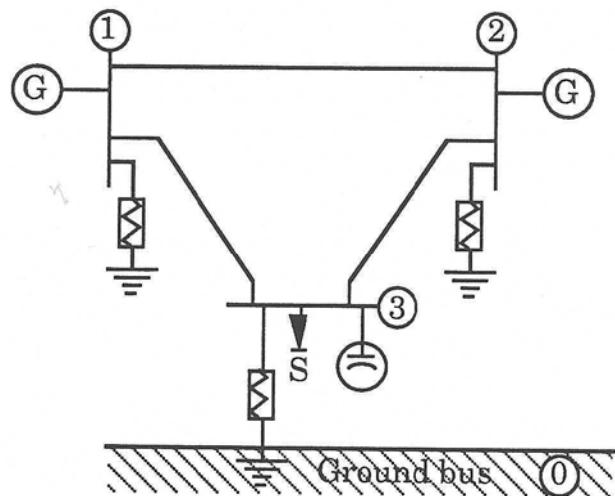


Power Flow Problem:

- 1) A balanced three-phase power system is assumed, and the transmission system is represented by its positive-phase sequence network of linear lumped series and shunt branch.
- 2) The generators are assumed to be three-phase balanced voltage sources and only the generator positive voltages are present.
- 3) The generators are shown as the constant P-Q models, i.e. as the injected powers into the system. Therefore, the internal impedance do not enter in the Y bus or Z bus matrix formulation.
- 4) The load on each bus is assumed to be a three-phase balanced load. The load models are constant P-Q models.
- 5) Each bus of the system is described by four parameters: P, Q, |V|, and θ . Two of the parameters are known and the other two parameters are unknown.
- 6) Slack (or swing) bus is a fictitious concept, created and defined in the problem formulations. It arises because the system I^2R losses are not known precisely in advance of the load flow calculation.

Power Flow Problem

Given: Schedule of generations, Loads
and the system model

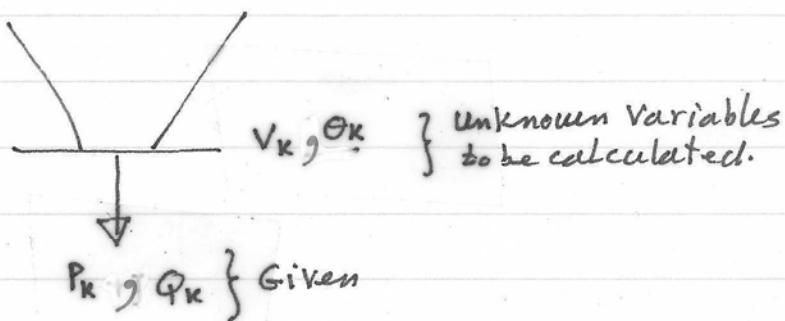
Find:

- Bus voltages $V_i | \theta_i, i=1..n$
- Power flow through transmission lines
- Active and reactive Losses

$P_G^{\text{sch}}, Q_G^{\text{sch}}$,
 $P_L^{\text{sch}}, Q_L^{\text{sch}}$
 $\mathbf{Y}_{\text{bus model}}$

Load buses:

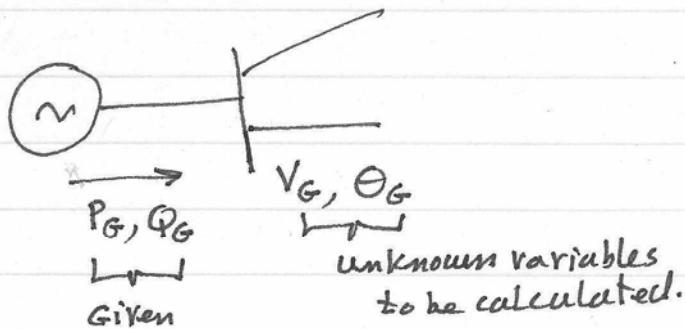
Load buses are modeled as constant P and Q power consumption.



Generator buses: TWO types of models are used:

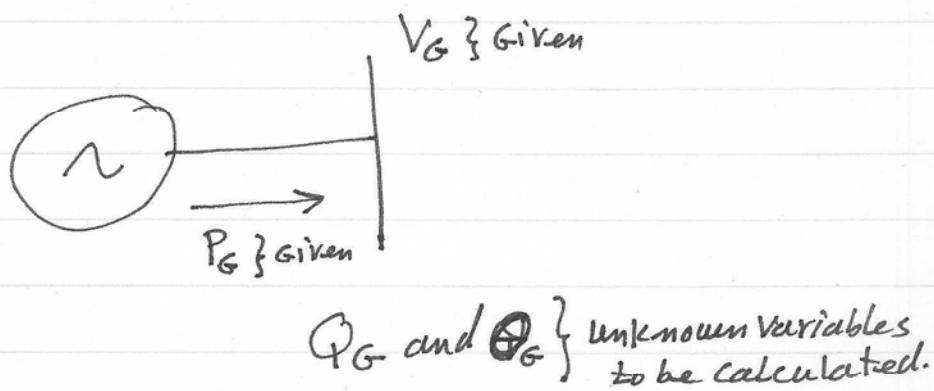
- Constant P_G and Q_G model
- Constant P_G and V_G model.

constant P_G and Q_G model

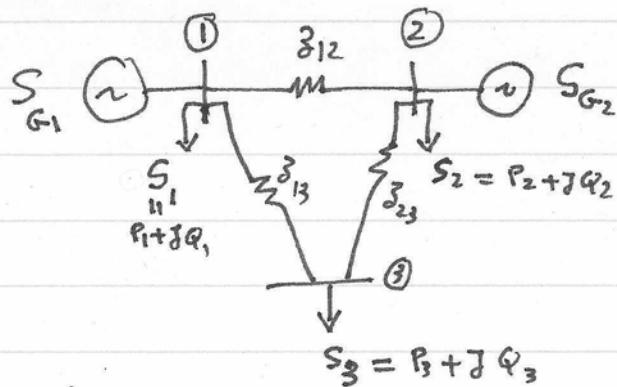


Note: For power flow studies, the internal impedance of generators is not included in the system model.

constant P_G and V_G model



consider the power system given below:
for a power system problem



Assume inductive loads.

The following conditions must be satisfied for all power flow problems:

$$P_{G1} + P_{G2} = P_1 + P_2 + P_3 + \text{Losses}$$

$$Q_{G1} + Q_{G2} = Q_1 + Q_2 + Q_3 + Q_{\text{Losses}}$$

How do we calculate the losses? . We need to know the losses in order to schedule the needed generation from generators (i.e. G₁ or G₂) so that to balance the generation with consumption

- To calculate the losses, we need to calculate the current flow through transmission lines
- Then, $P_{\text{Loss}}_{12} = I_{12}^2 R_{12}$ $Q_{\text{Loss}}_{12} = I_{12}^2 X_{12} \dots$
- To calculate the current, we need to calculate the voltages. i.e. $I_{12} = \frac{V_1 - V_2}{Z_{12}}$. However, the voltages are the unknown variables of power flow problems.
We have a problem!

Swing bus. A swing bus is a system bus other than ground at which the bus voltage is fixed in both magnitude and phase angle. The swing bus is also known as the slack bus.

$$V_1 = 1 \text{ p.u.} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ Given.}$$



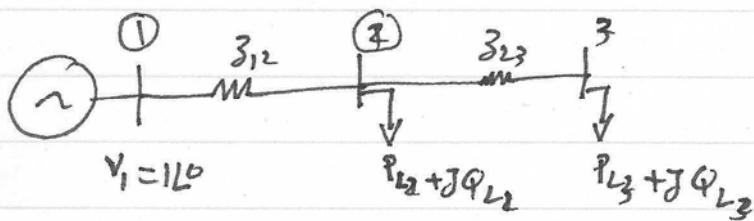
$$\left. \begin{array}{l} P_G, Q_G \\ \text{variables} \\ \text{to be calculated} \end{array} \right\}$$

Note: Since the swing bus voltage is fixed, it acts as an ideal voltage source and the swing generator would supply the active and reactive power **losses** and extra generation needed in order to balance active and reactive power of the system.

$$P_{G_1} + P_{G_2} + \dots = P_{L_1} + P_{L_2} + \dots + \text{Losses}$$

$$Q_{G_1} + Q_{G_2} + \dots = Q_{L_1} + Q_{L_2} + \dots + \text{Losses.}$$

swing bus can also be considered as an infinite bus.



$$P_{G_1} = \text{Losses} + P_{L_2} + P_{L_3}$$

$$Q_{G_2} = \text{Losses} + Q_{L_2} + Q_{L_3}$$

Load Flow Study as an Engineering Tool:

Application-Planning: The main objective of the load flow study for planning is to find out whether or not a specific system design alternative produces bus voltages within acceptable limits. Or, simply an answer to problems like these.

- For expected loads and generators several years ahead. Does this line need to be built?
- To correct a given system voltage profile to within acceptable limits, how much reactive power supply is required?

Application-Operation:

- For tomorrow's expected loads and available generation; can this transformer be taken out of service?
- For sudden loss of this line; can the system load demand be satisfied without any overloads?

Load flowstudies are performed in power system planning, operational planning, and operation and control. Also, load flow calculations are needed in:

- Outage security assessment.
- Within power system optimization problem.
- Within power system stability problem.

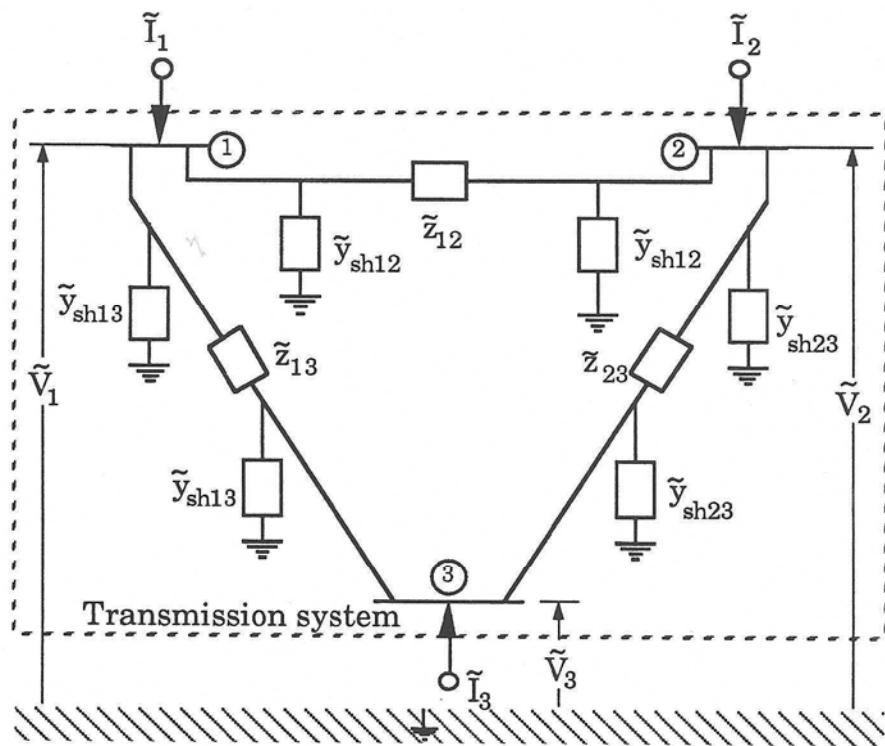


fig. 1

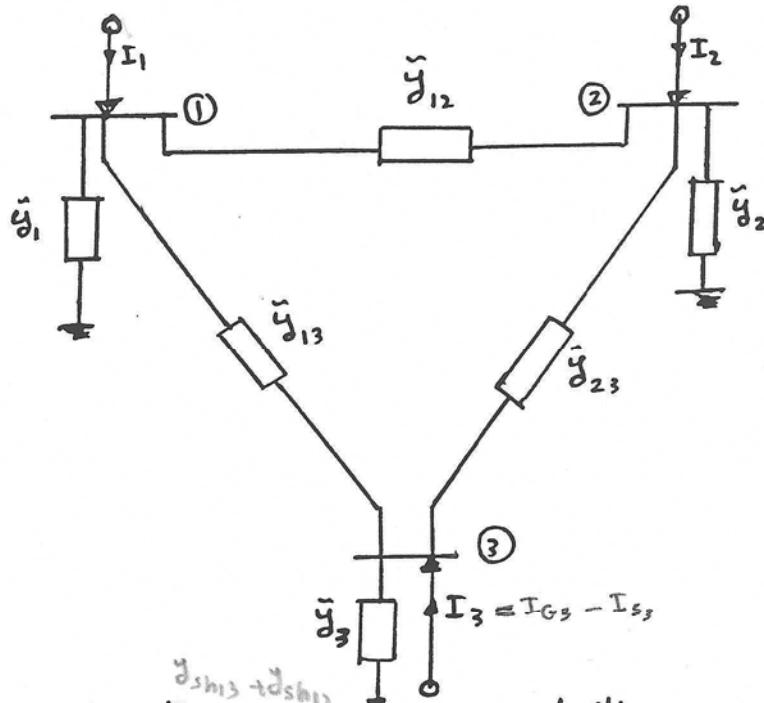
Where:

- The bus voltage \sim are actually bus-to-ground voltages.
- The bus currents \tilde{I} are currents flowing into the transmission system from generators, loads, or other power system elements connected to the transmission system, but not shown in fig. 1.
- All currents are assigned a positive direction into their respective buses, that is, all the generators inject positive currents and all the loads inject negative currents.
- The \tilde{z}_{ij} is the positive sequence impedance between bus i and j.
- The \tilde{y}_{ij} is the half of the positive sequence shunt and admittance between bus i and j.

Power system Modeling

6-Power system Bus Admittance Matrix Model. Cont.

The power system shown by Fig. 1 can be redrawn as:



Where :

- $\bar{y}_1 = \bar{y}_{13} + \bar{y}_{12}$ Total shunt admittance connected to bus 1
- $\bar{y}_2 = \bar{y}_{23} + \bar{y}_{12}$ Total shunt admittance connected to bus 2
- $\bar{y}_3 = \bar{y}_{13} + \bar{y}_{23}$ Total shunt admittance connected to bus 3.
- $\bar{y}_{12} = \frac{1}{\bar{Z}_{12}}$, $\bar{y}_{23} = \frac{1}{\bar{Z}_{23}}$, $\bar{y}_{13} = \frac{1}{\bar{Z}_{13}}$

Assuming the ground bus as the reference bus, Kirchhoff's current law for each bus (node) gives:

$$\bar{I}_1 = \bar{V}_1 \bar{y}_1 + (\bar{V}_1 - \bar{V}_3) \bar{y}_{13} + (\bar{V}_1 - \bar{V}_2) \bar{y}_{12}$$

$$\bar{I}_2 = \bar{V}_2 \bar{y}_2 + (\bar{V}_2 - \bar{V}_3) \bar{y}_{23} + (\bar{V}_2 - \bar{V}_1) \bar{y}_{12}$$

$$\bar{I}_3 = \bar{V}_3 \bar{y}_3 + (\bar{V}_3 - \bar{V}_2) \bar{y}_{23} + (\bar{V}_3 - \bar{V}_1) \bar{y}_{13}$$

The above equations can be written as

$$\bar{I}_1 = \bar{V}_1 (\bar{y}_1 + \bar{y}_{13} + \bar{y}_{12}) + \bar{V}_2 (-\bar{y}_{12}) + \bar{V}_3 (-\bar{y}_{13})$$

$$\bar{I}_2 = \bar{V}_1 (-\bar{y}_{12}) + \bar{V}_2 (\bar{y}_2 + \bar{y}_{12} + \bar{y}_{23}) + \bar{V}_3 (-\bar{y}_{23})$$

$$\bar{I}_3 = \bar{V}_1 (-\bar{y}_{13}) + \bar{V}_2 (-\bar{y}_{23}) + \bar{V}_3 (\bar{y}_3 + \bar{y}_{23} + \bar{y}_{13})$$

(8)

Power system Modeling

6 Power system \mathbf{Y}_{bus} Admittance Matrix Model. Cont.

In matrix form.

$$\begin{bmatrix} \bar{\mathbf{I}}_1 \\ \bar{\mathbf{I}}_2 \\ \bar{\mathbf{I}}_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_1 \\ \bar{\mathbf{V}}_2 \\ \bar{\mathbf{V}}_3 \end{bmatrix}$$

where : $\tilde{Y}_{11} = y_1 + \tilde{y}_{13} + \tilde{y}_{12}$, $\tilde{Y}_{12} = -\tilde{y}_{12}$, $\tilde{Y}_{13} = -\tilde{y}_{13}$
 $\tilde{Y}_{21} = \tilde{y}_{12}$, $\tilde{Y}_{22} = \tilde{y}_2 + \tilde{y}_{12} + \tilde{y}_{23}$, $\tilde{Y}_{23} = -\tilde{y}_{23}$
 $\tilde{Y}_{31} = \tilde{y}_{13}$, $\tilde{Y}_{32} = \tilde{y}_{23}$, $\tilde{Y}_{33} = \tilde{y}_3 + \tilde{y}_{23} + \tilde{y}_{13}$

In a more compact form:

$$\begin{bmatrix} \bar{\mathbf{I}}_{\text{bus}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\text{bus}} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_{\text{bus}} \end{bmatrix}$$

↓ Bus Injected current vector
 ↓ Bus Admittance matrix
 ↓ Bus voltage Profile vector.

Algorithm for Formulation of \mathbf{Y}_{bus} matrixThe elements of \mathbf{Y}_{bus} matrix can be calculated from the following

Algorithm.

$$\left(\mathbf{Y}_{\text{bus}} \right)_{ij} = \begin{cases} 1) \text{ If } i=j, Y_{ii} = \sum \tilde{y} \text{ i.e the } \Sigma \text{ of admittance connected to bus } i \\ 2) \text{ If } i \neq j \text{ and bus } i \text{ is not connected to bus } j, \text{ then the element } Y_{ij} = 0 \\ 3) \text{ If } i \neq j \text{ and bus } i \text{ is connected to bus } j \text{ through the admittance } \tilde{y}_{ij}, \text{ then the element is } Y_{ij} = -\tilde{y}_{ij} \end{cases}$$

(29)

Power system Modelling

6- Power system \mathbf{Y}_{bus} Admittance matrix Model. Cont.

Properties of \mathbf{Y}_{bus} :

- Symmetric matrix
- Complex.
- The \mathbf{Y}_{bus} matrix is sparse
- The row sum (or column sum) corresponding to each bus, is equal to the admittance to reference bus.
- If there is no connection to reference bus, every row sum is zero. For this case, the \mathbf{Y}_{bus} matrix is singular and

$$\det[\mathbf{Y}_{\text{bus}}] = 0$$

such a \mathbf{Y}_{bus} matrix can not be inverted.

7- Power System \mathbf{Z}_{bus} Impedance Matrix Model

BUS voltages are related to bus currents by the bus impedance matrix.

$$\begin{bmatrix} \tilde{\mathbf{V}}_1 \\ \tilde{\mathbf{V}}_2 \\ \tilde{\mathbf{V}}_3 \end{bmatrix} = \begin{bmatrix} \tilde{z}_{11} & \tilde{z}_{12} & \tilde{z}_{13} \\ \tilde{z}_{21} & \tilde{z}_{22} & \tilde{z}_{23} \\ \tilde{z}_{31} & \tilde{z}_{32} & \tilde{z}_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

In compact form.

$$[\mathbf{V}_{\text{bus}}] = [\mathbf{Z}_{\text{bus}}] [\mathbf{I}_{\text{bus}}]$$

↓ ↓ ↓
 Bus voltage profile vector Bus Impedance matrix Bus Injected current vector

The \mathbf{Z}_{bus} matrix is the inverse of the \mathbf{Y}_{bus} matrix:

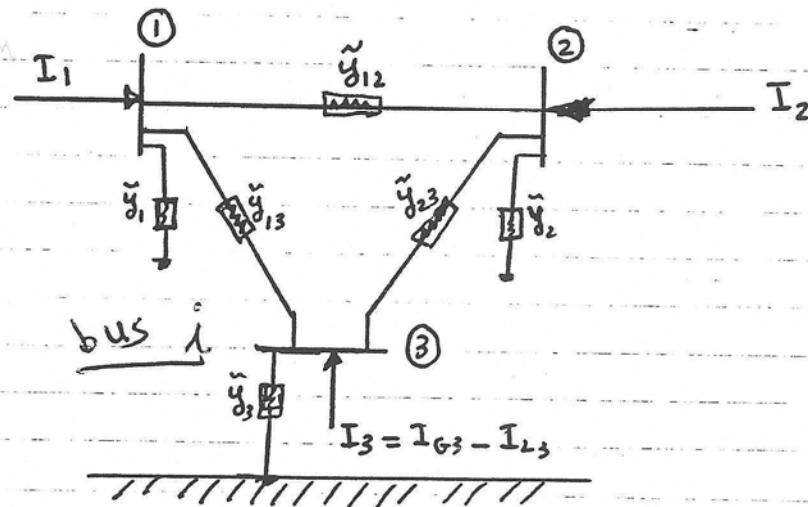
$$[\mathbf{Z}_{\text{bus}}] = [\mathbf{Y}_{\text{bus}}]^{-1}$$

Power System Methods of Analysis

3. Nature of Load Flow Problem.

The injection (power) form of the power system discussed

before is:



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \end{bmatrix}$$

$$Y_{11} = \tilde{Y}_1 + \tilde{Y}_{12} + \tilde{Y}_{13}$$

$$Y_{12} = Y_{21} = -\tilde{Y}_{12}$$

$$Y_{13} = Y_{31} = -\tilde{Y}_{13}$$

$$Y_{22} = \tilde{Y}_2 + \tilde{Y}_{12} + \tilde{Y}_{23}$$

$$Y_{33} = \tilde{Y}_3 + \tilde{Y}_{13} + \tilde{Y}_{23}$$

$$Y_{32} = Y_{23} = -\tilde{Y}_{23}$$

For each bus "i":

$$\tilde{S}_i = \tilde{V}_i \tilde{I}_i^* , \quad \tilde{S}_i = P_i + jQ_i$$

~~$$\frac{\tilde{S}_i}{\tilde{V}_i^*} = \frac{P_i - jQ_i}{\tilde{V}_i^*} = I_i \quad i=1,2,3$$~~

Therefore for bus "1":

$$\frac{P_1 - jQ_1}{V_1^*} = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3$$

$$\frac{P_2 - jQ_2}{V_2^*} = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3$$

$$\frac{P_3 - jQ_3}{V_3^*} = Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3$$

or:

$$P_1 - jQ_1 = Y_{11} V_1^2 + Y_{12} V_1^* V_2 + Y_{13} V_1^* V_3$$

$$P_2 - jQ_2 = Y_{21} V_1^2 + Y_{22} V_2^2 + Y_{23} V_2^* V_3$$

$$P_3 - jQ_3 = Y_{31} V_1^2 + Y_{32} V_2^2 + Y_{33} V_3^2$$

ECE System Methods of Analysis

3. Nature of Load Flow Problem.

Comments:

- The resulting equations are complex and nonlinear.
- Assuming all the bus voltages are the unknown variables, and all the injected powers are known variables, there are 3 - nonlinear complex equations to be solved for V's.

In general for a bus "i"

$$P_i = V_i \bar{V}_i / \delta_i$$

$$Y_{ij} = Y_{ij} / \delta_{ij}$$

$$P_i = \sum_{j=1}^n V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \delta_{ij})$$

$$Q_i = \sum_{j=1}^n V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \delta_{ij})$$

The above equations are in the form: Assume $V_i = 110$

$$1 \quad f_1(V_1, \dots, V_n, \delta_1, \dots, \delta_n) = 0 \quad i=1$$

$$2 \quad f_2(V_2, \dots, V_n, \delta_2, \dots, \delta_n) = 0 \quad i=2$$

$$\vdots \quad \vdots \quad \vdots$$

$$2n \quad f_{2n}(V_2, \dots, V_n, \delta_2, \dots, \delta_n) = 0$$

In compact form:

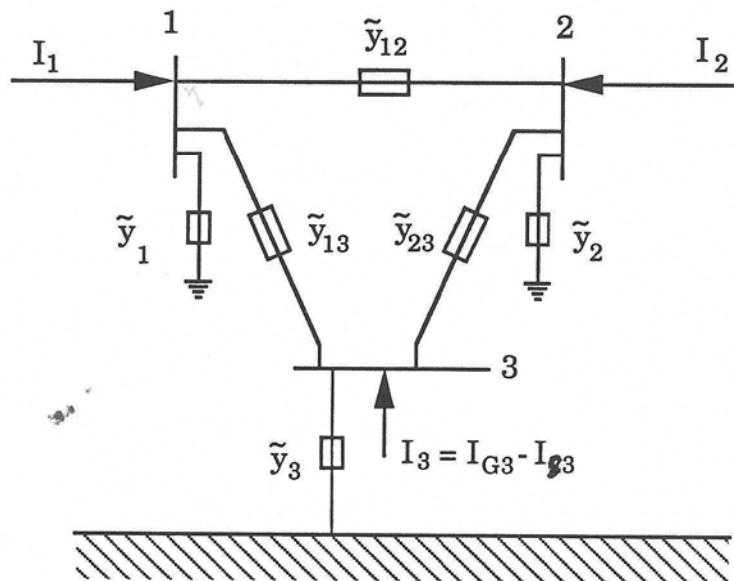
$$F(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_n(\mathbf{x}) \end{bmatrix}$$

where

$$[\mathbf{x}]^t = [V_2, \dots, V_n, \delta_2, \dots, \delta_n] = [x_1, \dots, x_n, x_{n+1}, \dots, x_{2n}]$$

For an n -bus system, there are $2n$ nonlinear equations to be solved.

EE740



Gauss-Seidel

$$\text{Model} \quad V_1 = 110$$

$$\Pi_{\text{BUS}} = Y_{\text{BUS}} \nabla_{\text{BUS}}$$

$$S_i = V_i \cdot I_i \quad i=1, 2, \dots, n$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\left(\frac{S_1}{V_1} \right)^* = I_1 = Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3$$

$$\left(\frac{S_2}{V_2} \right)^* = I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3$$

$$\left(\frac{S_3}{V_3} \right)^* = I_3 = Y_{31} V_1 + Y_{32} V_2 + Y_{33} V_3$$

Gauss-Seidel.

Step 1 $V_1 = 120$

Step 2. Gauss $V_2 = 120, V_3 = 120$

Step 3

$$\left(\frac{S_2}{V_2}\right)^* = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3$$

$$Y_{12}V_2 = \left(\frac{S_2}{V_2}\right)^* - (Y_{11}V_1 + Y_{13}V_3)$$

or

$$V_2 = \frac{\left(\frac{S_2}{V_2}\right)^* - (Y_{11}V_1 + Y_{13}V_3)}{Y_{12}}$$

or

$$V_2 = \frac{\left(\frac{S_2}{V_2}\right)^* - \sum_{j=1}^3 (Y_{1j}V_j)}{Y_{12}} \quad \begin{matrix} i=1, 2, \dots, n \\ j \neq 2 \end{matrix}$$

or

$$V_i = \frac{\left(\frac{S_i}{V_i}\right)^* - \sum_{j=1}^n (Y_{ij}V_j)}{Y_{ii}} \quad \begin{matrix} i=1, \dots, n \\ j \neq i \end{matrix}$$

$$\Delta P_i = P_i(\text{calculated}) - P_i(\text{scheduled}) \leq C_P$$

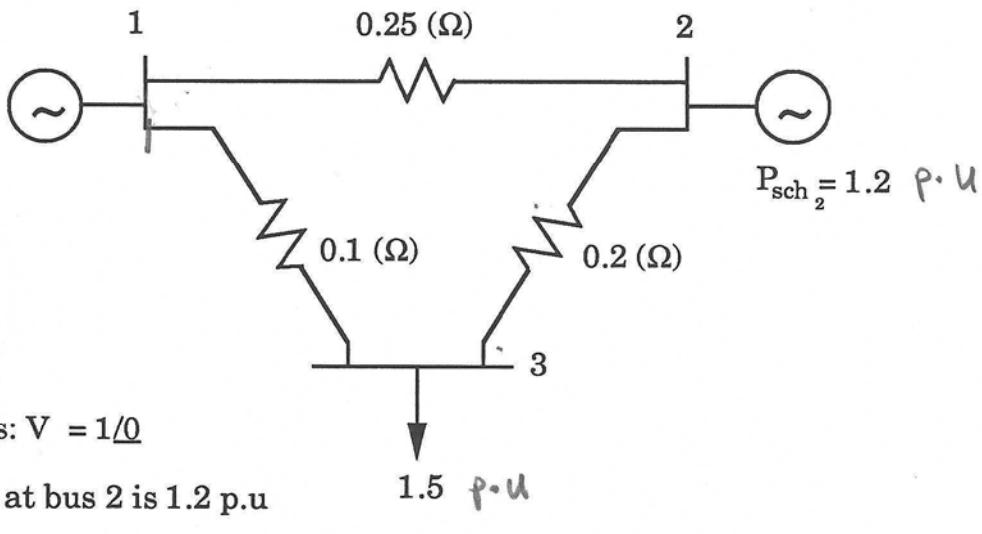
$$\Delta Q_i = Q_i(\text{calculated}) - Q_i(\text{scheduled}) \leq C_Q$$

 \rightarrow No Go To Step 3

\downarrow
yes

Set #10

EE740

Problem

Given:

- Bus 1 is slack bus: $V = 1/\sqrt{3}$
- Scheduled power at bus 2 is 1.2 p.u.
- Schedule Load at bus 3 is 1.5 p.u.

Compute:

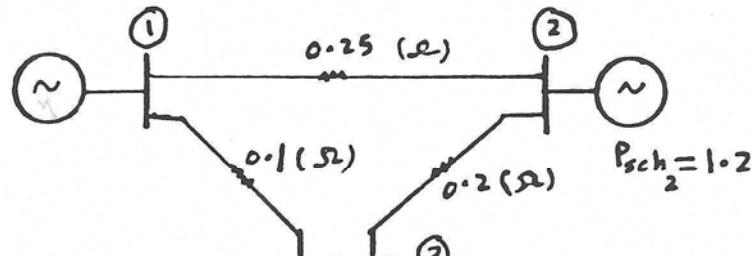
- 1) Y Bus model
- 2) Bus voltages using Gauss-Seidel Method
- 3) Power Mismatch at bus 2 and bus 3
- 4) Power supplied by the swing bus
- 5) Power Loss of transmission lines.

Due Date: November 29. (Monday)

(16)

Power system Methods of Analysis

Methods of solution. Example 2. Gauss-Seidel Δ_{bus} Method.



Comments:

- Bus 1 is slack bus: $V_1 = 1 \angle 0^\circ$
- Scheduled power at bus 2 is 1.2 p.u 1.5
- D.C case is analyzed for sake of convenience.

The Δ_{bus} matrix with respect to ground is

$$\Delta_{\text{bus}} = \begin{bmatrix} 14 & -4 & -10 \\ -4 & 9 & -5 \\ -10 & -5 & 15 \end{bmatrix}$$

model:

$$V_1 = 1 \angle 0^\circ$$

$$\mathbf{I}_{\text{bus}} = \mathbf{V}_{\text{bus}} \Delta_{\text{bus}}$$

$$1.2 = V_2 I_2''$$

$$-1.5 = V_3 I_3''$$

Gauss (zero iteration)

$$\Delta_{\text{bus}}^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For bus 2 calculate:

$$\sum_{\substack{j=1 \\ j \neq 2}}^3 Y_{2j} V_j = Y_{21} V_1 + Y_{23} V_3 = (-4)(1) + (-5)(1)$$

17
~~(1)~~

Power system Methods of Analysis

Example 2. Gauss-Seidel Bus Method.

update v_2

$$v_2 = \frac{\frac{P_2 - iQ_2}{V_2^{(0)}} - \sum_{\substack{j=1 \\ j \neq i}}^n Y_{2j} v_j}{Y_{22}} \quad i=1, \dots$$

$$V_2 = \frac{1}{Y_{22}} \left[\frac{1.2}{V_2^{(0)}} - [(-4) \times (1.0) + (-5) \times (1)] \right]$$

$$V_2^{(1)} = \frac{1}{9} \left[\frac{1.2}{1} + 4 + 5 \right] = 1.1333 \text{ P.U}$$

For bus 3 calculate:

$$\sum_{\substack{j=1 \\ j \neq 3}}^3 Y_{3j} v_j = Y_{31} v_1 + Y_{32} v_2$$

update v_3

$$V_3 = \frac{1}{Y_{33}} \left\{ \frac{-1.5}{V_3^{(0)}} - [Y_{31} v_1 + Y_{32} v_2] \right\}$$

$$V_3 = \frac{1}{15} \left\{ \frac{-1.5}{1.0} - [(-10)(1) + (-5)(1.1333)] \right\}$$

$$V_3^{(1)} = \frac{1}{15} \left\{ -1.5 + 10 + 5.666 \right\} = \frac{14.1666}{15} = 0.9444$$

The voltage tolerance for each bus is:

$$\Delta V_2^{(1)} = V_2^{(1)} - V_2^{(0)} = 1.1333 - 1.0$$

$$\Delta V_3^{(1)} = V_3^{(1)} - V_3^{(0)} = 0.9444 - 1.0$$

The mismatch at bus "1" is

$$\Delta P_1 = \sum_{\substack{j=1 \\ j \neq 1}}^3 V_1 I_{1j} - V_1 I_1 = V_1 I_{10} + V_1 I_{12} + V_1 I_{13} - V_1 I_1$$

$$P_1 = V_1 I_1 = V_1 \left(\sum_{j=1}^n Y_{1j} V_j \right) = V_1 (Y_{11} V_1 + Y_{12} V_2 + Y_{13} V_3)$$

$$P_1 = 1.0 [(14)(1.0) + (-4)(1.1333) + (-10)(0.9444)]$$

(13)

Power System Methods of Analysis

Example 2: Gauss-Seidel Bus Method.

$$\Delta P_1 = 0 + 1 \cdot \left(\frac{V_1 - V_2}{0.25} \right) + 1 \cdot 0 \left(\frac{V_1 - V_3}{0.1} \right) - P_1$$

$$\Delta P_1 = 0 + 1 \cdot 0 \left(\frac{1.0 - 1.133}{0.25} \right) + 1 \cdot 0 \left(\frac{1.0 - 0.944}{0.1} \right) - P_1$$

The mismatch at bus "2" is

$$\Delta P_2 = \sum_{\substack{j=0 \\ j \neq 2}}^3 V_2 I_{2j} - P_2(\text{scheduled})$$

$$\Delta P_2 = V_2 I_{20} + V_2 I_{21} + V_2 I_{23} - P_2(\text{scheduled})$$

$$\Delta P_2 = 0 + 1.133 \left(\frac{1.133 - 1.0}{0.25} \right) + 1.133 \left(\frac{1.133 - 0.944}{0.2} \right) - 1.2$$

$$\Delta P_2 = 0.474 \quad (\text{P.u})$$

The mismatch at bus "3" is

$$\Delta P_3 = \sum_{\substack{j=0 \\ j \neq 3}}^3 V_3 I_{3j} - P_3(\text{scheduled})$$

$$\Delta P_3 = V_3 I_{30} + V_3 I_{31} + V_3 I_{32} - (-1.5)$$

$$\Delta P_3 = 0 + 0.944 \left(\frac{V_3 - V_1}{0.1} \right) + 0.944 \left(\frac{V_3 - V_2}{0.2} \right) + 1.5$$

$$\Delta P_3 = 0.944 \left(\frac{0.944 - 1.0}{0.1} \right) + 0.944 \left(\frac{0.944 - 1.0}{0.2} \right) + 1.5$$