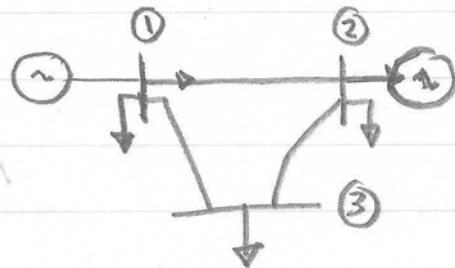


Lecture #9 A. Keyhani

Newton-Raphson Method



Power Flow Problem.

$$S_{Ti} = P_{Gi} + jQ_{Gi} = V_i I_{i,n} \quad i=1, 2, 3$$

Given $P_{Gi}^{\text{sch}}, Q_{Gi}^{\text{sch}}$

ΔV_{bus} model

Find ΔV_{bus}

Losses, ΔQ_{losses}

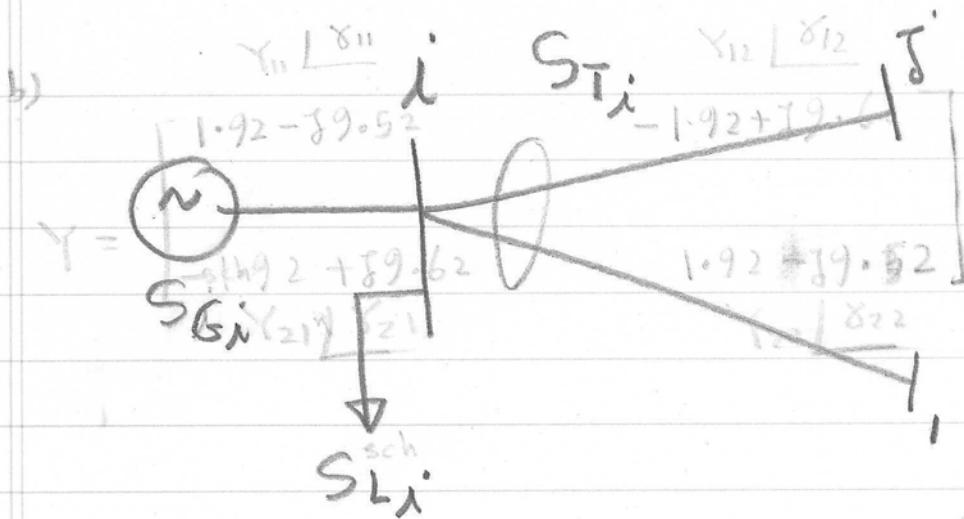
P_{ij}, Q_{ij} Power flow through
Trans. lines and Transformers

model

$$\Delta V_{\text{bus}} = \Delta Y_{\text{bus}} \Delta V_{\text{bus}}$$

$$I_i = \sum_{j=1}^n Y_{ij} V_j \quad i=1, \dots$$

$$S_i = V_i I_i^*$$



c) $\bar{S}_{T_i} = \bar{P}_{T_i} + j\bar{Q}_{T_i}$ Transmitted Power

$$= V_i I_{T_i}^* \quad i=1, 2$$

but

$$\begin{bmatrix} I_{T_1} \\ I_{T_2} \end{bmatrix} = \begin{bmatrix} \tilde{Y}_{11} & \tilde{Y}_{12} \\ \tilde{Y}_{21} & \tilde{Y}_{22} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{bmatrix}$$

or $I_{T_i} = \sum_{j=1}^2 \tilde{Y}_{ij} V_j \quad i=1, 2$

let $\tilde{V}_i = V_i \angle \delta_i \quad \tilde{Y}_{ij} = Y_{ij} \angle \delta_{ij}$

Then

$$I_{T_i}^* = \sum_{j=1}^n Y_{ij} V_j \angle -\delta_{ij} - \delta_j$$

~~Since~~ $S_{T_i} = V_i \sum_{j=1}^n Y_{ij} V_j \angle \delta_i - \delta_j - \delta_{ij}$

$$P_{Ti} = \sum_{j=1}^n V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij})$$

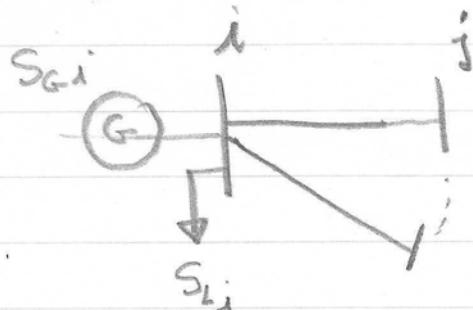
$$Q_{Ti} = \sum_{j=1}^n V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \gamma_{ij})$$

I like to call P_T

$$P_i^{\text{cal}} (\mathbf{V}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) = P_{Ti}$$

$$Q_i^{\text{cal}} (\mathbf{V}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) = Q_{Ti}$$

Consider Bus "i"



$$P_{Gi} = P_{Li} + P_i^{\text{cal}} (\mathbf{V}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) \quad \text{Eq. 7.19a}$$

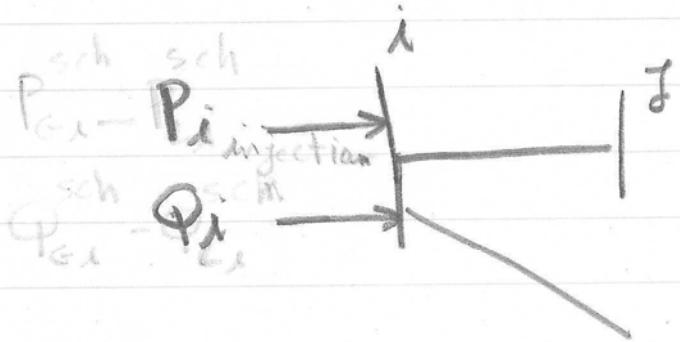
$$Q_{Gi} = Q_{Li} + Q_i^{\text{cal}} (\mathbf{V}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) \quad \text{Eq. 7.19b}$$

I like to call

$$P_{Gi}^{\text{sch}} = P_{Li}^{\text{sch}} + P_i^{\text{cal}}$$

$$Q_{Gi}^{\text{sch}} = Q_{Li}^{\text{sch}} + Q_i^{\text{cal}}$$

Note that the net-injection to bus i



$$P_i^{\text{mij}} = P_{Gi}^{\text{sch}} - P_{Li}^{\text{sch}}$$

$$Q_i^{\text{mij}} = Q_{Gi}^{\text{sch}} - Q_{Li}^{\text{sch}}$$

Therefore, Your book

$$P_i^{\text{mij}} = P_{Gi}^{\text{sch}} - P_{Li}^{\text{sch}} = P_i^{\text{cal.}} (\mathbf{v}_{\text{bus}}, \mathbf{Y}_{\text{bus}})$$

$$Q_i^{\text{mij}} = Q_{Gi}^{\text{sch}} - Q_{Li}^{\text{sch}} = Q_i^{\text{cal.}} (\mathbf{v}_{\text{bus}}, \mathbf{Y}_{\text{bus}})$$

~~Various net-injection equations:~~

$$\Delta P_i = P_i^{\text{mij}} - P_i^{\text{cal.}} (\mathbf{v}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) \leq \epsilon$$

$$\Delta Q_i = Q_i^{\text{mij}} - Q_i^{\text{cal.}} (\mathbf{v}_{\text{bus}}, \mathbf{Y}_{\text{bus}}) \leq \epsilon$$

(5)

$$P_i^{\text{inj}} - P_i^{\text{cal}}(\Delta_{\text{bus}}, Y_{\text{bus}}) = 0$$

$$Q_i^{\text{inj}} - Q_i^{\text{cal}}(\Delta_{\text{bus}}, Y_{\text{bus}}) = 0 \quad i=1, \dots$$

$$P_1^{\text{inj}} - P_1^{\text{cal}}(v_1, \dots, v_m) = 0 \quad f_1(\mathbf{x}) = 0$$

$$P_2^{\text{inj}} - P_2^{\text{cal}}(v_1, \dots, v_m) = 0 \quad f_2(\mathbf{x}) = 0$$

$$P_m^{\text{inj}} - P_m^{\text{cal}}(v_1, \dots, v_m) = 0 \quad f_m(\mathbf{x}) = 0$$

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_m(\mathbf{x}) \end{bmatrix} = 0$$

$$f_1(\mathbf{x}) = f_1(x_1, x_2, \dots, x_m) = f_1(x_1^{(0)}, \dots, x_m^{(0)})$$

$$+ \left. \frac{\partial f_1}{\partial x_1} \right|_{\mathbf{x}^{(0)}} \Delta x_1 + \left. \frac{\partial f_1}{\partial x_2} \right|_{\mathbf{x}^{(0)}} \Delta x_2 + \dots$$

$$\left. \frac{\partial f_m}{\partial x_m} \right|_{\mathbf{x}^{(0)}} \Delta x_m = 0$$

(6)

$$f_1(\mathbf{x}) = f_1(\mathbf{x}^{(0)}) + \sum_{k=1}^n \frac{\partial f_1}{\partial x_k} \Big|_{\mathbf{x}^{(0)}} \Delta x_k = 0$$

$$f_2(\mathbf{x}) = f_2(\mathbf{x}^{(0)}) + \sum_{k=1}^n \frac{\partial f_2}{\partial x_k} \Big|_{\mathbf{x}^{(0)}} \Delta x_k = 0$$

$$f_m(\mathbf{x}) = f_m(\mathbf{x}^{(0)}) + \sum_{k=1}^n \frac{\partial f_m}{\partial x_k} \Big|_{\mathbf{x}^{(0)}} \Delta x_k = 0$$

or in vector notation

$$\mathbf{F}(\mathbf{x}) = \mathbf{F}(\mathbf{x}^{(0)}) + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_m} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \dots & \frac{\partial f_m}{\partial x_m} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_m \end{bmatrix} = 0$$

\mathbf{J}

$$\begin{aligned} \Delta \mathbf{x} &= \mathbf{x}_{\text{old}} - \mathbf{x}_{\text{new}} \\ &= \mathbf{x}^0 - \mathbf{x}^m \end{aligned}$$

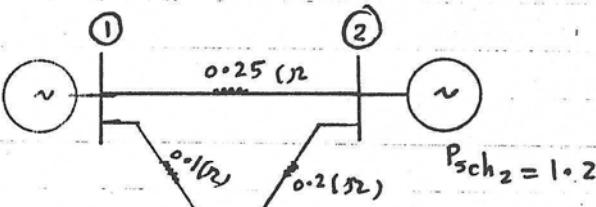
$$\mathbf{F}(\mathbf{x}^{(0)}) + [\mathbf{J}]_{\mathbf{x}^{(0)}} [\Delta \mathbf{x}] = 0$$

↳ Jacobian matrix

$$[\Delta \mathbf{x}] = - [\mathbf{J}]^{-1} \mathbf{F}(\mathbf{x}^{(0)})$$

Power system methods of Analysis

24. Newton-Raphson method. Example 1 - D-C system.



Comments:

- Bus 1 is slack bus
- $\tilde{V}_1 = 1 \text{ p.u}$
- Scheduled injected power at bus 2 is 1.2 p.u
- Scheduled Load (negative injection) at bus 3 is 1.5 p.u
- D-C case is analyzed for sake of convenience.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$S_i = V_i I_i \quad i = 1, 2, 3$$

The bus powers can be expressed as non-linear functions of bus voltages in residue form as

D-C

$$P_1^{\text{cal}} = V_1 I_1 \quad \text{or} \quad P_1^{\text{cal}} - V_1 I_1 = 0$$

$$P_1(V_1, V_2, V_3) - V_1(Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3) = 0$$

$$P_2^{\text{cal}} = V_2 I_2 \quad P_2(V_1, V_2, V_3) - V_2(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3) = 0$$

$$P_3^{\text{cal}} = V_3 I_3 \quad P_3(V_1, V_2, V_3) - V_3(Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3) = 0$$

$$F(X) = 0$$

Using Taylor Series expansion about guess solution, i.e. $V_2^{(0)}, V_3^{(0)}$ and $V_1 = 1$.

Recall:

$$F(x^*) + [J]_{x^*} [\Delta x] = 0$$

Note here

$$\Delta P = F(x^*) = P_{\text{sch}} - P_{\text{cal}}$$

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial V_2} & \frac{\partial P_1}{\partial V_3} \\ \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial V_1} & \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} \Rightarrow \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

OR in compact notation

$$[\Delta P] = -[J] \times [\Delta V]$$

where

$$\Delta P_2 = P_2^{\text{sch}} - P_2^{\text{cal}}$$

$$\Delta P_3 = P_3^{\text{sch}} - P_3^{\text{cal}}$$

$$\Delta P_2 = P_2^{\text{sch}} - V_2^{(0)} (Y_{11}V_1^{(0)} + Y_{12}V_2^{(0)} + Y_{13}V_3^{(0)})$$

$$\Delta P_3 = P_3^{\text{sch}} - V_3^{(0)} (Y_{31}V_1^{(0)} + Y_{32}V_2^{(0)} + Y_{33}V_3^{(0)})$$

Power System Methods of Analysis

2.4. Example 1. Network-Raphson method. cont.

Note that $V_1 = 1 \angle 0^\circ$ (slack bus). Therefore $\Delta V_1 = 0$. The system of equations become:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix} = - \begin{bmatrix} \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} \end{bmatrix} \begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix} \Big|_{V^{(0)}}$$

ΔV_2 and ΔV_3 can be calculated from:

$$\begin{bmatrix} \Delta V_2 \\ \Delta V_3 \end{bmatrix} = - \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix} = - \begin{bmatrix} \mathcal{J} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix} \Big|_{V^{(0)}}$$

Note that

$$\begin{aligned} \Delta V_2 &= V_2(\text{old}) - V_2(\text{new}) & V_2(\text{new}) &= V_2(\text{old}) - \Delta V_2 \\ \Delta V_3 &= V_3(\text{old}) - V_3(\text{new}) & V_3(\text{new}) &= V_3(\text{old}) - \Delta V_3 \end{aligned}$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}_{\text{new}} = \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}_{\text{old}} + \begin{bmatrix} \mathcal{J} \end{bmatrix}^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix}_{\Delta V(\text{old})}$$

The elements of \mathcal{J} matrix are:

$$\frac{\partial P_2}{\partial V_2} = Y_{21} V_1 + 2Y_{22} V_2 + Y_{23} V_3$$

$$\frac{\partial P_2}{\partial V_3} = V_2 Y_{23}$$

$$\frac{\partial P_3}{\partial V_2} = V_3 Y_{32}$$

$$\frac{\partial P_3}{\partial V_3} = Y_{31} V_1 + Y_{32} V_2 + 2Y_{33} V_3$$

Power system Methods of Analysis

4. Example 1. Newton - Raphson method. cont.

Assume $V_2^{(0)} = 1$, $V_3^{(0)} = 1$, the elements of Jacobian matrix

$$\frac{\partial P_2}{\partial V_2} = -4 + 2 \times 9 - 5 = 9$$

$$\frac{\partial P_2}{\partial V_3} = -5$$

$$\frac{\partial P_3}{\partial V_2} = -5$$

$$\frac{\partial P_3}{\partial V_3} = -10 - 5 + 2 \times 15 = 15$$

Therefore, the Jacobian matrix is:

$$[J] = \begin{bmatrix} 9 & -5 \\ -5 & 15 \end{bmatrix}$$

and, the $[J]^{-1}$ is

$$[J]^{-1} = \frac{1}{110} \begin{bmatrix} 15 & 5 \\ 5 & 9 \end{bmatrix}$$

The mismatch power at each bus is

$$\Delta P_2 = 1.2 - 1.0(-4 + 9 - 5) = 1.2$$

$$\Delta P_3 = -1.5 - 1.0(-10 - 5 + 15) = -1.5$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} + \frac{1}{110} \begin{bmatrix} 15 & 5 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 1.2 \\ -1.5 \end{bmatrix}$$

$$\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1.095 \\ 0.932 \end{bmatrix}$$