

Synchronous Machine Parameter Estimation Using the Hartley Series

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Abstract—This paper presents a novel alternative to estimate armature circuit parameters of large utility generators using real time operating data. The proposed approach uses the Hartley series for fitting operating data (voltage and currents measurements). The essence of the method is the use of linear state estimation to identify the coefficients of the Hartley series. The approach is tested for noise corruption likely to be found in measurements. The method is found to be suitable for the processing of digital fault recorder data to identify synchronous machine parameters.

Index Terms—Fault detection in field windings, Hartley series, on-line tracking, operational matrices, orthogonal series expansion, parameter estimation.

I. INTRODUCTION

TRADITIONAL methods of obtaining synchronous machine parameters are specified in IEEE and IEC Standards and other national electrical standards from many countries. These methods are often conducted under offline conditions. The parameters obtained by these methods may not truly characterize the synchronous machine under various loading conditions. Many researchers have addressed the issues and problems associated with off-line parameter measurements. However, the interest and need for on-line estimation of synchronous generator parameters has arisen in recent years. On-line methods of obtaining machine parameters are most attractive due to minimal site/system impact and principally because they do not involve service interruption. On-line methods also represent a way to take into account parameter deviation due to changes in load levels, saturation, and machine aging.

On-line methods have been studied by many researchers [1]–[8]. Most of these studies are conducted mainly on conventional $dq0$ frame of reference models of synchronous generators. In spite of its simple structure, $dq0$ models are not capable of simulating unbalanced, rectifier type loading conditions and to include higher space harmonics, which exist in real time machines. A more accurate approach to this problem is put forward in [9]. In reference [9] the authors simulate a large utility generator in the abc frame of reference

in order to generate small excitation data is generated; the data is then used to identify machine parameters in $dq0$ axis.

In this study, the utilization of the Hartley series in a linear state estimator for synchronous generator parameters is illustrated. The method is based on a $dq0$ model. The proposed method is verified using both synthetic data and operational data from a digital fault recorder. The effect of measurement noise corruption, always present in real time data acquisition, is also investigated.

A distinctive feature of the proposed approach is the use of the Hartley series. This allows writing a set of linear algebraic equations that can be solved using a pseudoinverse to obtain the unknown parameters. During the last two decades algebraic methods have been established for the solution of problems described by linear differential equations, such as analysis, model reduction, optimal control and system identification. Typical examples are the applications of Laguerre polynomials [10], [11], Legendre polynomials [12], [13], Chebyshev polynomials of the first [14], and second kind [15], Fourier series [16], Walsh series [17], block-pulse series [18], Haar series [19], and Hartley series [20]. The utilization of these series have the common objective of representing models efficiently, and calculating intermediate parameters rapidly for the given problem. It can be said that the various domains cited are alternative “windows” to view a problem, and one wishes to employ the window that gives the best view of the problem and the most efficient calculation. Although *solutions* are the same in all domains, the bandwidth of the problem and numerical characteristics of intermediate matrices used are different, e.g., the set of equations of a particular problem may be sparser (or fewer) in one domain than in others.

The use of the all real Hartley series for parameter estimation is unique in this paper: the concept offers most of the advantages of a frequency based approach without the use of complex quantities.

II. PROBLEM DEFINITION

There are numerous applications for which the parameters of a synchronous generator need to be measured or estimated. For example, transient stability studies are routinely done within reliability areas in order to study the consequences of line switching and unit outages. In many cases, “postmortem” studies are done to determine why the system responded the way it did perhaps during a fault, unit outage, or component failure. For these studies, accurate models are needed. In most cases, manufacturers data are used to satisfy the parameter needs of synchronous generator models. However, saturation,

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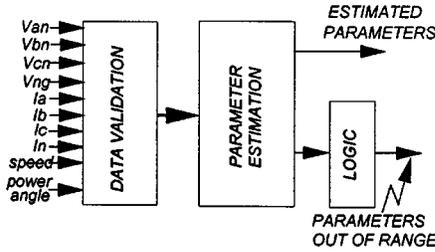


Fig. 1. Parameter estimation of a synchronous generator.

nonlinearities, and machine aging may result in inaccuracies when manufacturers' data are used. In addition, there is an increasing need to use parameter identification to determine either incipient or existent failures: for example, a turn-to-turn short in a field winding of a large generator may be determined through the use of an on-line parameter estimator. When values of field resistance and inductance, and mutual inductances relating to the field winding are estimated as "out of range," an alarm may be issued to indicate the failure.

In general, the process of parameter estimation is shown in Fig. 1. On-line measurements of armature and field voltages and currents are verified using a data verification method as described in [9]. This step is used to determine the confidence in the measurement. If the validation of a specific datum passes, the confidence is high (e.g., near 1.0), and if the measurements does not check with simple physical considerations [e.g., $v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0$], the datum is assigned a low confidence value (e.g., 0.1). The measurements are used with a parameter estimator as shown in Fig. 1, and selected parameters are read by digital logic to identify parameters out of range. Estimated parameters are also used to perform transient stability studies.

III. THE HARTLEY SERIES

In this section a brief review of the Hartley series is given. It can be stated that the kernel function of the where familiar Fourier transform and Fourier series is the complex exponential, $e^{-j\omega t}$. The Hartley transform and series utilizes a similar frequency based kernel, the function $\cos(\omega x) + \sin(\omega t)$, also known as the *cosine- and-sine* function or $\text{cas}(\omega X)$. Thus Hartley technology does not employ complex numbers. Reference [24] is a definitive work on the subject.

The Hartley series basis function $T(t)$ is denoted as (note that the prime notation indicates transposition),

$$T(t) = [T_{-n} \ \cdots \ T_{-1} \ T_0 \ T_1 \ \cdots \ T_n]$$

$$T_n = \text{cas}(n\omega t) = \cos(n\omega t) + \sin(n\omega t).$$

Thus a periodic function of period T_p can be approximated by a $2n + 1$ term truncated Hartley series as

$$f(t) = FT(t)$$

$$F = [F_{-n} \ \cdots \ F_{-1} \ F_0 \ F_1 \ \cdots \ F_n]_{(1, 2n+1)}.$$

Each coefficient of vector F , F_n is calculated using (1),

$$F_n = \frac{1}{T_p} \int_0^{T_p} f(t) \text{cas}(n\omega t) dt. \quad (1)$$

Similar to other orthogonal series expansions, the Hartley series possesses operational properties, namely an operational matrix of integration and differentiation. These can be defined as follows: consider the integral,

$$\int_0^t T_0(\tau) d\tau = \int_0^t d\tau = t \quad (2)$$

$$\int_0^t T_n(\tau) d\tau = \int_0^t \text{cas}(n\omega\tau) d\tau = \frac{1}{2n\pi} (1 - \text{cas}(-n\omega t)). \quad (3)$$

Use of the Hartley series for approximating (2) and (3) yields,

$$\int_0^t T_0(\tau) d\tau = t$$

$$= \frac{T(t)}{2\pi} \begin{bmatrix} \frac{1}{n} & \cdots & \frac{1}{2} & 1 & \pi & -1 & \frac{1}{2} & \cdots & \frac{1}{n} \end{bmatrix}$$

$$\int_0^t T_{-n}(\tau) d\tau = \int_0^t \text{cas}(n\omega\tau) d\tau$$

$$= \frac{1}{2\pi} \begin{bmatrix} 0 & 0 & \cdots & 0 & -\frac{1}{n} & 0 & \cdots & 0 & \frac{1}{n} \end{bmatrix} \cdot T(t)$$

$$\int_0^t T_n(\tau) d\tau = \int_0^t \text{cas}(-n\omega\tau) d\tau$$

$$= \frac{1}{2\pi} \begin{bmatrix} -\frac{1}{n} & 0 & \cdots & 0 & \frac{1}{n} & 0 & \cdots & 0 & 0 \end{bmatrix} \cdot T(t).$$

These integral expressions imply that the integral of a basis function can be also expanded in a Hartley series. This concept is further developed in the Appendix for the integral of a function.

IV. THE HARTLEY ESTIMATOR FORMULA

Consider a deterministic time-invariant single-input single-output (SISO) system with zero initial conditions of the type

$$A_n(\rho)y(t) = B_n(\rho)u(t) \quad (4)$$

where

$$A_n(\rho) = \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + a_1 \frac{d}{dt} + a_0$$

$$B_n(\rho) = \frac{d^n}{dt^n} + b_{n-1} \frac{d^{n-1}}{dt^{n-1}} + \cdots + b_1 \frac{d}{dt} + b_0. \quad (5)$$

The problem is to determine (compute or estimate) the $2n$ parameters $a_0, a_1, \dots, a_{n-1}, b_0, b_1, \dots, b_{n-1}$, by using a record of input-output measurements $\{u(t), y(t)\}$ over an interval T . Without loss of generality, assume that

$$T = \{t; 0 < t < 1\}.$$

If

$$T = \{t; 0 < t < t_f\}$$

the time interval is normalized by $\sigma = t/t_f$ to obtain the normalized interval,

$$T = \{\sigma, 0 < \sigma < t_f\}.$$

At this point, the synchronous machine model (Park's equations) is introduced in form by referring to one of many references, e.g., [21],

$$FF = -AZ - B\dot{Z}.$$

To solve the problem of parameter identification, the differential which involves the derivatives of the available input output data must first be converted to an algebraic model. Functions $y(t)$ and $u(t)$ are expanded in a Hartley series and the operational matrix of differentiation is successively employed to yield,

$$\begin{aligned} \frac{d^n}{dt^n} y(t) &= D^n Y T(t), & \frac{d^n}{dt^n} u(t) &= D^n U T(t), \\ \frac{d^{n-1}}{dt^{n-1}} y(t) &= D^{n-1} Y T(t), & \frac{d^{n-1}}{dt^{n-1}} u(t) &= D^{n-1} U T(t), \\ \dots & & \dots & \\ \frac{d}{dt} y(t) &= D Y T(t) & \frac{d}{dt} u(t) &= D U T(t). \end{aligned} \quad (6)$$

In these expressions, Y and U are the Hartley series coefficients of $y(t)$ and $u(t)$ respectively, i.e.,

$$\begin{aligned} U &= [U_{-n} \ \dots \ U_0 \ \dots \ U_1] \\ Y &= [Y_{-n} \ \dots \ Y_0 \ \dots \ Y_1]. \end{aligned} \quad (7)$$

Using (7)–(9), the machine model becomes,

$$A_n(D) Y T(t) = B_n(D) U T(t). \quad (8)$$

Defining the vectors

$$\begin{aligned} \theta_a &= \begin{bmatrix} a_{n-1} & a_{n-1} & \dots & a_0 \end{bmatrix} \\ \theta_b &= [\theta_a \ \theta_b] \end{aligned}$$

yields a model appropriate for the pseudoinverse solution,

$$\theta [D^{n-1} Y \ D^{n-2} Y \ \dots \ Y \ D^n U \ D^{n-1} U \ \dots \ U]^T = D^n Y.$$

Or in compact form,

$$\theta Q = q. \quad (9)$$

Finally, solving for the parameters,

$$\theta = q Q^+ \quad (10)$$

where the “+” superscript means the pseudoinverse [22]. Similar equations may be derived using the operational matrix of integration and the basic formula can be readily extended to account for multiple input multiple output (MIMO) systems as well as for accounting for noise effects without knowing its stochastic nature [23].

V. ESTIMATION OF ARMATURE CIRCUIT PARAMETERS

To illustrate the identification approach a one machine infinite bus is simulated for a specific steady state operating point.

TABLE I
EXAMPLE MACHINE PARAMETERS

Parameter	Value (per unit)	Description
$r_a = r_b = r_c = r$	0.0027	Stator phase resistance
r_n	100	Equivalent neutral resistance
$L_q = x_q$	1.72	Equivalent quadrature axis inductance
$M_Q = L_{AQ}$	1.27	Mutual inductance between phase and damper winding Q
$L_d = x_d$	1.80	Equivalent direct axis inductance
M_F	1.339	Mutual inductance between phase winding and field winding
M_D	1.339	Mutual inductance between phase winding and damper winding D
r_F	$9.722 \cdot 10^{-4}$	Field resistance
r_D	0.01254	Damper winding resistance, direct axis
r_Q	0.01632	Damper winding resistance, quadrature axis
$L_0 = x_0$	0.150	Zero sequence inductance of stator
L_n	100	Neutral inductance
$L_F = l_F$	0.1353	Field inductance
$M_R = L_{AD}$	1.64	Mutual inductance between field windings and damper winding D
$L_D = l_D$	0.1321	Damper inductance, direct axis
$L_Q = l_Q$	0.03059	Damper inductance, quadrature axis.

The constants in Park's state voltage equation [21], [22] are obtained directly (or by simple calculation) from the manufacturers' stability study data sheet and saturation curves. The detailed derivation of these constants for the example shown here can be found in [22]. Table I lists the final per unit quantities that are used in exemplar studies to follow.

The estimation of the armature parameters employs the Moore–Penrose pseudoinverse. This method is identical to the least squares estimation for linear systems.

After the one-machine infinite bus system is simulated and steady state measurable states and inputs are collected for estimation, Park's state voltage equations are written in the compact form

$$V = -AI - B\dot{I} \quad (11)$$

where V refers to the forcing functions ($dq0$ and field voltages), I is the state vector and A and B are the matrices of machine parameters. Since the armature parameters are estimated from steady state data the following model can be established in terms of dq axis variables taking into account that in this operating

TABLE II
ESTIMATES OF ARMATURE PARAMETERS (PER UNIT, STEADY STATE COMPONENTS USED IN ESTIMATION PROCESS)

Case		R_a	M_F	L_d	L_q
SNR	Actual Values	0.0027	1.64	1.80	1.72
∞	Estimate	0.0027	1.64	1.8	1.72
	Error	0	0	0	0
10^5	Estimate	0.0027	1.64	1.80	1.72
	% Error	0	0	0	0
10^3	Estimate	0.00273	1.6401	1.8002	1.7199
	% Error	1.133	0.006	0.0125	0.0017
500	Estimate	0.002638	1.6403	1.8004	1.72005
	% Error	2.26	0.0183	0.026	0.003
200	Estimate	.00271639	1.6402	1.80034	1.720007
	% Error	0.607	0.0125	0.0212	0
100	Estimate		1.6426	1.8030	1.72144
	% Error		0.1576	0.183	0.0877
50	Estimate		1.63075	1.7893	1.7144
	% Error		0.567	0.6551	0.339

point damper winding currents and the rate of change of stator flux linkages are zero,

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = - \begin{bmatrix} r & \omega L_q & 0 \\ -\omega L_d & r & -\omega k M_F \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_F \end{bmatrix} \quad (12)$$

that for estimation purposes should be rewritten as

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = - \begin{bmatrix} i_d & & \omega i_q \\ i_q & -\omega k i_F & -\omega i_d \end{bmatrix} \begin{bmatrix} r \\ M_F \\ L_d \\ L_q \end{bmatrix}. \quad (13)$$

Having collected measurable variables $v_d, i_d, v_q, i_q, v_F, i_F$ and the Hartley coefficients of each state variable in (16) in the frequency domain becomes

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = - \begin{bmatrix} I_d & & \omega I_q \\ I_q & -\omega k I_F & -\omega I_d \end{bmatrix} \begin{bmatrix} r \\ M_F \\ L_d \\ L_q \end{bmatrix} \quad (14)$$

where V_d, I_d, V_q, I_q , are the vectors containing the Hartley coefficients of variables v_d, i_d, v_q, i_q . Parameters are obtained from solution of (10).

In order to obtain parameters from (14), the one machine infinite bus system was simulated for v_{fd} disturbances within 2% and steady state data before and after the disturbances are utilized for estimation. The estimation results obtained from this experiment are outlined in Table II. At this point the algorithm was also tested for noise corruption that often appears in real time measurements. The algorithm is tested for different

TABLE III
ESTIMATED VALUES FOR AN ACTUAL SYNCHRONOUS MACHINE
(NOTE: "a" IS THE FIELD/STATOR TURNS RATIO)

Parameter	Estimate (mH)	Manufacturer's value (mH)
aL_{ad}	46.2	43.59
L_d	3.5	4.7
L_q	1.7	4.5

values of SNR (signal/noise ratio). It can be seen that in absence of noise the algorithm is able to recover armature parameters without error.

It can be also seen from Table II that performance is very good even for cases of high levels of noise. Parameters were recovered with an error of 1% for SNR levels up to 200 : 1. For larger levels of noise the parameter with the smallest value is impossible to recover. However, the remainder of the parameters can be recovered within remarkably low error even for considerable levels of noise such as SNR 10 : 1. In this case, the largest detected estimation error was 1.2%. The Hartley estimator formula appears to be noise resistant because the series truncation plays the role of a low pass filter. For this reason, the minimum SNR levels that allow accurate estimation appear to be lower than minimum SNR levels for comparable estimation techniques.

VI. ESTIMATION OF ARMATURE CIRCUIT PARAMETERS FROM OPERATING DATA

For this section, experimental on-line operating data provided by the electric utility is used to estimate a large utility generator armature circuit parameters. The measurements provided include $v_{ab}, v_{bc}, v_{ca}, i_{as}, i_{bs}, i_{cs}, v_{fd}$ and i_{fd} . The data sets correspond to two different steady-state operating points. The steady state equations used in Section V are in the rotor reference frame. In order to use this model, measurable quantities are converted to $dq0$ axis equivalents using the following expressions,

$$\begin{aligned} V_t &= \sqrt{(v_{as}^2 + v_{bs}^2 + v_{cs}^2)}/1.5 \\ v_d &= V_t \sin \delta \quad v_q = V_t \cos \delta \\ i_d &= (P \sin \delta + Q \cos \delta)/1.5V_t \\ i_q &= (P \sin \delta - Q \cos \delta)/1.5V_t \\ P &= i_{as}v_{as} + i_{bs}v_{bs} + i_{cs}v_{cs} \\ Q &= (v_{ab}i_{cs} + v_{bc}i_{as} + v_{ca}i_{bs})/\sqrt{3} \\ E_q \angle \delta &= V_t \angle 0 + I_t \angle \phi (R_a + jX_a). \end{aligned}$$

Where the subscripts a, b , and c refer to phase coordinates.

It was impossible to recover r from the limited sets of data available but considering that the sensitivity of the parameters M_f, L_d , and L_q is negligible for changes in r , this value was set constant at the manufacturer's value. Table III summarizes the estimated parameters for the machine cited earlier. The manufacturer's data are shown for comparison only: there may be disagreement between estimated and manufacturer's data due to estimation error and parameter variation with operating point, saturation, and age.

