) - (\text{original value})}{(\text{original value})} \times 100\%$$

In both *d*- and *q*-axis estimation, the initial values of the unknown parameters are selected at 20% of the actual values. The armature resistance R_a is computed using the steady-state current prior to the flux decay test, 200 points are used for this. For the estimation of *d*-axis parameters, R_a is set to the specified values, and L_l is assumed to be one of the unknown parameters which needs to be estimated. For the estimation of *q*-axis parameters, R_a and L_l are set to the values obtained in the steady-state test and the *d*-axis parameter estimation.

As shown in Table 1, the *d*-axis parameters can be estimated quite accurately under all noise levels considered. For *q*-axis model estimation, the second *q*-axis damper winding inductance L_{2q} cannot be estimated quite accurately under all noise levels considered. For *q*-axis model estimation, the second *q*-axis damper winding inductance L_{2q} cannot be estimated accurately when the S/N ratio is 200 : 1 (see Table 2). In order to keep the *q*-axis estimated parameters to be within 95% of the actual values, the

Table 1: Estimated *d*-axis parameter values from 150 A initial current flux decay data

Machine parameters p.u.	Original values p.u.	Estimated parameter values, p.u.							
		S/N ratio I, I_{fd} – 2000 : 1		S/N ratio I, I_{fd} – 1000 : 1		S/N ratio I, I_{fd} – 500 : 1		S/N ratio I, I_{fd} – 200 : 1	
		Est. value	% error	Est. value	% error	Est. value	% error	Est. Value	% error
R_a	0.00411	(0.00410)	−0.24	(0.00411)	−0.24	(0.00409)	−0.49	(0.00409)	−0.49
R_{1d}	0.01865	0.01855	−0.54	0.01853	−0.64	0.01851	−0.75	0.01843	−1.18
R_{fd}	0.00105	0.00104	−0.95	0.00104	−0.95	0.00104	−0.95	0.00104	−0.95
L_l	0.19000	0.18909	−0.48	0.18911	−0.47	0.18914	−0.45	0.18925	−0.39
L_{ad}	1.73000	1.72204	−0.46	1.72214	−0.45	1.72233	−0.44	1.72293	−0.41
L_{1d}	0.13050	0.13035	−0.11	0.13081	0.24	0.13173	0.94	0.13451	3.07
L_{fd}	0.16380	0.16300	−0.49	0.16299	−0.49	0.16297	−0.51	0.16291	−0.54

Table 2: Estimated *q*-axis parameter values from 150 A initial current flux decay data

Machine parameters p.u.	Original values p.u.	Estimated parameter values, p.u.							
		S/N ratio I – 2000 : 1		S/N ratio I – 1000 : 1		S/N ratio I – 500 : 1		S/N ratio I – 200 : 1	
		Est. value	% error	Est. value	% error	Est. value	% error	Est. value	% error
R_a	0.00411	(0.00410)	−0.24	(0.00410)	−0.24	(0.00409)	−0.49	(0.00409)	−0.49
R_{1q}	0.00585	0.00583	−0.34	0.00584	−0.17	0.00584	−0.17	0.00585	0.00
R_{2q}	0.02475	0.02475	0.00	0.02477	0.08	0.02481	0.24	0.02493	0.73
L_l	0.19000	(0.18909)	−0.48	(0.18911)	−0.47	(0.18914)	−0.45	(0.18925)	−0.39
L_{aq}	1.66000	1.65596	−0.24	1.65562	−0.26	1.65492	−0.31	1.65290	−0.43
L_{1q}	0.54450	0.54323	−0.23	0.54325	−0.23	0.54324	−0.23	0.54333	−0.21
L_{2q}	0.08441	0.08577	1.61	0.08672	2.74	0.08865	5.02	0.09464	12.12

lowest S/N ratio for the q-axis flux decay test may have to be limited above 500 : 1.

9 Conclusions

The parameters of the standard synchronous machine model have been estimated using the time-domain flux decay test data. The results of the estimation show that this type of standstill test data excites the machine dynamic modes and can be used to estimate the machine parameters provided that the signal/noise levels in various measured signals are kept within a range of 200 : 1. If the signal/noise ratio is increased above 500 : 1, the q-axis damper winding parameter L_{2q} may also be accurately estimated.

10 Acknowledgment

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Errata

ALOLAH, A.I.: 'Capacitance requirements for three phase self-excited reluctance generators', *IEE Proc. C*, 1991, **138**, (3), pp. 193–198

The author wishes to point out the following errors in his paper:

In Section 2 assumptions (ii) and (iii) should read as follows:

(ii) Although saturation may happen in both d-q axes, it is assumed that q-axis saturation is negligible.

(iii) The effect of saturation on parameters other than X_d is neglected.

In Section 3 on the upper right of page 195 X_0 is 1.73 p.u.

On upper right of page 196 (line three) should read as follows:

results in another situation at a speed a_{c2} where $E_1^2 \ldots$

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