A Greedy Link Scheduler for Wireless Networks with Gaussian Multiple Access and Broadcast Channels

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***Abstract*—Information theoretic Broadcast Channels (BC) and Multiple Access Channels (MAC) enable a single node to transmit data simultaneously to multiple nodes, and multiple nodes to transmit data simultaneously to a single node respectively. In this paper, we address the problem of link scheduling in multihop wireless networks containing nodes with BC and MAC capabilities. We first propose an interference model that extends *protocol interference models*, originally designed for point to point channels, to include the possibility of BC and MAC. Due to the high complexity of optimal link schedulers, we introduce the Multiuser Greedy Maximum Weight algorithm for link scheduling in multihop wireless networks containing BCs and MACs. Given a network graph, we develop new *local pooling conditions* and show that the performance of our algorithm can be fully characterized using the associated parameter, the *multiuser local pooling factor*. We provide examples of some network graphs, on which we apply local pooling conditions and derive the multiuser local pooling factor. We prove optimality of our algorithm in tree networks and show that the exploitation of BCs and MACs improve the throughput performance considerably in multihop wireless networks.**

I. Introduction

The link scheduling problem for multi-hop wireless networks has received significant attention in the past few years [1]-[11]. The common assumption in these studies is that, a node transmits to only one node at any instant, and the possibilities arising from the development of network information theory has not been incorporated. In this paper we expand the scope of link schedulers to include multi-user communication scenarios using techniques developed in **multi-user information theory**. We first propose a generalized interference model to allow for such multi-user communication scenarios. We then introduce the **Multiuser Greedy Maximum Weight (MGMW) scheduler** for the proposed interference model and analyze its performance for arbitrary network graphs. For that purpose, we derive special conditions, known as **local pooling conditions**.

In a network with multiple nodes, interference from nearby nodes prevents all nodes from communicating simultaneously at full link capacity. The general objective of the scheduling problem is to determine which links to activate simultaneously in a network. A scheduling algorithm is said to be throughput optimal if it can keep all the queues stable under any arrival rate that can possibly be stabilized for the given network. In a multi-hop network with multiple flows and fixed link capacities, the throughput optimal scheduling problem was initially proposed in [10]. The complexity of the optimal scheme is however very high, making it highly impractical to implement. Recently, researchers have focussed on certain classes of network interference models which impose constraints on the set of links that can be simultaneously active in a network. One such model is the *node exclusive interference model* under which a node cannot simultaneously transmit and receive, and also cannot communicate simultaneously with more than one node. An optimal schedule for the node exclusive interference model, known as Maximum Weighted Matching, has a high complexity, $O(N^3)$ [16], where N is the number of nodes in the network. A more general interference model is the k-hop interference model, k being the the minimum number of hops between any two active links (when $k = 1$, we end up with node exclusive interference model). Maximum weight matching is NP-hard for $k \geq 2$ [7].

To address the complexity issue, low complexity suboptimal algorithms like greedy maximal scheduling have been proposed. An example of greedy scheduling is Greedy Maximal Matching (GMM) for node exclusive interference models [2]. Apart from being suitable for distributed implementation [13], GMM has the property that at each time slot the sum of the weight of the scheduled links is no less than a fraction $1/2$ of the maximum weight [11], [14]. This also leads to the conclusion that it achieves at least a fraction $1/2$ of the capacity region of the network [2]. However, the performance of the GMM scheme turns out to be far better than this lower bound in many scenarios, as shown in [9] and [1]. The authors in [1] characterized the performance of the GMM scheme using a parameter called the local pooling factor, which is obtained from the knowledge of the network topology, link capacities and interference constraints. It was shown using this local pooling factor that GMM was in fact throughput optimal for many classes of network graphs including all tree networks, under the node exclusive interference model [4].

The past work on scheduling mainly focused on orthogonal

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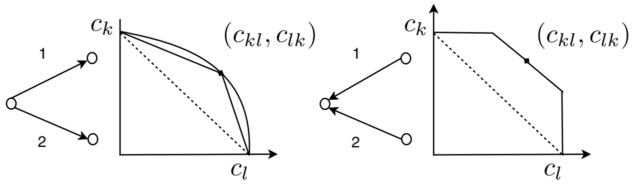

(a) Broadcast capacity region. (b) MAC capacity region.

Figure 1: The capacity regions of the two-user Gaussian BC and MAC are illustrated in this figure.

resource sharing, i.e. if a link is active no other interfering link can be active simultaneously. Scenarios arising from the development of network or multi-user information theory have not been incorporated. For example, using *superposition coding*, a node could simultaneously transmit to two or more nodes at a rate lower than the individual link capacities, but higher than what could be achieved by time sharing between the individual links [15], [17]. Similarly by using *successive interference cancellation* techniques at the receiver node in a Multiple Access Channel, two or more nodes could transmit simultaneously to a receiver node with the achievable rate region being larger than the time sharing region.

In this work, we develop an interference model, which incorporates the possibility of nodes to communicate simultaneously using an information theoretic broadcast or multiple access channel. We design a simple greedy scheduling algorithm, MGMW and analyze its performance, based on our interference model. For that purpose, we develop certain conditions, which we refer to as the **σ_M-local pooling conditions** and show that the performance of MGMW can be fully characterized using the associated parameter, the **multiuser local pooling factor** σ_M^*. We show that σ_M^* is the largest fraction of the capacity region of the network that can be stabilized by MGMW. Based on this notion of *efficiency*, we are interested in answering the following questions: What are the network topologies for which MGMW is throughput optimal? What is the efficiency of the MGMW in graphs where it is not throughput optimal? How does the performance of MGMW scheduler compare to that of GMM scheduler, which was designed for the scenario with sole point-to-point links?

II. SYSTEM MODEL

We model the wireless network as a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the vertex set representing the nodes and $\mathcal{E}$ is the set of edges. Each edge represents a directed point-to-point link over which a sender node transmits to a receiver node. To present the ideas clearly, we assume a single-hop network traffic model. The extension to multi-hop traffic models is readily possible using techniques in [1], [5]. We assume a time slotted model indexed by t in which the packets arrive at the start of every time slot. Each node keeps a separate queue for every edge it transmits over. Let $Q_l(t)$ represent the queue length for the packets to be transmitted over edge l and λ_l denote the arrival rate for edge l.

We assume that a node can communicate to a single node or a *pair of nodes* simultaneously, using information theoretic broadcast and multiple access channels. The information theoretic Broadcast Channel (BC) occurs when a transmitting node sends jointly encoded data using a suitable codebook, intended to more than one receiver at the same time, from which the receivers can decode their respective information. Similarly, a Multiple-Access Channel (MAC) occurs when a node receives simultaneously from more than one transmitter nodes . The MAC (or BC) capacity region for a given power constraint, is the closure of the convex hull of the set of rate vectors such that there exists codebooks at this rate allowing the receivers to decode with arbitrarily small probability of error [17]. Fig. 1 illustrates the capacity region of a two user AWGN Broadcast Channel and an AWGN MAC. Given the channels, given certain coding schemes, we have certain BC and MAC rate regions that our rates belong to. In the rest of the development, for the ease of exposition, we shall refer to these as *capacity region*s. The important observation is that the MAC and BC rate regions are strictly larger than the respective time-sharing achievable rate regions. Any point in the interior of the capacity region is achievable by choosing the appropriate set of codes.

While the model is generalizable to N-user BCs and MACs, the complexity of the scheduling algorithm increases and the performance analysis for N-users is more complicated. Therefore, in this paper, we restrict attention to BCs and MACs comprised of only two edges, i.e. a sender can transmit to two receivers (BC) or two senders can simultaneously transmit to one receiver (MAC). In the sequel, we use the term link in a generic sense to include point-to-point links as well as links made up of broadcast and muliple access channels. A *point-to-point link* consists of a sender node transmitting to a receiver node over one edge. A *multiuser link* is formed when a node transmits to two receiver nodes over a pair of edges (BC), or when a pair of nodes transmit to a common receiver node over two edges (MAC). We will specify the nature of the link wherever necessary. We assume that each transmitter, capable of multiuser communication chooses a single rate pair from the boundary of the capacity region for the associated multiuser link (BC or MAC), and whenever it chooses to transmit over that link, it uses the associated rate pair. In Fig. 1, (c_{kl}, c_{lk}) is the rate pair associated with the multiuser link (k, l). The pair (c_{kl}, c_{lk}) can be chosen as any point on the boundary of the capacity region of the multiuser link (k, l). We assume that, once a link specifies the pair (c_{kl}, c_{lk}), it is used whenever (k, l) is scheduled. This assumption is highly practical and it simplifies the system design significantly, since only a single codebook needs to be kept for every multiuser communication session. For a Gaussian BC or MAC, as seen in Fig. 1, $c_{kl} < c_k$, and $c_{lk} < c_l$ and $\frac{c_{kl}}{c_k} + \frac{c_{lk}}{c_l} > 1$. This means that we can achieve rates strictly larger than that achieved by time sharing the two point-to-point links. We emphasize that, this does not mean that the same message is being sent to both receivers, but different messages (packets) are sent to the receivers connected by the edges l and k. Also, we do not rule out the possibility of any edge being utilized as a point-to-point link, even if it is a part of some multiuser link.

To incorporate the possibility of such information theoretic

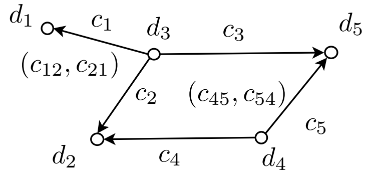

Figure 2: Five edge network with point-to-point link capacities c_1, c_2, c_3, c_4 and c_5. Links (1,2) and (4,5) each form broadcast links with rates (c_{12}, c_{21}) and (c_{45}, c_{54}) respectively.

Broadcast and MAC links, we are motivated to introduce a generalized binary interference model. Similar to classical binary interference models, each edge $l \in \mathcal{E}$ is associated with a set consisting of all edges that cannot be scheduled when edge l is scheduled. We call this set the *main interference set* and denote it by X_l. For edge l, let Y_l denote the set of edges that can be paired with l to form a multiuser link. We call Y_l the *secondary interference set* of l to distinguish it from the main interference set, and also due to the fact that each edge of the multiuser link operates at a lower rate with respect to its point-to-point capacity. Note that Y_l and X_l are mutually exclusive sets and if edge $k \in Y_l$, then $l \in Y_k$. Let c_l be the individual capacity of the point-to-point link l. Link l can be active at rate c_l, only if no other edge $k \in X_l \cup Y_l$ is active. If edge $k \in Y_l$ is active simultaneously as edge l, then it implies that they are active as multiuser link (k, l), with the maximum rate at which data can be transmitted over the multiuser link (k, l) being (c_{kl}, c_{lk}), as illustrated in Fig. 1. We also observe that the notion of *main* and *secondary* interference sets serve a more general purpose than allowing for multiuser links. This is because, in our interference model an edge could belong to the secondary interference set of another link, and not share a common node.

We define a rate allocation vector $\vec{r}^{1 \times |\mathcal{E}|}$ of link rates where $\vec{r}_l$ represents the rate of transmission over the edge l. A rate allocation vector must satisfy the following constraints:

(i) If $r_l > 0$ then $r_k = 0, \ \forall k \in X_l$. This condition describes the main interference constraint for a point-to-point link l.
(ii) If $r_l > 0$ and $r_k > 0$, and also if $k \in Y_l$ and $l \in Y_k$, then $r_l = c_{lk}$, and $r_k = c_{kl}$. Furthermore, $r_j = 0$ for all $j \in Y_l$ where $j \neq k, l$. This condition captures the constraints arising from the secondary interference set Y_l: If a multiuser link is scheduled then the edges belonging to the secondary interference sets of either of the two edges that constitute the multiuser link cannot be scheduled. Thus a node is allowed to transmit or receive simultaneously over at most two edges.
(iii) There exists no $j \in \mathcal{E}$ such that j does not interfere with any link and yet is not scheduled.

As an example, a network graph comprised of five edges is shown in Fig. 2 along with the set of rate allocation vectors. We show in Table I the interference sets that we could define for this network under a node-exclusive interference model. Node d_3 can set up a multiuser link to send data to (d_1, d_2) at rate (c_{12}, c_{21}). Similarly, node d_4 can transmit to (d_2, d_5) at rate (c_{45}, c_{54}) The rate allocation vectors for this network are $[0\, c_2\, 0\, 0\, c_5]$, $[0\, 0\, c_3\, c_4\, 0]$, $[c_1\, 0\, 0\, 0\, c_5]$, $[c_1\, 0\, 0\, c_4\, 0]$, $[c_{12}\, c_{21}\, 0\, 0\, c_5]$, and $[c_1\, 0\, 0\, c_{45}\, c_{54}]$. These are elements of the set S. Note that in the absence of the BC links only the first four rate vectors would be available.

link l	X_l	Y_l
1	{3}	{2}
2	{3,4}	{1}
3	{1,2,5}	$\emptyset$
4	{2}	{5}
5	{3}	{4}

Table I: Interference sets for the five edge network of Fig. 2.

The *capacity region of a network* is the set of all arrival rate vectors such that for any arrival vector in this set, there exists some scheduling scheme that can keep the queue lengths from growing unbounded. Let S denote the set of all rate allocation vectors $\vec{r}$. The capacity region of the network [10] is given by the interior of the set $\Lambda = \{\vec{\lambda} : \vec{\lambda} \preceq \vec{\phi}, \text{ for some } \vec{\phi} \in Co(S)\}$, where $Co(S)$ denotes the convex hull of the vectors in S and $\preceq$ represents componentwise inequality. Let $\pi : \vec{Q}(t) \rightarrow S$ be a scheduling policy that selects a rate allocation vector for every time slot, based on the queue length state vector $\vec{Q}(t)$. Let Π denote the set of all such scheduling schemes π. For this model, the entire capacity region can be achieved by the Maximum Weight scheduler [10], which at every time slot t, selects the rate vector which has the highest sum of queue weighted rates. To compare the advantage of using multiuser links, we also define a set of scheduling policies that cannot utilize the possibility of multiuser links. Let $\hat{S}$ denote the set of rate vectors for a network with sole point-to-point communication. Set $\hat{S}$ satisfies the following interference constraints: $r_l > 0 \Rightarrow r_k = 0, \ \forall k \in X_l \cup Y_l$. Let $\hat{\pi} : Q(t) \rightarrow \hat{S}$ be a scheduling scheme designed based on this constraint and let $\hat{\Pi}$ denote the set of all such schemes $\hat{\pi}$. Note that $\hat{\Pi} \subset \Pi$. Our objective is to find a simple scheme that belongs to the set Π and characterize its performance with respect to the capacity region, as well as to compare its performance to that of other schemes chosen solely from $\hat{\Pi}$. In the next section we describe MGMW for our network model.

III. Multiuser Greedy Maximum Weight (MGMW) algorithm

For the network model defined above, we present a "greedy" scheduling algorithm, MGMW which selects a rate allocation vector every time slot. MGMW, in principle is similar to the GMM, discussed in [1]. Before giving the details, we first summarize the operation of MGMW briefly. Each link is assigned a weight, which is basically the queue size weighted link rates. At each time slot, MGMW first greedily picks the link (point-to-point or multiuser) with the highest weight. Then it removes all interfering links and picks the link with the highest weight from the remaining links. This process goes on until there are no more links left to pick. More precisely, let $\mathcal{E}_B$ denote the set of all links (point-to-point as well as multiuser), i.e.,

$$\mathcal{E}_B = \left\{ \mathcal{E} \cup \left\{ (k, l) \in \mathcal{E}^2 \mid k \in Y_l \text{ and } l \in Y_k \right\} \right\}.$$

For any element $n \in \mathcal{E}_B$, We define the weight w_n as follows:

$$w_n = \begin{cases} Q_j c_j & n \text{ is a point-to-point link } j \\ Q_k c_{kl} + Q_l c_{lk} & n \text{ is a MAC/BC link } (k,l) \end{cases}. \quad (1)$$

1. Let Z denote the set of remaining links that satisfy the interference constraints. MGMW initializes Z to $\mathcal{E}_B$ and repeats (1)-(2) until $Z = \emptyset$. First it selects the link in the set Z with the highest weight.
$$m = \underset{n \in Z}{\text{argmax}} \{w_n\}. \quad (2)$$
2. After the selection, all links interfering with m are removed, *i.e* their rates are set to zero. If m is a point-to-point link j then the scheduler sets $r(k) = 0$, for all $k \in X_j \cup Y_j$. If m is a multiuser link (k,l) then it sets $r(i) = 0$, for all $i \in X_k \cup Y_k \cup X_l \cup Y_l$ except $i = k, l$. The Set Z is updated to consist of only non-interfering links. The maximum weight is then calculated by examining the non-interfering links in step 1.

At the end of the procedure MGMW yields a rate vector that belongs to the set S. Also, if $Y_l = \emptyset, \forall l \in \mathcal{E}$, MGMW reduces to the GMS of [1].

Example 1. MGMW for the five edge network of Fig. 2.

The set of point-to-point, and multiuser links for this case is given by $\mathcal{E}_B = \{1, 2, 3, 4, 5, (1, 2), (4, 5)\}$. Let the link rates be $c_1 = 4$, $c_2 = 6$, $c_3 = 2$, $c_4 = 8$, $c_5 = 5$, $(c_{45}, c_{54}) = (4, 3)$ and $(c_{12}, c_{21}) = (3, 4)$. Let $Q_1(t) = 20$, $Q_2(t) = 5$, $Q_3(t) = 2$, $Q_4(t) = 12$ and $Q_5(t) = 1$. Applying the MGMW algorithm, link 4 is observed to have the highest weight of 96. Link 1 as well as $(1, 2)$ each have weight 80. Link 4, having the highest weight is picked first and following step 2, the interfering edges 2 and 5 given in Table I are removed from set Z. Among the remaining links, highest weight is seen to be 80 for link 1. Node d_3 is hence selected to transmit over link 1. The chosen rate allocation vector is then $[4\,0\,0\,8\,0]$. Thus, at time t, no multiuser link is chosen to transmit.

Since the link weights are calculated locally, the distributed implementation of MGMW can be performed in the same manner as that proposed for GMS [13], with some added complexity resulting from considering multi-user links.

We note that although the possibility of multi-user links increases the capacity region, it is not clear that by using MGMW we improve the performance over GMS. This is because selecting multi-user links eliminates the primary interference set of both edges of the links. Therefore, we need to explore the performance of MGMW further which we do in the subsequent section.

A. Performance Characterization of MGMW Scheduler

We adopt the definition of efficiency given in [1] to describe the performance of the MGMW algorithm. The efficiency of the MGMW scheduling algorithm γ is defined as the largest fraction of the capacity region such that any arrival rate vector inside this region can be stabilized under the MGMW scheme. i.e,

$$\gamma^* := \sup \{\gamma \mid \text{the system is stable under MGMW for all arrival rate vectors } \vec{\lambda} \preceq \gamma\Lambda.\} \quad (3)$$

We study the efficiency of the MGMW algorithm for any network by relating it to a parameter that depends on the network topology, the link capacities and the interference sets. We call this parameter the *multiuser local pooling factor*. In the no multiuser link scenario, i.e, when the set $Y_l = \emptyset$ for all l, [9] showed that the GMM scheduler is throughput optimal for network graphs which satisfy certain conditions. These conditions, known as local pooling conditions are based on the network topology and the link capacities. In [1], a more general condition called σ-local pooling was introduced to characterize the performance of GMM for arbitrary interference graphs, including those for which GMM was not throughput optimal.

In this section we identify new network conditions in the presence of multiuser links, which we call *multiuser local pooling (σ_M-local pooling) conditions*. We will use these conditions to define the multiuser local pooling factor for any network graph. Recall that $\mathcal{E}_B$ is the set of all links for a given network graph. To describe the σ_M-local pooling conditions, we focus on certain subsets of $\mathcal{E}_B$, called candidate maximum weight (MW) subsets. A subset $L_{MW} \subset \mathcal{E}_B$ is called a candidate MW subset if both individual edges k and l of any multiuser link (k,l) do not appear as point-to-point links in L_{MW} separately. In other words, if an edge j appears as a point-to-point link in the set L_{MW}, then no other edge in the secondary interference set of j appears as a point-to-point link in L_{MW}: $\{j \in L_{MW} \Rightarrow l \notin L_{MW}$ for all $l \in Y_j\}$.

Example 2. Consider a network graph consisting of just two edges 1 and 2, which apart from being point-to-point links, also function as a broadcast link (1,2). The possible candidate MW subsets are $\{1\}, \{2\}, \{(1, 2)\}, \{1, (1, 2)\}, \{2, (1, 2)\}$. Note that $\{1, 2\}$ is not a candidate MW subset as the both edges of a multiuser link cannot appear at the same time.

We now introduce the idea of σ_M-local pooling conditions applied to candidate MW subsets.

Definition 3. Let L_{MW} be any candidate MW subset. Then L_{MW} contains point-to-point and multiuser links as its elements. Let $S_{L_{MW}}$ denote the set of all rate allocation vectors for the set L_{MW}. Then L_{MW} satisfies σ_M*- local pooling* if, for any given pair of convex combinations $\vec{\mu}, \vec{\nu}$ of the rate vectors in $S_{L_{MW}}$, either of the following hold.
(i) There exists a point-to-point link $j \in L_{MW}$ such that $\sigma_M \mu_j < \nu_j$, or
(ii) There exists a multiuser link $(k,l) \in L_{MW}$ such that $\sigma_M(\mu_k c_{kl} + \mu_l c_{lk}) < \nu_k c_{kl} + \nu_l c_{lk}$.

Condition (i) becomes the standard σ-local pooling condition for GMS, given in [1], except that, unlike GMS, where (i) is verified over all subsets of edges in $\mathcal{E}$, the σ_M-local pooling condition requires (i) to be verified over candidate MW subsets. We introduce (ii) to generalize it to the case where multi-user channels are possible. From the structure of condition (ii), we observe that for a multiuser link (k,l) to satisfy (ii), either $\sigma_M \mu_k < \nu_k$ or $\sigma_M \mu_l < \nu_l$ must be true, although the point-to-point links k and l might not belong to L_{MW}. Therefore, a candidate MW subset L_{MW} *does not* satisfy σ_M*- local pooling* if there exists a convex combination $\vec{\mu}, \vec{\nu}$ of the rate vectors in $S_{L_{MW}}$, which satisfy $\sigma_M \vec{\mu} \succeq \vec{\nu}$. The *multiuser local pooling factor* σ_M^* of a network is the supremum of all σ_M such that every candidate MW subset L_{MW} of $\mathcal{E}_B$ satisfies σ_M-local pooling.

$$\sigma_M^* = \sup\{\sigma_M \mid \forall L_{MW} \in \mathcal{E}_B, \text{ conditions (i) or (ii) are satisfied for every } \vec{\mu}, \vec{\nu} \in S_{L_{MW}}\}.$$

To show throughput optimality when local pooling conditions were satisfied, the authors in [9] argued that if a set of links alternately had the highest queue weighted rate in a small interval of time, and if they satisfied local pooling, then GMM served to bring down the highest weights in that interval. The proof used a fluid limit argument to find a Lyapunov function whose drift was then shown to be negative. A similar approach is followed in the proof of Lemma 1 in [1].

When multiuser links are included, one needs to consider the weight of both point-to-point and multiuser links. This leads to local pooling conditions being defined over a fixed class of subsets, i.e., candidate MW subsets of links and not over all subsets of links. The reason for this will become evident in the proof of Theorem 1, where it is seen that while considering the set with links having maximum queue weighted rates, one may exclude sets containing both individual edges of a multiuser link. This is due to the fact that under the convexity condition $\frac{c_{kl}}{c_k} + \frac{c_{lk}}{c_l} > 1$, the two edges of a multiuser link , when treated as point-to-point links, cannot simultaneously have the highest weight in the network. We now give Theorem 1 and Theorem 2 to prove the main result of the paper, which is that for network graphs in which no two multi-user links share a common edge, the efficiency of MGMW is equal to the multiuser local pooling factor, i.e,

$$\sigma_M^* = \gamma^*. \tag{4}$$

Theorem 1. *If the* multiuser local pooling factor *of a graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ *is* σ_M^**, then the network is stable under MGMW algorithm for all arrival rate vectors* $\vec{\lambda}$ *satisfying* $\vec{\lambda} \in \sigma_M^* \Lambda$.

Proof: The Proof is given in Appendix A. ■

While Theorem 1 shows that any arrival rate within $\sigma_M^* \Lambda$ is stabilizable by the MGMW algorithm, we further link the performance of the MGMW to σ_M^* in Theorem 2 by showing that there exist arrival rates, arbitrarily close but strictly outside of $\sigma_M^* \Lambda$, for which the system is unstable under the MGMW scheme. Theorem 1, together with Theorem 2 implies that σ_M^* is the highest fraction of the capacity region that can be stabilized by the MGMW algorithm. The result in (4) then follows from the definition of efficiency in (3).

Theorem 2. *Consider a a network graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ *with* $Y_k \cap Y_l = \emptyset$ *for all* $k, l \in \mathcal{E}$*. Let there exist a candidate MW subset* $L_{MW} \in \mathcal{E}_B$ *and a positive number* σ*, such that for a pair of vectors* $\vec{\mu}, \vec{\nu}$*, which are convex combination of the elements in* $S_{L_{MW}}$*,* $\sigma \vec{\mu} \succeq \vec{\nu}$ *holds. Then, for any positive* $\epsilon > 0$*, there exists a* $\vec{k} \succ 0$ *such that the arrival rate* $\vec{\lambda} = \vec{\nu} + \epsilon \vec{k}$ *makes the system unstable under the MGMW scheme.*

Proof: The proof is given in Appendix B. ■

IV. Examples of the Performance of MGMW

In this section we analyze the performance of MGMW in some sample wireless networks. We evaluate the multiuser local pooling factor for these network graphs to show the throughput gain by leveraging multiuser links. We first show that MGMW is throughput optimal for certain tree networks.

A. Tree Networks

In the following theorem, we show that under the node exclusive interference assumption, MGMW is throughput optimal for directed tree graphs in which no two multiuser links have a link in common. The node exclusive interference assumption for our model only restricts a node from transmitting and receiving at the same time. It does not restrict a node from transmitting simultaneously to multiple nodes, or receiving simultaneously from multiple nodes.

Theorem 3. *Let* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ *be a directed tree graph such that* $Y_k \cap Y_l = \emptyset$ *for all* $k, l \in \mathcal{E}$*. Then, if the primary and secondary interference sets are constructed under the node exclusive interference assumption, MGMW is throughput optimal, or equivalently* $\sigma_M^* = 1$ *for such tree networks.*

Proof: In this paper, we provide a detailed sketch of the proof and illustrate the theorem with an example. The complete proof is provided in [18]. Let L_{MW} be any candidate MW subset of the graph $\mathcal{G}$. Since L_{MW} is a set of links from a tree network graph, it satisfies one or both of the following conditions:

1) L_{MW} has an isolated point-to-point link, i.e., it consists of two nodes that are only connected to each other.
2) L_{MW} has at least one node with degree 1, i.e. it has only one edge connecting it.

For the set L_{MW}, the largest value of σ_M-local pooling that it satisfies can be represented as the solution to the following linear program: Let $M_{L_{MW}}$ be a matrix whose columns are the rate vectors in the set $S_{L_{MW}}$. Also let $\vec{e}$ be the column vector of all '1' entries with a dimension identical to the number of edges in L_{MW}.

$$\inf_{\sigma,\mu,\nu \geq 0} \sigma_M \tag{5}$$

$$\text{subject to } \sigma \mu_{L_{MW}} \geqslant \nu_{L_{MW}}, \tag{6}$$

$$\nu_{L_{MW}} = M_{L_{MW}} \vec{\beta}, \quad \mu_{L_{MW}} = M_{L_{MW}} \vec{\alpha}, \tag{7}$$

$$\vec{\alpha}' \vec{e} = 1, \ \vec{\beta}' \vec{e} = 1, \vec{\alpha}, \ \vec{\beta} \succeq 0. \tag{8}$$

Using Lemma 5 and Lemma 11 in [3], one can rewrite it as a solution to the following dual problem.

$$\max_{x \geq 0, \tau} \ \tau \tag{9}$$

$$\text{subject to } x' M_{L_{MW}} \leq e', \quad x' M_{L_{MW}} \geq \tau e'. \tag{10}$$

The dual problem is to find an $x \geq 0$ that maximizes the value of τ for which (10) is satisfied. From the dual problem, the set L_{MW} satisfies 1-local pooling if $\tau = 1$ is a solution to the problem. This happens if there exists an $x \geq 0$, such that constraint (10) is satisfied with equality so that $x' M_{L_{MW}} = e'$. We show that for every L_{MW}, one can find an x to satisfy

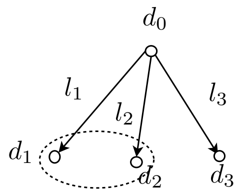

Figure 3: Figure shows an example of a possible L_{MW} for a tree network. The broadcast links are enclosed in ellipses.

the equality constraint in (10) in [18]. Here we illustrate the construction of x using an example of a possible L_{MW} shown in Fig. 3. In the figure, $L_{MW} = \{l_2, (l_1, l_2), l_3\}$, with the rates of the links being c_2, (c_{12}, c_{21}), and c_3 respectively. The column vectors of $M_{L_{MW}}$, under the node exclusive interference assumption are given by $[0\, c_2\, 0]$, $[c_{12}\, c_{21}\, 0]$, and $[0\, 0\, c_3]$. We note that L_{MW} satisfies condition 2, with d_1, d_2 and d_3 being the nodes of degree 1. For this L_{MW}, setting x to be $\left[\left(1 - \frac{c_{21}}{c_2}\right)/c_{12} \;\; \frac{1}{c_2} \;\; \frac{1}{c_3}\right]$ satisfies $x' M_{L_{MW}} = e'$, yielding $\tau = 1$. To see why this works, we note that only one of the links connected to d_0 is scheduled at a time. Hence, the non-zero entries in any column vector of $M_{L_{MW}}$, correspond to one of the links connected to d_0. Also, due to the assumption that the multiuser links in $\mathcal{G}$ do not share a common edge, each index of x corresponds uniquely to an edge of a point-to-point or multiuser link connected to d_0. This allows us to compute the entries in x corresponding to each link independently, in order to ensure that the product of x with every column vector in $M_{L_{MW}}$ is 1. ■

Theorem 3 also shows that MGMW is throughput optimal for **downlink cellular networks** without intercell interference with multiuser links consisting of broadcast channels. This is because the downlink cellular model is an instance of a tree network.

B. A Network with $\sigma_M^* < 1$.

Consider the network graph shown in Fig. 4. All links have a rate of 1 when used as point-to-point links. All multiuser links have a rate of (0.75,0.75) each. To study the performance of MGMW, we again consider the highest value of σ_M-local pooling satisfied by every L_{MW} of the network graph. Lemma 14 in [3] applied to the linear program in (9) gives the following relation for $\sigma_{L_{MW}}$:

$$\sigma_{L_{MW}} \geqslant \frac{\min_i \left\|\vec{r}^i\right\|_1}{\max_i \left\|\vec{r}^i\right\|_1}, \tag{11}$$

where $\vec{r}^i \in S_{L_{MW}}$, $i \in \{1, 2, 3, \cdots, |S_{L_{MW}}|\}$. One can show for all candidate MW subsets L_{MW} that $\sigma_{L_{MW}} \geq 2/3$. Here we will only show this for one candidate MW subset and will not repeat the same operation for all candidate MW subsets. Consider the candidate MW subset $L_{MW} = \{(1,2), (3,4), (5,6), (7,8), (9,10), (11,12)\}$. The set of rate allocation vectors $S_{L_{MW}}$ is given by

$$\begin{bmatrix} \vec{\nu}_1 \\ \vec{\nu}_2 \\ \vec{\nu}_3 \\ \vec{\nu}_4 \\ \vec{\nu}_5 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{3}{4} & 0 & 0 & 0 & 0 & \frac{3}{4} & \frac{3}{4} \end{bmatrix}$$

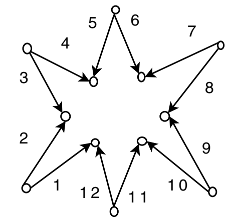

Figure 4: Figure shows a twelve edge network consisting of six broadcast links.

For this set, applying (11) yields $\sigma_{L_{MW}} \geqslant 2/3$, since the minimum and maximum values of $\left\|\vec{r}^i\right\|_1$ are $2 \times (3/4)$ (for $\vec{\nu}_3, \vec{\nu}_4, \vec{\nu}_5$) and $3 \times (3/4)$ (for $\vec{\nu}_1, \vec{\nu}_2$). This implies that the local pooling factor and hence the efficiency ratio of MGMW for this network graph is at least $2/3$. For the same network graph, if we exclude the possibility of multiuser links, it can be shown that GMM algorithm has an efficiency ratio of 2/3 (with respect to the capacity region in the no multiuser link scenario). To see this, we only need to consider the set $L = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. The set L forms a cycle, and in the same manner as that shown for the six-link cycle network in [1], one obtains the local pooling factor as 2/3. Hence, in this example, by having a multiuser local pooling factor of 2/3, MGMW guarantees a larger stability region compared to GMM. This is because the network capacity region for the multi-user case is a superset of the network capacity region in the no multiuser link scenario.

C. Simulation of the Performance of MGMW

In this section, we simulate the performance of MGMW and GMM in a randomly connected network graph. Figure 5a shows an arbitrary network graph having point-to-point as well as multiuser links. The multiuser (BC) links in this graph are links (1,2), (4,7), (3,8), (5,6), and (9,14). In this example we chose the rates of the point-to-point links at random, uniformly between 3 and 10 units. The rates of the multiuser links are chosen to make sure the convexity of the rate region is assured. The arrival process for each edge is Bernoulli and we denote the arrival rate with λ_l.

In Fig. 5b, we plot the total queue size (sum of queue lengths at each edge) as we increase the arrival rate in edges 9 and 14, from 2 to 2.5 as we keep $\lambda_l = 1$ for all other links. Here, the rates of links 9, 14 and (9,14) are 6, 4 and (4,3) respectively. The graph suggests that MGMW yields a constant gain in arrival rate for the multi-user links as each edge of the multiuser link (9,14) is seen to sustain around 5% more traffic, as compared to the GMM case.

In Fig. 5c, we simultaneously increase the arrival rate across edges 1, 2, 9, 14, 3 and 8 while keeping the arrival rates at other edges fixed at 1. Here, the rates of these links are 10, 8, 6, 4, 12 and 8, and the rates of the BCs $(1, 2)$, $(9, 14)$ and $(3, 8)$ are (9,5), (4,3) and (10,6) respectively. Similar to the previous scenario, the plots again show that the total queue size with MGMW is lower than that with GMM. Here, each edge of the multiuser links is able to sustain 10% more traffic than in the no multiuser link case. Thus, for the network in

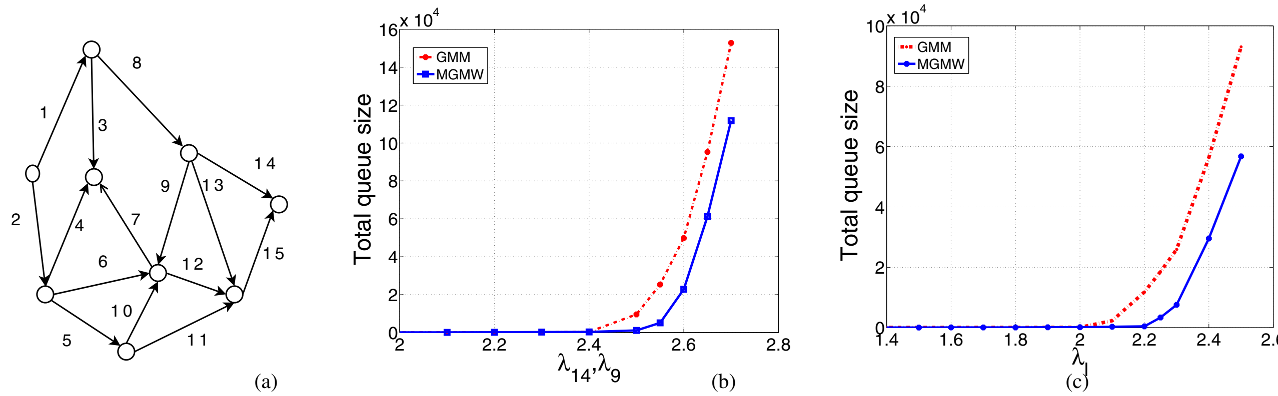

Figure 5: An Arbitrary network graph with 15 edges and 5 broadcast links.

Fig. 5a, MGMW appears to stabilize larger range of arrival rates.

V. Conclusions

In this paper we explored the problem of link scheduling in a setting that allows for the use of techniques from multi-user information theory. To this end, we proposed a modified version of the binary interference model by introducing the notion of a secondary interference set for each link of the network. The interference model proposed in this paper could be thought of in a loose sense as a hybrid of the binary interference model and the SINR model. Since the optimal algorithm is known to have high complexity (NP hard in many cases), we provided a suboptimal greedy algorithm called MGMW for our interference model. We characterized the performance of MGMW by deriving local pooling conditions and relating the multiuser local pooling factor to the efficiency of MGMW. For a network with capacity region Λ and a multiuser local pooling factor σ_M, we showed that MGMW stabilizes every arrival rate vector in $\sigma_M\Lambda$ and that there exists a non-stabilizable arrival rate vector, arbitrarily close to, but strictly outside of $\sigma_M\Lambda$. We gave examples of certain network graphs where MGMW was throughput optimal and a graph where the multiuser local pooling factor is less than one. We also observed the performance of MGMW in an arbitrary graph and compared it to that of GMM. We plan to extend this work to further explore the performance of MGMW, for instance in k-hop interference models, using the σ_M-local pooling conditions.

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Appendix A
Proof of Theorem 1

Proof: The proof shows stability of the network under MGMW by finding a Lyapunov function and showing that it has a negative drift for the fluid limit of the system. The idea of the proof is similar to the stability proof in [1], which is for the scenario of no broadcast links. We assume that the arrival process is IID and mutually independent across links. It is also a stationary and ergodic process satisfying conditions for the fluid limit to exist, which is that the number of arrival packets should be IID across time slots for each link. Let $\vec{A(t)}$ denote the cumulative arrival process and $\vec{S(t)}$ denote the cumulative service process until time slot t. For the arrival and service processes, we use $A_l(t) = A_l(\lfloor t\rfloor)$, and $S_l(t) = S_l(\lfloor t\rfloor)$. For the queue process $Q_l(t)$, we employ linear interpolation. We now consider a sequence of scaled queuing systems $(\vec{Q^n}(\cdot), \vec{A^n}(\cdot), \vec{S^n}(\cdot))$. where we apply the scaling $Q_l^n(nt)/n,\ A_l(nt)/n,$ and $S_l(nt)/n, \forall l \in \mathcal{E}$ with the queue process satisfying $\sum_{l\in\mathcal{E}} Q_l^n(0) \leq n$. Then, using the techniques to establish fluid limit in [12], one can show that a fluid limit exists almost surely. i.e, for almost all sample paths and for any positive $n \to \infty$, there exists a sub-sequence n_j with $n_j \to \infty$ such that following convergence holds uniformly over compact sets. For all $l \in \mathcal{E}$, $\frac{1}{n_j} A_l^{n_j}(n_j t) \to \lambda_l t$, $\frac{1}{n_j} S_l^{n_j}(t) \to s_l(t)$, and $\frac{1}{n_j} Q_l^{n_j}(n_j t) \to q_l(t)$. $q_l(t)$ and $s_l(t)$ are the fluid limits for the queue length processes and the service rate processes respectively. The fluid limit is absolutely continuous and hence the derivative of $q_l(t)$ exists almost everywhere [12] satisfying:

$$\frac{d}{dt} q_l(t) \ = \begin{cases} [\lambda_l - \pi_l(t)]^+ & q_l(t) > 0. \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

where $\pi_l(t) = \frac{d}{dt} s_l(t)$. We now show that the largest queue weighted rate (taken over point-to-point link or broadcast links) of the fluid limit model always decreases under the MGMW algorithm. This allows us to define the Lyapunov function for the system as the maximum weight over all links and establish its drift to be negative. Consider the times t when the derivative $\frac{d}{dt} q_l(t)$ exists for all $l \in \mathcal{E}$. Let $L_0(t)$ denote the set of links with the largest weight, i.e

$$L_0(t) = \{n \in \mathcal{E}_B \mid w_n = \max_{m \in \mathcal{E}_B} w_m\}.$$

Define the derivative of the weights of links in $\mathcal{E}_B$ as follows:

$$\hat{w}_n(t) = \begin{cases} \frac{d}{dt} q_j(t) c_j & n \text{ is a point-to-point link } j \\ \frac{d}{dt} q_k(t) c_{kl} + q_l(t) c_{lk} & n \text{ is a} \\ & \text{multiuser link } (k,l) \end{cases}.$$

Let $L(t)$ denote the set of links from $L_0(t)$, which have the maximum derivative of the weights,

$$L(t) = \{n \in \mathcal{E}_B \mid \hat{w}_n(t) = \max_{m \in L_0(t)} \hat{w}_m(t)\}.$$

Then, one can find a small δ such that in the interval $(t, t+\delta)$, links in $L(t)$ will have the highest weights in that time interval. i.e. $\min_{n\in L(t)} \hat{w}_n(\tau) > \max_{m \in \mathcal{E}_B \setminus L(t)} \hat{w}(\tau)$ for all τ in $(t, t+\delta)$. MGMW will select links from the set $L(t)$ first in the interval $(t, t+\delta)$, since it picks the links in decreasing order of weights. If we focus on the links in $L(t)$, then any rate allocation vector selected by MGMW in $(t, t+\delta)$, projected on the set $L(t)$ would yield a rate allocation vector that is an element of $S_{L(t)}$. A formal argument to show this is mostly identical to that in [1] and is omitted here. It can be shown in a manner identical to the procedure in [1], [9] that $\vec{\pi(t)}$, the service rate vector under MGMW projected on $L(t)$ is a convex combination of the elements of $S_{L(t)}$. Because of the convexity condition $\frac{c_{kl}}{c_k} + \frac{c_{lk}}{c_l} > 1$, $L(t)$ is a candidate MW subset[1]. Now consider a $\vec{\lambda}$ lying strictly within $\sigma_M^* \Lambda$. Since σ_M^* is the local pooling factor, and $L(t)$ is a candidate MW set, there exists $n \in L(t)$ such that

$$\begin{aligned} \lambda_j &< \pi_j(t) && n \text{ is point-to-point link } j. \\ \lambda_k c_{kl} + \lambda_l c_{lk} &< \pi_k(t) c_{kl} && n \text{ is a multiuser link} \\ &\quad + \pi_l(t) c_{lk} && (k,l). \end{aligned} \tag{13}$$

which follows from the local pooling conditions. For any candidate MW set L_{MW}, let $\vec{s}_{L_{MW}}$ be any convex combination of the elements (rate allocation vectors) of the set $S_{L_{MW}}$. Since every $L_{MW} \subset \mathcal{E}_B$ satisfies σ_M^* local pooling, and $\vec{\lambda}$ lies strictly within $\sigma_M^* \Lambda$, the quantity

$$\epsilon_{\vec{s}_{L_{MW}}} = \max_{j \in L_{MW},\,(k,l)\in L_{MW}} \Big((s_j - \lambda_j), \\ (s_k c_{kl} + s_l c_{lk} - \lambda_k c_{kl} - \lambda_l c_{lk}) \Big)$$

is strictly positive for every L_{MW}. We then define the infimum of all such positive quantities over all such subsets L_{MW} and all vectors $\vec{s}_{L_{MW}}$ as

$$\epsilon^* = \inf_{\{\vec{s}_{L_{MW}} \forall L_{MW} \subset \mathcal{E}_B\}} \epsilon_{\vec{s}_{L_{MW}}}.$$

and we observe that that $\epsilon_* > 0$. Hence, from the relation in (13), there exists $n \in L(t)$ such that the following holds:

$$\begin{aligned} \lambda_j - \pi_j(t) &\leq -\epsilon^*, && n \text{ is a point-to-point link } j. \\ (\lambda_k - \pi_k(t)) c_{kl} & && \\ +(\lambda_l - \pi_l(t)) c_{lk} &\leq -\epsilon^*, && \text{multiuser link } (k,l). \end{aligned} \tag{14}$$

From (12), and the fact that all links in $L(t)$ have the same derivative, (14) implies that

$$\hat{w}_n \leq -\epsilon^*, \ \forall n \in L(t).$$

Hence, we observe that there exists no link in $L(t)$ with $\hat{w}_n \geq 0$, and $q_n > 0$, where $q_n = \max(q_k, q_l)$, if n is a multiuser link (k,l). Now, we can consider the following Lyapunov function $V(t) := \max_{n \in \mathcal{E}_B} w_n$. For $V(t) > 0$, we have that

$$\frac{d^+}{dt^+} V(t) \leq \max_{n \in \mathcal{E}_B} \hat{w}_n \leq -\epsilon_*. \tag{15}$$

where $\frac{d^+}{dt^+} V(t) = \lim_{\delta \downarrow 0} \frac{V(t+\delta) - V(t)}{\delta}$ is the right hand derivative of $V(t)$. This implies that the largest weight must decrease in the time interval $(t, t+\delta)$. Since the above inequality holds almost everywhere in t, the negative drift of the Lyapunov function implies that the fluid limit model of the system is stable and hence by the result in [12], the original system is also stable. ∎

Appendix B
Proof of Theorem 2

Proof: We construct a traffic pattern using the principle in [1] and show that under this traffic pattern the queue lengths grow to infinity under MGMW. Let J denote the number of possible rate allocations on the set L_{MW}. The vector $\vec{\nu}$, being a convex combination of the elements of the elements in $S_{L_{MW}}$, can be written as

$$\vec{\nu} = \sum_{i=0}^{J-1} \omega_i \vec{r_i}, \qquad \vec{r_i} \in S_{L_{MW}}$$

[1]Consider the set $L_0(t)$. Let $k \in L_0(t)$, $l \in L_0(t)$ and $l \in Y_k$. Then the weights of the links$k$ and l are equal, i.e, $q_k c_k = q_l c_l$. Suppose the weight of the broadcast link is lesser than the individual links, $q_k c_{kl} + q_l c_{lk} \leq q_k c_k$. Substituting for q_l from $q_k c_k = q_l c_l$ gives us the condition $\frac{c_{kl}}{c_k} + \frac{c_{lk}}{c_l} \leq 1$. This contradicts our earlier assumption on the rates for the broadcast channel that $\frac{c_{kl}}{c_k} + \frac{c_{lk}}{c_l} > 1$, and hence the weight of the broadcast link exceeds that of the individual links. Thus, $L_0(t)$ cannot be the set with the highest weight.

where $\omega_i \geq 0$ for all $0 \leq i \leq J-1$ and $\sum_{i=0}^{J-1} \omega_i = 1$. Let v_i be a rational number which satisfies $\sum_{i=0}^{J-1} |\omega_i - v_i| \leq \frac{\delta}{J}$ for any $\delta > 0$. Such a v_i clearly exists for every ω_i. To enable the construction of the traffic pattern we now define a new vector $\hat{\vec{\nu}} = \sum_{i=0}^{J-1} v_i \vec{r_i}$. Thus one can make the vector $\hat{\vec{\nu}}$ arbitrarily close to $\vec{\nu}$. We now construct a traffic pattern with load $\hat{\vec{\nu}} + \epsilon\vec{k}$, such that the system is unstable under MGMW. Packets are assumed to arrive at the beginning of a time slot. As L_{mw} is a candidate MW subset, without loss of generality we assume that if (k,l) is a multiuser link in L_{mw}, then k is the individual edge of the link (k,l) that can be in L_{mw}.

Let the initial queue state vector be $\vec{q_0} \succeq 0$. Let j denote any point-to-point link belonging to L_{MW}, and (k,l) denote any multiuser link in L_{MW}. Then we choose $\vec{q_0}$ to satisfy the following constraints:

$$q_j c_j = q_k c_{kl} + q_l c_{lk} = q_k c_k + c_k c_{kl} > q_l c_l + c_l c_{lk}, \quad \forall j,\ (k,l) \in L_{MW}. \tag{16}$$

These constraints are relations between the link weights that allow us to generate the desired traffic pattern. It is shown in Lemma 3 of [18] that a vector $\vec{q_0} \succeq 0$ satisfying (16) exists. Let T denote the smallest number such that, for all i, $t_i = v_i T$ is an integer. Let $t_i = v_i T$, and assume that $t_{N-1} \geq 1$ without loss of generality. We now outline the steps to generate the required traffic arrival pattern.

1. For the first t_0 time slots, we inject packets in every time slot only into those links which have non zero rates in the rate allocation vector $\vec{r_0}$. More precisely we inject packets as follows:
   (a) If $\vec{r_0}(j) > 0$ and $\vec{r_0}(m) = 0 \, \forall m \in Y_j$, then inject c_j packets every time slot.
   (b) If $\vec{r_0}(k) > 0$ and $\vec{r_0}(l) > 0$, $l \in Y_k$, then inject c_{kl} packets in link k and c_{lk} packets in link l every time slot.

In each time slot, the MGMW algorithm serves only the links that are allocated non zero rates by the rate vector $\vec{r_0}$. This is due to the fact that initially weights of links in L_{MW} satisfy the relation in (16). When packets are injected into certain links at the beginning of the time slot, then only those links have the largest weights and satisfy the interference constraints, and are served by MGMW. We show that MGMW only serves links scheduled by $\vec{r_0}$ below. At the end of t_0 time slots the queues will be such that all weights will continue to satisfy the relation in (16).

(i) When c_j packets are injected into a point-to-point link j scheduled by $\vec{r_0}$, then the weight of the link j is given by $q_j c_j + c_j^2$, which exceeds the weight of unscheduled links as seen from (16).

(ii) When c_{kl} and c_{lk} packets are injected into the queues of edges k and l, then the weight of the link (k,l) is $q_k c_{kl} + q_l c_{lk} + c_{kl}^2 + c_{lk}^2$, which exceeds the weight of the unscheduled links. It also exceeds the weight of the point-to-point links k and l, which are $q_k c_k + c_k c_{kl}$ and $q_l c_l + c_l c_{lk}$ respectively. This follows from relation (16).

(iii) If c_k packets are inserted into edge k of the link (k,l), then the weight of link k, given by $q_k c_k + c_k^2$ exceeds the weight of the unscheduled links, the multiuser link (k,l) and link l. Thus, MGMW serves link k in this case.

2. We repeat this process for rate vectors $\vec{r_1}, \vec{r_2}, \vec{r_3}, \ldots \vec{r}_{J-2}$. Corresponding to each rate vector $\vec{r_i}$, packets are injected into the links with non zero rates in $\vec{r_i}$ for t_i slots. The packets are injected as shown above for the first rate vector $\vec{r_0}$. Then, MGMW serves only those links in $\vec{r_i}$ and at the end of t_i slots the queues satisfy the relation in (16).
3. For the last rate vector $\vec{r}_{J-1}$, we inject packets using the above procedure for $t_{J-1} - 1$ time slots into links scheduled by the rate vector $\vec{r}_{J-1}$. In the next time slot, with probability $1 - \tilde{\epsilon}$, we inject packets according to the earlier procedure into the links scheduled by $\vec{r}_{J-1}$, and with probability $\tilde{\epsilon}$, inject packets in the following manner:
   (a) If $\vec{r}_{J-1}(j) > 0$ and $\vec{r}_{J-1}(m) = 0 \, \forall m \in Y_j$, then inject $c_j + \hat{c}_j$ packets.
   (b) If $\vec{r}_{J-1}(k) > 0$ and $\vec{r}_{J-1}(l) > 0$ for $l \in Y_k$, then inject $c_{kl} + \hat{c}_k$ packets into link k and $c_{lk} + \hat{c}_l$ packets into link l.
   (c) To other links $j \in L_{MW}$ such that $\vec{r}_{J-1}(j) = 0$, inject $\hat{c}_j$ packets. To the other multiuser links $(k,l) \in L_{MW}$ such that $\vec{r}_{J-1}(k) = 0$ and $\vec{r}_{J-1}(l) = 0$, inject $\hat{c}_k$ and $\hat{c}_l$ packets into links k and l respectively.

The quantities $\hat{c}_j$, $\hat{c}_k$, and $\hat{c}_l$ are such that they satisfy the following weight criteria:

$$\hat{c}_j c_j = \hat{c}_k c_k = \hat{c}_k c_{kl} + \hat{c}_l c_{lk} > \hat{c}_l c_l. \tag{17}$$

It can be shown using an argument identical to that used for (16) that there exist positive $\hat{c}_j$, $\hat{c}_k$, and $\hat{c}_l$ that satisfy relation (17). When packets are injected in this manner in the last time slot, MGMW still serves the links scheduled by $\vec{r}_{J-1}$ in all the t_{J-1} time slots. This is because (16) and (17) yield

$$\begin{aligned}(q_j + \hat{c}_j)c_j &= (q_k + \hat{c}_k)c_{kl} + (q_l + \hat{c}_l)c_{lk} \\ &= (q_k + \hat{c}_k)c_k + c_k c_{kl} \\ &> (q_l + \hat{c}_l)c_l + c_l c_{lk}.\end{aligned}$$

which again satisfies relation (16). However, with probability $\tilde{\epsilon}$, the queue length of each edge $j \in L_{MW}$ increases by $\hat{c}_j$, and the queue length of links $(k,l) \in L_{MW}$ increases by $\hat{c}_k$ and $\hat{c}_l$ respectively.

The above pattern is then repeated so that the same pattern of arrival occurs every $\sum_{i=0}^{J-1} t_i$ time slots. Let $\vec{k}$ denote the vector defined as follows:

$$\vec{k}(i) = \begin{cases} \hat{c}_j & i \text{ corresponds to point-to-point link } j \in L_{MW} \\ \hat{c}_k & i \text{ corresponds to edge } k \text{ of link } (k,l) \in L_{MW} \\ 0 & i \text{ corresponds to an edge not in } L_{MW} \end{cases}.$$

The arrival rate of this traffic pattern is now given by :

$$(\sum_{i=0}^{J-1} t_i \vec{r_i} + \tilde{\epsilon}\vec{k}) / (\sum_{i=0}^{J-1} t_i) = \hat{\vec{\nu}} + \epsilon\vec{k},$$

where $\epsilon = \tilde{\epsilon} / \sum t_i$. Since $\hat{\vec{\nu}}$ can be made arbitrarily close to $\vec{\nu}$ by choosing δ small, the system with rate $\vec{\nu} + \epsilon\vec{k}$ is unstable under the MGMW scheme. ∎