EFFICIENCY BASED OPTIMAL CONTROL OF KAPLAN HYDROGENERATORS

Philip Schniter, Graduate Student*
Department of Electrical & Computer Engineering

L. Wozniak, P.E., Fellow
Department of General Engineering

University of Illinois
Urbana, Illinois 61801

Abstract - This paper investigates an optimal strategy for controlling the speed response of Kaplan hydrogenerating systems to decreases in load. Typically, primary control gates restrict and redirect water through the turbine to stabilize and transfer the system to operating point demand. The adjustable turbine blade angle is used to return to maximum operating efficiency at the new load level. The over-speed reduction is limited by the conduit's ability to withstand the over-pressure caused by the flow restriction at the turbine.

A control scheme using gates and blades simultaneously and independently is developed. The initial action of the proposed control moves the gates and blades in opposite directions at maximum rates in an effort to reduce efficiency without altering flow, thus quickly decreasing the generated power while affecting the pressure minimally. The final control action moves the gates and blades simultaneously to restore maximum efficiency operation at the new power level demand.

Similarities are found when comparing the efficiency based control to the Bang-bang and Linear Quadratic Regulator results of optimal control theory, though attempting to apply either of these methods directly to this problem is impractical.

Simulation results show that the proposed scheme outperforms the traditional gate-dominant control in minimizing turbine over-speed and speed settling time under prescribed load change and conduit pressure limits. However, in its present form, the design is found to be more sensitive to system nonlinearities than its conventional counterpart.

KEY WORDS

hydrogenerator, governor, Kaplan turbine, gate-dominant, optimal control

NOMENCLATURE

- x Turbine Speed, *pud (per unit deviation)*
- h Pressure, *pud*
- q Flow, *pud*
- m Torque (generated), *pud*
- l Load Torque, *pud*
- y Gate Position, *pud*
- β Blade Position, *pud*
- η Efficiency, *pu*
- T_w Hydraulic Time Constant, sec
- T_a Rotor Time Constant, sec
- T_β Blade Filter Time Constant, sec
- t_x Speed Settling Time, sec
- x_{max} Maximum Over-speed, *pud*
- h_{max} Maximum Pressure Deviation, *pud*
- T_η Efficiency Return Time-Constant, sec

*Now with Tektronix Inc., Beaverton, Oregon.

94 SM 373-1 EC A paper recommended and approved by the IEEE Energy Development and Power Generation Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1994 Summer Meeting, San Francisco, CA, July 24 - 28, 1994. Manuscript submitted August 25, 1993; made available for printing May 3, 1994.

1. INTRODUCTION

One goal in the control of hydrogenerating plants is maintaining stable and efficient performance over various load levels. This work focuses on hydrogenerators supplied with *Kaplan* turbines, axial flow turbines controlled by variable pitch *blades* and adjustable guidevanes known as *gates*. It develops a control scheme that uses both parameters in an optimal fashion to control step decreases in load, termed *load rejections*.

Typically, control of hydrogenerating systems is implemented by gates which restrict and redirect the flow of water through the turbine. In the case of Kaplan turbines, the blade angle is used to maintain peak efficiency over a wide range of power levels. In such strategies, the blades are operated in *follow-up* mode, readjusting to maximum operating efficiency as or after the gates have stabilized the system at the new operating point. This is known as *gate-dominant* control. The *cam curve* defines the maximum turbine efficiency relation between blade and gate positions.

Some of the limitations of conventional gate-dominant control can be understood using a simplified model for the hydrogenerating system. A reservoir is coupled to the turbine through a water passage referred to as *conduit*. Acting on speed deviation, the gates alter flow reaching the turbine. Should the gates close to reduce turbine over-speed, pressure in the conduit will rise. Closing the gates too fast may result in conduit over-pressure. A similar case can be made for opening the gates, resulting in conduit under-pressure. Both cases are undesirable.

During load changes when the hydrogenerator is operating islanded, the system output frequency will deviate from 60 Hz. Frequency also deviates when the unit in question is a significant portion of network generation. In severe cases, system stability may be threatened and over-speeds may damage the generator.

These considerations impose the design challenge of minimizing speed deviations at allowed conduit pressure limits while returning the turbine to reference operating speed at new power levels in minimum time. From these considerations, one can define control design performance measures of *maximum over-speed, maximum pressure*, and *speed settling time*. An additional, though noncritical design consideration, is the time required to return to maximum operating efficiency. This can be described by an *efficiency return time constant*.

Gate-dominant control has an implicit tradeoff between over-speed and over-pressure: as the gates restrict the flow through the conduit to reduce over-speed, the pressure rises. Because the blade angle has an effect on flow similar to that of the gates, a blade-dominant control would pose no improvement. As a safety measure, it is customary to impose maximum allowable rates on both gate and blade travels.

The proposed control scheme is different from the above in that it is neither gate nor blade dominant. It is found that driving the gates and blades in opposite directions at maximum allowable rates provides the quickest transition to the new power level while keeping the system within safe speed and pressure deviation boundaries. A second action of the proposed control returns the system to maximum operating efficiency while maintaining the new power level, again by using the blades and gates simultaneously. As described later, the proposed control temporarily decreases efficiency instead of flow as a method of reducing generated power. Comparisons made with gate-dominant control as well as a form of *Linear Quadratic Regulator* (LQR) optimal control show the merits of the proposed control scheme. This paper is based on work by Schniter [1]. Previous works by Defenbaugh [2] and Wozniak [3] have investigated similar, but nonoptimal, control designs. A study using the MULTISIM[1] program, where nonlinear simulations function as a realistic test-bed for the design, follows.

2. SYSTEM MODELING

2.1. The Linear Model

A development of the basic model is presented here. A hydrogenerator turbine can be described using characteristics that relate speed (x), pressure (h), generated torque (m), flow (q), load torque (l), gate position (y), and blade position (β) [3]. It is convenient to deal with all quantities on a *per-unit-deviation* basis, referred to as *pud*. Deviations are from a normalized reference value of unity.

The use of a linear model [3] is generally valid when simulating small perturbations. Linearization simplifies derivations and allows for fast computer simulations. Once a reference operating point is chosen, partial derivative relationships can be used to calculate flow and torque.

$$q = \frac{\partial q}{\partial x}x + \frac{\partial q}{\partial y}y + \frac{\partial q}{\partial h}h + \frac{\partial q}{\partial \beta}\beta, \qquad (1)$$

$$m = \frac{\partial m}{\partial x}x + \frac{\partial m}{\partial y}y + \frac{\partial m}{\partial h}h + \frac{\partial m}{\partial \beta}\beta. \qquad (2)$$

The partial derivatives are obtained from prototype turbine data as functions of blade and gate angles. It was found that the partial derivatives do not change appreciably for speeds within 20% of reference.

Other important relations needed to complete the system description for the linear model are the conduit and the rotational inertia equations.

$$h = -T_w \frac{dq}{dt}, \qquad (3)$$

$$m = T_a \frac{dx}{dt} + l. \qquad (4)$$

The values T_W and T_a are the *hydraulic* or *water time constant* and the *mechanical* or *rotor time constant,* respectively. Equation (3) models a frictionless, inelastic conduit. Since l is taken to be a constant, Eq. (4) indicates that the load is isolated and speed independent (also called *speed neutral*). Other load representations can be selected.

Deriving the linear model results in a differential equation of the form

$$\ddot{x} + c_1\dot{x} + c_2 x = c_3\dot{y} + c_4 y + c_5\dot{\beta} + c_6\beta. \qquad (5)$$

Above, $c_1,...,c_6$ are constants defined by the partial derivatives, the water time constant, and the rotor time constant.

The fourth-order differential equation in (5) can be expressed as a system of linear differential equations written in state-space form. It is convenient to choose the state, input, and output vectors, respectively, as

$$\mathbf{x} = [\, x \;\; \tfrac{dx}{dt} \;\; y \;\; \beta \,]^T, \qquad (6)$$

$$\mathbf{u} = [\, \tfrac{dy}{dt} \;\; \tfrac{d\beta}{dt} \,]^T, \qquad (7)$$

$$\mathbf{y} = [\, x \;\; m \;\; y \;\; \beta \;\; h \;\; q \,]^T, \qquad (8)$$

and the state-space system as

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}, \qquad (9)$$

$$\mathbf{y} = \mathbf{Cx}. \qquad (10)$$

[1]Hydrogenerator nonlinear simulation program. Written for the Woodward Governor Company, Stevens Point, Wisconsin, by Schniter and Wozniak.

The outputs q and h are included to obtain an understanding of system behavior, but should not be used in a non-observer control design since they are not easily transduced. The dimensions of the system matrices **A**, **B**, and **C** are 4×4, 4×2, and 6×4, respectively.

2.2. The Quasi-linear Parameter-varying Model

Comparisons between linear simulations of the gate-dominant and the proposed control and of nonlinear simulations provided by the MULTISIM program show significant differences even for small load changes. This is brought about in part by the relatively large changes in turbine partial derivatives over the range of blade and gate motions.

Model inaccuracy was addressed by extension of the linear model to a quadratic one. This was done by defining functions that describe the partials.

$$\frac{\partial q}{\partial x} = f_1(y,\beta), \quad \frac{\partial q}{\partial y} = f_2(y,\beta), \quad \frac{\partial q}{\partial h} = f_3(y,\beta), \quad \frac{\partial q}{\partial \beta} = f_4(y,\beta),$$
$$\frac{\partial m}{\partial x} = f_5(y,\beta), \quad \frac{\partial m}{\partial y} = f_6(y,\beta), \quad \frac{\partial m}{\partial h} = f_7(y,\beta), \quad \frac{\partial m}{\partial \beta} = f_8(y,\beta). \qquad (11)$$

Each function is linear in y and β and valid in a region around an operating point, (y_o, β_o). Graphically, this is equivalent to approximating a local region in the partial derivative surfaces by planes.

The variables $c_1,...,c_6$ in Eq. (5) are no longer constants, but instead functions of the parameters y and β. This leads to system matrices that are system parameter dependent, and, hence, a *parameter-varying* state-space model is obtained.

$$\mathbf{x} = \mathbf{A}(y,\beta)\,\mathbf{x} + \mathbf{B}(y,\beta)\,\mathbf{u}, \qquad (12)$$

$$\mathbf{y} = \mathbf{C}(y,\beta)\,\mathbf{x}. \qquad (13)$$

One way to implement such a parameter-varying system in a digital simulation is to update the system matrices at each time increment, but assume that the system is linear-time-invariant (LTI) between increments. This can then be referred to as a *quasi-linearized* parameter-varying system.

In control design, one is motivated to use quasi-linear parameter-varying systems based on results by Shamma [4]. The system in Eqs. (12) and (13) demonstrates a special case of the above, where the parameters are system outputs. Shamma's studies assure the validity of *gain-scheduling* using *frozen-parameter* designs in the case that the parameters change slowly enough. More specifically, if optimal control designs are known for a range of LTI approximations to the original plant, and at any given time the scheduled control is the one whose LTI reference operating point is closest to the system's operating point, the resulting response approaches the optimal as the time between parameter changes approaches zero [4]. In terms of digital simulations, this suggests a time discretization that ensures slowly changing parameters.

2.3. Modeling Gate-dominant Control

Described below is the modeling of the gate-dominant controlled system. It is important as a reference of comparison to any proposed control scheme since it represents conventional governors.

The gate is driven by deviations of speed from reference. Usually, a proportional-integral (PI) controller of the form

$$y = -\left(K_P + \frac{K_I}{s}\right)x \qquad (14)$$

dominates. The "s" refers to the Laplace operator.

The blade position follows the gate position according to the cam curve to keep the system at peak efficiency. For stability reasons, its motion may be slowed by a rate filter, resulting in a blade movement that is slower than the gate's. Because the cam curve in this example is nearly linear, it was implemented by using the best linear fit. Blade action is weighted by a blade gain constant, K_β.

It is desirable to present the best performance of such a design in terms of simulation results so that it may be used as a reference by the control design described next. Figure 1 shows the closed-loop gate-dominant system's response to a 2% load

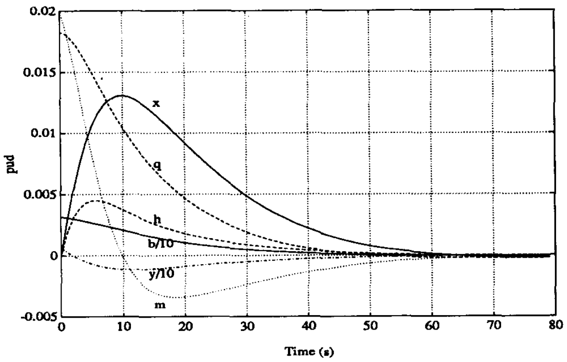


Fig. 1. Best performance of gate-dominant control scheme with a 2% load rejection. Traces represent flow (q), torque (m), speed (x), pressure (h), one-tenth of gate angle (y/10), and one-tenth of blade angle (b/10). All quantities are per-unitized deviations from reference (*pud*).

rejection. The controller parameters were chosen by exhaustive search to achieve the best results over load rejections up to 10%.

3. CONTROL DESIGN

3.1. The Efficiency Based Design Concept

The goal of this control design is to achieve optimal performance in the case of load rejections. More specifically, it aims to minimize speed deviations at allowed conduit pressures while returning the turbine to reference operating speed at the new power level in minimum time. The proposed control can be divided into two parts: an initial action which transfers the system to the new power level by means of efficiency reduction, and a secondary action which regains maximum operating efficiency without deviating from the new power level. Due to the nonlinearity of the data, the quasi-linear modeling restricts the analysis to largest load rejections on the order of 10%.

The proposed control is unique in that the blade and gate initially move at maximum rates in opposite directions in an effort to quickly reduce efficiency. By defining efficiency proportional to the ratio of the product of torque and speed to the product of flow and pressure

$$\eta \approx \frac{MX}{QH}, \tag{15}$$

and defining power as

$$P = MX, \tag{16}$$

one can gain insight into the motivations for such a control. Because changes in speed should be avoided, Eq. (16) indicates that a change in generated torque would be the preferred method for a change in power level. Furthermore, Eq. (3) shows that a change in flow results in a displacement from reference pressure. Thus, in order to avoid pressure deviations, a goal-designed control scheme must avoid flow changes as well. Equation (15) then implies that a change in efficiency is necessary to transfer the system to a new power level while avoiding deviations from reference speed and pressure.

An optimal scheme would reduce efficiency in the quickest manner possible, so as to lessen the degree of inevitable speed and pressure deviations. As a result, this implies operating the blades and gates at the allowed maximum rates.

In addition to lowering the efficiency, the intermediate position must satisfy the new system power level. It has been determined [1] that the best trajectory would involve decreasing the gate opening and increasing the blade angle.

For a step decrease in load, some over-speed is inevitable. To remove this over-speed, generated torque must be held to a level below the load torque. Before the over-speed has been completely removed, however, it is important to begin returning the system to the final power level; otherwise, speed oscillations will ensue [1]. This involves reversing the directions of the blade and gate movements.

3.2. Selection of Maximum Blade and Gate Rates

The optimal maximum blade and gate rates are determined from resulting pressure deviations. For simplicity, positive and negative pressure deviations are taken to be equally undesirable, leaving the task of constraining absolute pressure deviation (h_{max}) from reference. In the way that closing the gates causes an increase in conduit pressure, opening the blades causes a decrease. It is then possible to select blade and gate rates that nearly cancel each other's effect on pressure.

All simulations show the common feature of an initial under-pressure, followed by a peak in over-pressure, followed by a return to reference (see Fig. 2). Matched blade and gate rates, with nearly identical over- and under-pressures, result in the minimum deviation from reference.

Though the exact timing of peak over-pressure is governed by the rate reversal discussed in Section 3.3, it is important that this peaking be restricted to a relatively small neighborhood in time. This justifies the claim that the maximum control rates, not the timing of the rate reversal, determine the peak pressure deviation and thus the pressure peak.

It is reasonable that quick matched rates reduce maximum over-speed (x_{max}) and speed settling time (t_X) more than slower ones without affecting pressure deviations. Though one is motivated to choose the quickest matched rates possible, rates are limited for safety reasons and restrictions must be imposed. For the purpose of this paper, a gate time (T_Y) of 30 seconds was selected as the design restriction. The resulting blade times (T_β) are within safe limits selected on the basis of overpressure.

Selected gate and blade rates result in speed and pressure deviations much improved over those for gate-dominant designs (see Tab. 1). Testing load rejections of different magnitudes, however, evidences that blade and gate rates are specific to load rejection magnitude [1]. If the gate rate is held fixed, a blade rate selection procedure sensitive to the level of load change is necessary.

The blade rate selection must be accomplished *on-line* since the controller can not sense the load change in advance. In addition, the selection of rates must be accomplished before any action has taken place, since the first control action is dependent upon knowledge of these rates. One solution involves using the assumption that the load rejection is of a step nature. By measuring initial acceleration, the control can determine load change and select a pretabulated optimal blade rate. The control logic can be determined by simulation and verified empirically from experimental data [1].

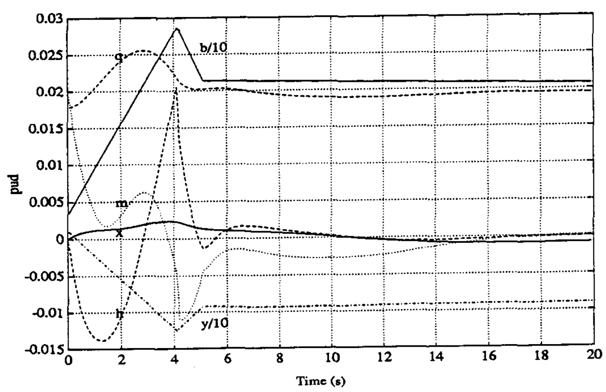


Fig. 2. Performance of the proposed control scheme using the gate-dominant version of the final control action, 2% rejection.

3.3. Selection of Rate-reversal Time

Selecting the optimal time for the reversal of maximum rates is necessary when removing over-speed by reduction in generated torque. If the generated torque is kept below the load torque for an incorrect time period, the speed will be reduced by more or by less than the optimal amount. It is desirable to obtain a method of selecting the reversal time that is independent of the level of load change and that can be implemented on-line.

Switching logic based on speed was found to perform successfully for all load rejections simulated. It was determined by simulation [1] that for this study the rates should be reversed when the speed has decreased to approximately 0.8 of its peak value.

3.4. Switching to the Final Control Action

The second significant control switching is required when the purpose in control changes from reducing power to maximizing efficiency. It may seem desirable to use the rate reversal to drive speed and torque close to zero (*pud*) before switching to local control action, but this results in significant pressure deviations which later act to disturb the speed from reference [1]. It is necessary to determine a way to zero the over-speed and torque without causing large pressure disturbances thus driving states toward reference. Compared to the requirement on speed, the time necessary to return the system to maximum efficiency operation is not of paramount importance. Gate and blade motions are kept relatively slow to avoid unwanted speed deviations.

Considering rate reversal, an opportune switching time exists when the torque and over-speed are appropriately related. If the system is permitted to run freely at this time, the speed and torque decay quickly to equilibrium positions without perturbing other states. That time is explicitly stated in the form of Eq. (17).

$$x_{sw} = \int_{t_{sw}}^{\infty} \dot{x}(t)\, dt = \frac{1}{T_a} \int_{t_{sw}}^{\infty} [m(t) - l(t)]\, dt. \qquad (17)$$

The pressure is conveniently near reference at the switching time [1] and does not disturb the speed or torque for the remainder of the simulation.

Once again, an on-line implementation of the control is necessary. In this case, it involves determining the switching time based on measurements of speed and acceleration. Assuming that the speed and acceleration monotonically decay to reference, only one instant in time exists when they obey the relationship Eq. (17), which can be approximated by

$$x_{sw} = f(x_{sw}) \cdot \dot{x}_{sw}^{2}. \qquad (18)$$

A quadratic fit to experimental data resulted in the function used in Eq. (18). This technique performs well over the full range of load rejections tested [1].

3.5. The Final Control Action

Once the switch has been made, the control enters into its final action. The objective of this phase is to bring the system back to maximum operating efficiency in the quickest manner possible without disturbing speed from reference. Two methods are suggested, each with particular advantages. One results in better performance than the other but requires more computational resource.

The first method uses a gate-dominant proportional-integral (PI) control system, identical in structure to that described in Section 2.3. The control parameters chosen for the best performance for this design slow the blade and gate rates considerably as a means of preventing disturbances in speed and pressure. The use of an efficiency-return time constant (T_η) is adopted to compare different final control actions and weigh relative merits. It is defined using the length of time required for flow decay.

An alternative method is suggested that results in better overall performance but requires a microprocessor supporting on-line calculations of matrix inversion complexity. This method uses a version of Linear Quadratic Regulator (LQR) [5] optimal control that requires calculating the optimal control action over successive intervals of time. The LQR optimal control is discussed in more detail in Section 5.2. A discussion of gain-scheduled frozen-parameter systems in Section 2.2 validates this implementation of LTI optimal control.

Simulations show that the LQR version of the final control action improves over the gate-dominant in speed settling time and efficiency return time [1]. The initial control action, which determines maximum speed and pressure deviations, remains unchanged, as does the switch time to the final control action. Benefits are clearly shown in a comparison of the proposed control scheme with LQR as the final action to the use of LQR optimal control alone (see Section 5.2). Table 1 indicates that the proposed control scheme performs better than straight LQR in all areas.

Table 1. Comparison of various control scheme performances for a 2% load rejection.

Control	x_{max}	h_{max}	t_x	T_η	T_y	T_β
Gate-Dominant Only	1.3×10^{-2}	4.5×10^{-3}	55 s	---	---	---
Proposed + Gate-Dominant	1.9×10^{-3}	5.7×10^{-3}	5.4 s	120 s	30.0 s	19.9 s
Proposed + LQR	1.9×10^{-3}	5.7×10^{-3}	3.3 s	50 s	30.0 s	19.9 s
LQR Only	4.3×10^{-3}	5.7×10^{-3}	10.8 s	250 s	30.1 s	20.0 s

4. DESIGN VERIFICATION

4.1. Nonlinear Simulations

The MULTISIM program is a hydrogenerator nonlinear simulation program which functions as an accurate analysis tool for investigating the performance of the proposed design in a fully nonlinear setting and indicates how the control may perform if implemented in the field. All nonlinear simulations of the proposed control scheme represent the version with the gate-dominant final control action.

An examination of the nonlinear system shows a number of features. One is the torque response shown in Fig. 3, which, unlike the monotonic decay in the quasi-linear simulations, changes direction during the initial control action. This is a result of large variations in the partial derivatives $\partial m/\partial y$ and $\partial m/\partial \beta$ [1].

The reversal in torque delays the over-speed peak, and therefore the rate-reversal, by about 2 sec. Extending the maximum travel time by a factor of two has a detrimental effect on the flow. Both nonlinear and quasi-linear simulations indicate that it is impossible to keep the flow from fluctuating when driving the blade and gate at constant rates due to changing values of $\partial q/\partial y$ and $\partial q/\partial \beta$. The longer the controls move at constant rates, the more there exists potential for pressure deviations from reference. This is manifested in the nonlinear simulation by pressure peaks much higher than those of the quasi-linear simulation.

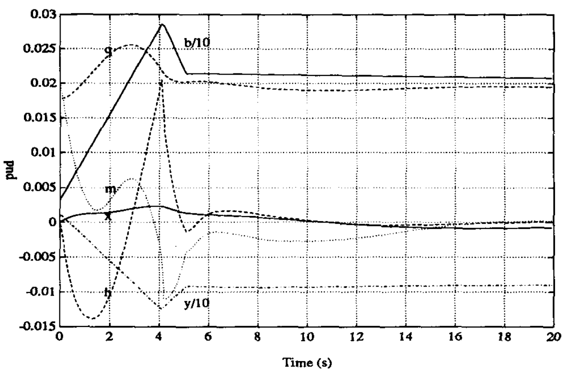


Fig. 3. Close-up of the MULTISIM nonlinear simulation of the proposed control scheme, 2% rejection.

Comparing nonlinear simulations of the proposed control design to a gate-dominant design, the efficiency-based control performance gain over the gate-dominant control is less than expected from the quasi-linear simulations. The proposed control has a much better speed response at the expense of pressure response. The over-speed is reduced by a factor of 4, and the speed settling time is approximately 10 sec as compared to over 80 sec for the gate-dominant simulation. The pressure deviation, however, is a factor of 5 higher than for the gate-dominant design. The abnormally high pressure peaks are the results of partial derivatives that change with the operating point, and detune the preset rates [1].

It is clear that a method of locally tuning the efficiency-based design is necessary in order to guarantee success with nonlinear characteristics. When faced with smoothly varying partials, as in the quasi-linear simulations, the proposed control has been shown to clearly outperform the gate-dominant.

4.2. Load Acceptances

In controlling a load acceptance, efficiency reduction is not feasible. If an *increase* in generated torque is sought without a change in flow, only an efficiency enhancement scheme would be effective. The fact that a system at reference is assumed to be operating at maximum efficiency renders the method impossible. This constraint is mitigated by a decreased generator damage potential caused by under-speeds.

5. RELATIONS TO OPTIMAL CONTROL THEORY

5.1. Time-optimal Control

This paper attempts to design an optimal form of hydrogenerator control. Discussed here are the relations between optimal control theory and the proposed design and other possible contributions derived from optimal control theory.

The efficiency reduction phase of the control problem bears similarities to the *Bang-bang* form of time-optimal control [6]. Bang-bang control is the time-optimal control for systems with constrained inputs and a cost function that involves only time. Since the system inputs are given as limited rates, and since speed recovery is desired in minimum time, there are strong similarities between the typical bang-bang time-optimal control problem and the problem at hand.

It is not surprising that the initial control action in the proposed design also bears similarities to the optimal solution of the constrained optimization problem formulated above: each control input is operated at maximum level with switches in direction. It is difficult to determine, however, if the heuristically chosen trajectory is indeed theoretically optimal for reasons explained below.

It is known that n^{th} order systems with real eigenvalues have at most $n - 1$ switchings of control [6]. For systems with complex eigenvalues, such as the one dealt with here, determining the optimal number of switchings is more involved. The hydrogenerating system has multiple inputs and nonminimum-phase zeros [6], as well as a parameter-varying approximation of system nonlinearities, which further complicate mathematical tractability.

The final heuristic design, using a bang-bang approximation, was accomplished in a gain-scheduled empirical manner.

5.2. Linear Quadratic Regulator Optimal Control

Linear Quadratic Regulator (LQR) [5] optimal control minimizes a pre-defined cost function over a certain interval of time. The *infinite-horizon* case of optimal control applies the cost function over all finite time, and has been chosen as the desired cost application for this study of hydrogenerating systems. The cost function can be expressed as

$$J = \int_0^\infty (\mathbf{y}^T\mathbf{Q}\mathbf{y} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt. \quad (19)$$

The diagonal weighting matrices **Q** and **R** enable the designer to select what variables will be minimized. In this case, the **Q** and **R** cost coefficients were chosen to result in the same blade and gate rates as well as the same pressure deviations of the proposed control design. The vectors **u** and **y** are the input and output state vectors, respectively.

Figure 4 demonstrates the effects of applying LQR control to a 2% load rejection. It is found to be a clear improvement over gate-dominant control in the areas of speed settling time, efficiency return time, and final speed error. As compared to quasi-linear simulations of the proposed control, it does not fare as well.

Perhaps the most interesting outcome of these simulations is the similarity between the LQR and the efficiency-based control responses. Both move blade and gate in opposite directions at quick rates before gradually returning to reference. Both keep the generated torque below the load torque as a method of decreasing over-speed, and slowly change blade and gate rates after pressure, speed, and torque have achieved reference positions. This last behavior is identical to the final control action of the proposed control, where efficiency is regained slowly at the expense of keeping speed and pressure at reference. The remarkable thing about the LQR control is that no intuitive strategy was involved in arriving at the control actions, only setting the cost functions.

It is apparent that there are close similarities between the proposed control and two types of optimal control theory. The fact that the proposed control achieves better results than LQR shows that a combination of two optimal control strategies has been effectively synthesized in a gain-scheduled implementation. The fact that the final control design was achieved heuristically implies that results of optimal control theory can still be useful when an exact solution to the problem is unattainable.

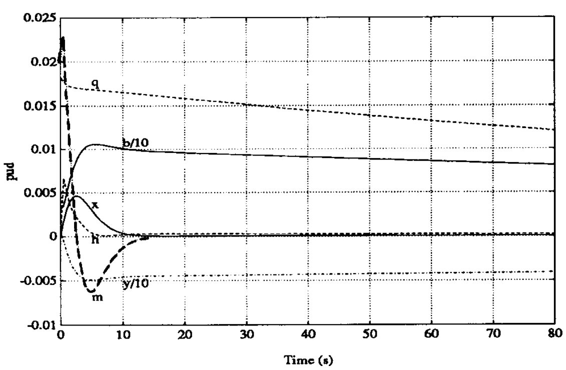


Fig. 4. Simulation of LQR optimal control applied to a 2% load rejection.

6. CONCLUSIONS

Through simulations based on a quasi-linear modeling technique, the performance of the efficiency-based control design has shown to be a significant improvement over gate-dominant control. The results of the proposed control presented in Table 1 show peak over-speed reduced by a factor of 7, and speed settling time by a factor of 10, for a equivalent peak pressure deviation. More specifically, it has improved Kaplan turbine response to load rejections in maximum speed and pressure deviations, speed settling time and efficiency return time.

Comparing the proposed control actions to the bang-bang and LQR results of optimal control theory shows close similarities. The maximum rate initial control actions can be thought of as an optimal solution to the constrained minimum-time problem where the objective was to decrease the generated torque in minimum time subject to maximum allowable gate and blade rates. The proposed control is also reflected in the results of a gain-scheduled frozen-parameter implementation of LQR optimal control, where initially quick gate and blade motions are subsequently slowed as a means of regaining efficiency without disturbing pressure and speed from reference.

Nonlinear simulations have shown that the quasi-linear based implementation of the proposed control design is sensitive to quickly changing partials along the system's trajectory. Using current blade and gate positions to describe the system operating point, it is found that the maximum rate control actions quickly move the operating point through regions of substantial variation in the system characteristics. This presents a challenge for an implementation that was not developed to encounter such rapid changes.

With further work, improvements in performance can be gained at the expense of added detailed system knowledge involving mapping the turbine characteristics and tuning control actions based on current operating point and predicted trajectory. This study also indicates that if an accurate way of monitoring conduit pressure were available, incorporating the pressure signal into an enhanced control strategy should be further investigated.

REFERENCES

[1] P. Schniter, "An efficiency based technique for the optimal control of Kaplan hydrogenerating systems," M.S. thesis, Department of Electrical Engineering, University of Illinois, Urbana, IL, 1993.

[2] J. Defenbaugh, "Improved transient response for Kaplan hydrogenerators," M.S. thesis, Department of General Engineering, University of Illinois, Urbana, IL, 1983.

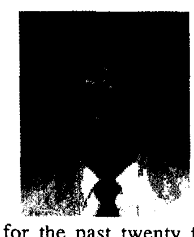
[3] L. Wozniak, "Load level tuning of governor gains for sub-optimal speed control of hydroelectric installations," ASME Paper No. 74-WA/FE-4, 1974.

[4] J. Shamma, "Analysis and design of gain scheduled control systems," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, 1988.

[5] A. Sage and C. White, Optimum Systems Control, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1977.

[6] M. Athans and P. Falb, Optimal Control. New York, NY: McGraw Hill, 1966.

Philip Schniter was born July 22, 1970 in Evanston, Illinois. He received the B.S. and M.S. degrees in Electrical Engineering from the University of Illinois at Urbana in 1992 and 1993, respectively. He is currently designing signal processing test and measurement equipment with Tektronix, Inc. in Beaverton, Oregon.



Louis Wozniak was born in Rome, Italy in 1938 and immigrated to the United States at the age of 13. He received the B.S. in Mechanical Engineering and the Ph.D. in Electrical Engineering in 1967 at the University of Illinois. He is presently Associate Professor of General Engineering at the Urbana campus of the University of Illinois. Professor Wozniak has been involved with hydro speed control research for the past twenty five years. He has authored a number of publications and has served as consultant for several hydro projects.