A Message-Passing Receiver For BICM-OFDM Over Unknown Clustered-Sparse Channels

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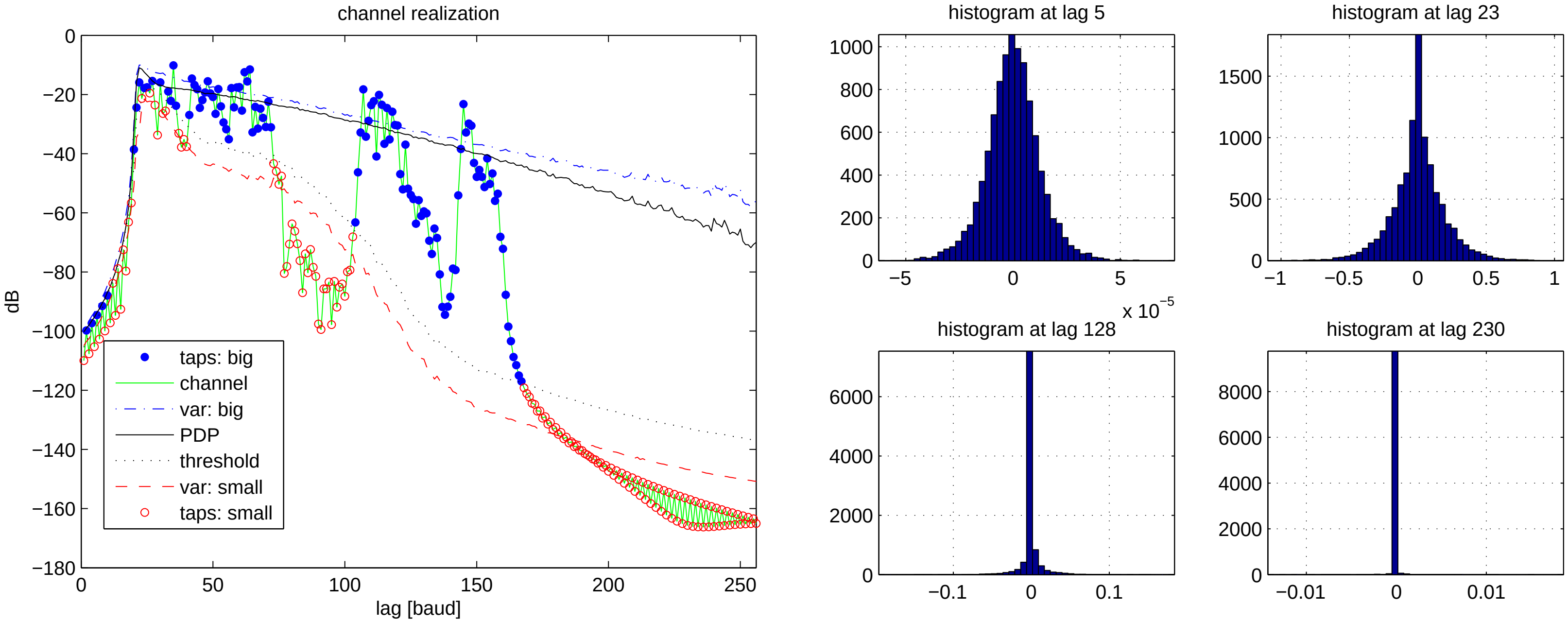
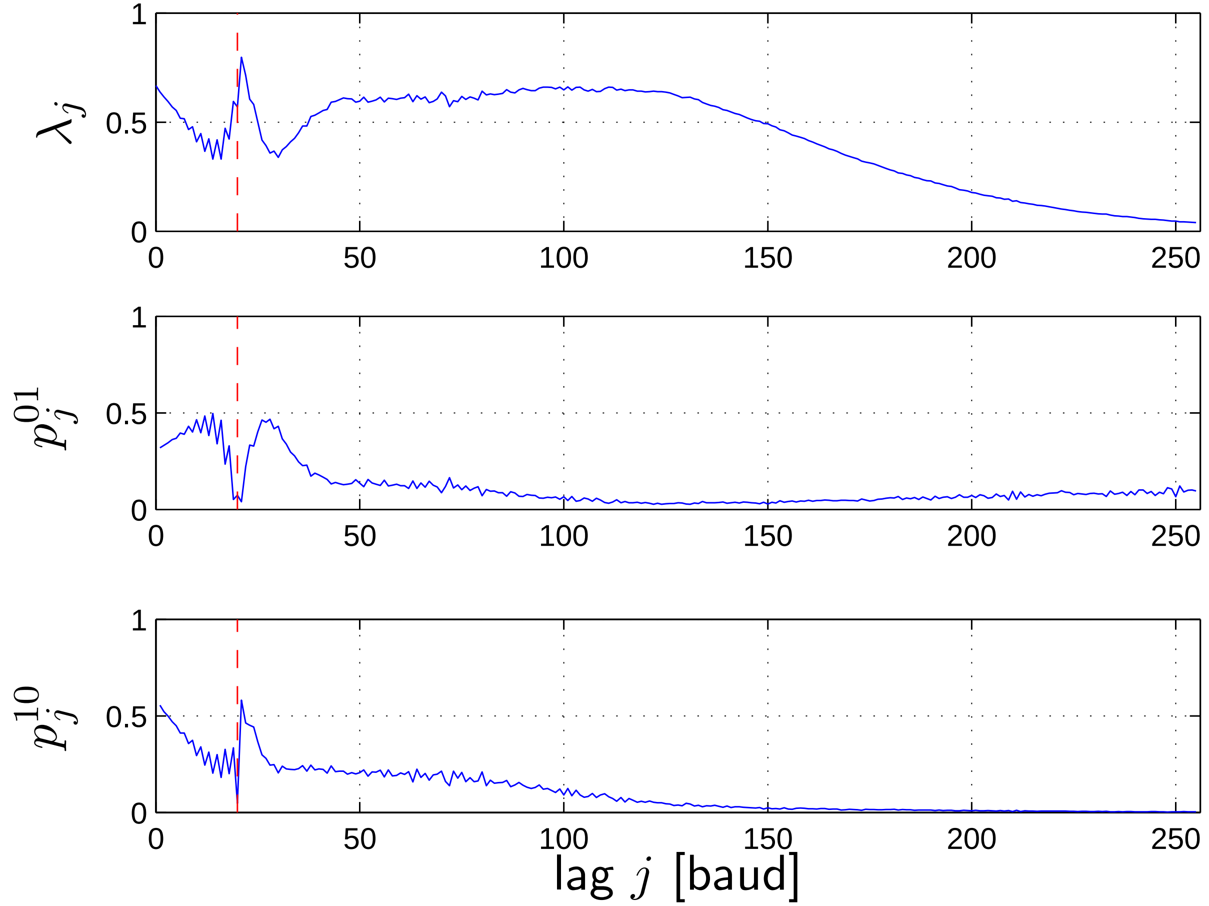
Motivation:

- At large communication bandwidths, communication channels are not only frequency selective but *sparse*.
- For example, consider channel taps $\boldsymbol{x} = [x_0, \ldots, x_{L-1}]$, where
  - $x_n = x(nT)$ for bandwidth $T^{-1} = 256$ MHz,
  - $x(t) = h(t) * p_{\text{RC}}(t)$, and
  - $h(t)$ is generated randomly using 802.15.4a outdoor NLOS specs.



Lag-dependent statistics:

- Note that the tap energy, sparsity rate, and clustering are *lag-dependent*:



(Empirically estimated using 10000 realizations 802.15.4a outdoor NLOS.)

Proposed channel model:

- Saleh-Valenzuela (e.g., 802.15.4a) models are accurate but difficult to exploit in receiver design.
- We propose a structured-sparse channel model based on a *2-state Gaussian Mixture* model with *Markov-chain* structure on the state:

$$p(x_j \,|\, d_j) \;=\; \begin{cases} \mathcal{CN}(x_j; 0, \mu_j^0) & \text{if } d_j\!=\!0 \quad \text{“small”} \\ \mathcal{CN}(x_j; 0, \mu_j^1) & \text{if } d_j\!=\!1 \quad \text{“big”} \end{cases}$$

$$\Pr\{d_{j+1} = 1\} \;=\; p_j^{10} \Pr\{d_j = 0\} + (1 - p_j^{01}) \Pr\{d_j = 1\}$$

- Our model is parameterized by the lag-dependent quantities:

$\{\mu_j^1\}$: big-state power-delay profile
$\{\mu_j^0\}$: small-state power-delay profile
$\{p_j^{01}\}$: big-to-small transition probabilities
$\{p_j^{10}\}$: small-to-big transition probabilities

- Can learn these statistical params from observed realizations via the EM alg.

Optimal communication over unknown sparse channels:

For the unknown N-block-fading, L-length, S-sparse channel, Kannu & Schniter [1] established that

1. In the high-SNR regime, the ergodic capacity obeys

$$C_{\textsf{sparse}}(\textsf{SNR}) = \frac{N-S}{N}\log(\textsf{SNR}) + \mathcal{O}(1).$$

2. To achieve the prelog factor $R_{\textsf{sparse}} = \frac{N-S}{N}$, it suffices to use
   - pilot-aided OFDM (with N subcarriers, of which S are pilots)
   - with (necessarily) *joint* channel estimation and data decoding.

Key points:

- The effect of *unknown channel support* manifests only in the $\mathcal{O}(1)$ offset.
- While [1] uses constructive proofs, the scheme proposed there is impractical.

[1] A. P. Kannu and P. Schniter, "On communication over unknown sparse frequency selective block-fading channels," *IEEE Trans. Info. Thy.*, to appear [arXiv 1006.1548].

Practical communication over unknown clustered-sparse channels:

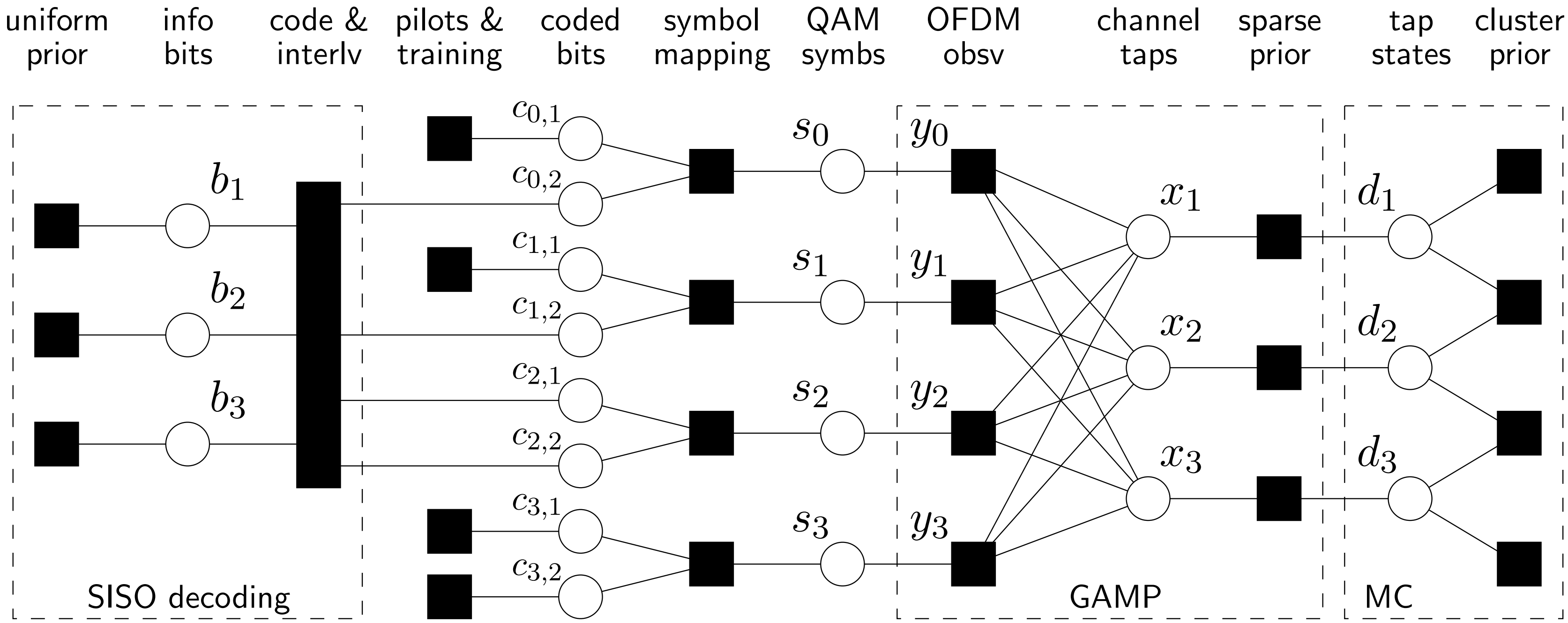
Transmitter:

- BICM OFDM
- carefully placed training bits

Receiver:

- joint data decoding / clustered-sparse channel estimation / cluster detection
- based on loopy *belief propagation* (BP)
- key enablers:
  1. "**generalized AMP**" algorithm [Rangan 10]
     - builds on AMP [Donoho/Maleki/Montanari 09]
     - near-optimal in large-system limit [Bayati/Montanari 10]
     - extremely low complexity: $\mathcal{O}(N \log N)$ per iteration
  2. "**turbo**" message scheduling [Schniter 10]

Factor graph for pilot-aided BICM-OFDM:



○ = random variable ■ = posterior factor

To jointly infer all random variables, we perform loopy-BP via the sum-product algorithm, using carefully chosen message approximations in each dashed box.

(See SPAWC paper for details, or longer version in arXiv:1101.4724.)

Numerical results:

Transmitter:

- OFDM with $N = 1024$ subcarriers.
- 16-QAM with multi-level Gray mapping
- LDPC codewords with length ~ 10000 yielding spectral efficiency of 2 bpcu.
- P "*pilot subcarriers*" and T "*training MSBs*."

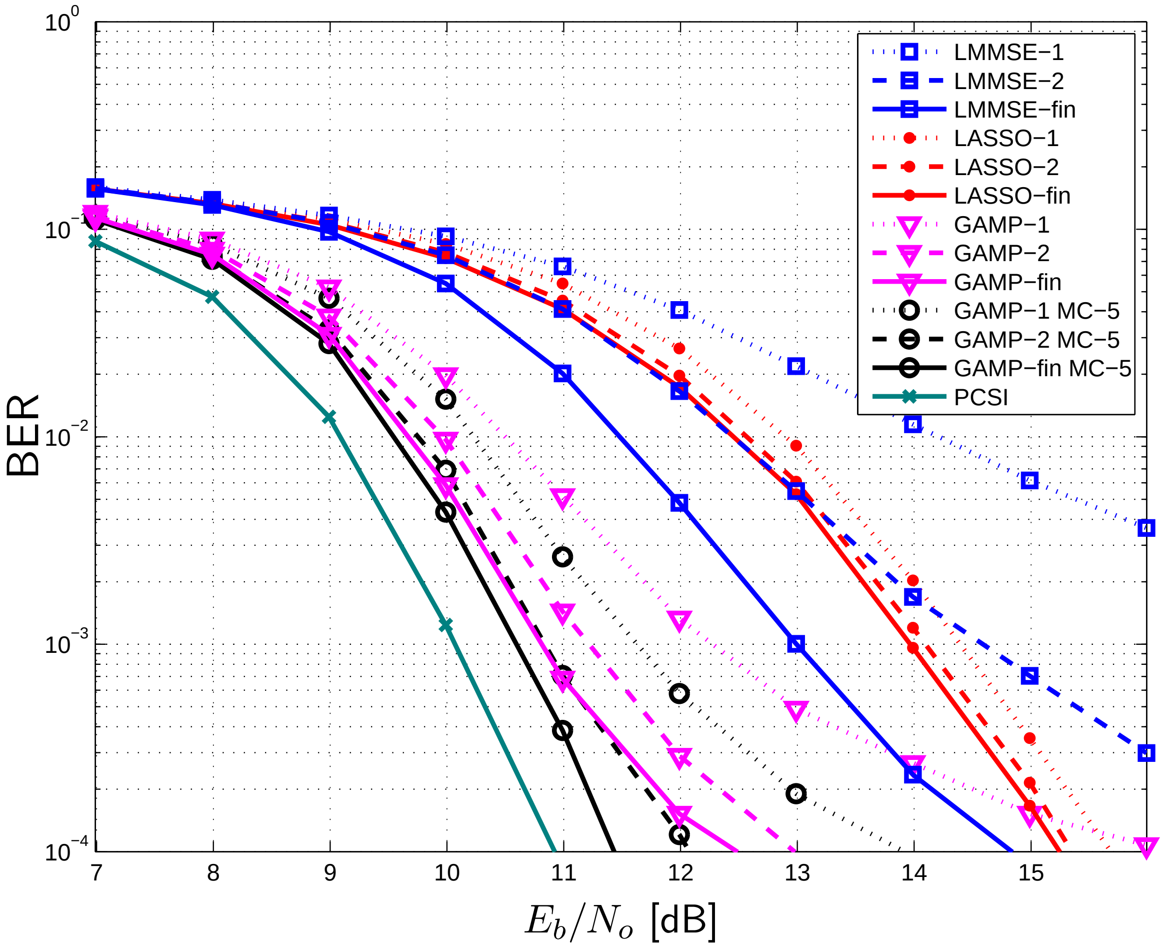
Channel:

- 802.15.4a outdoor-NLOS (not our Gaussian-mixture model!)
- Length $L = 256 = N/4$.

Reference Channel Estimation / Equalization Schemes:

- soft-input soft-output (SISO) versions of LMMSE and LASSO.
- perfect-CSI genie.

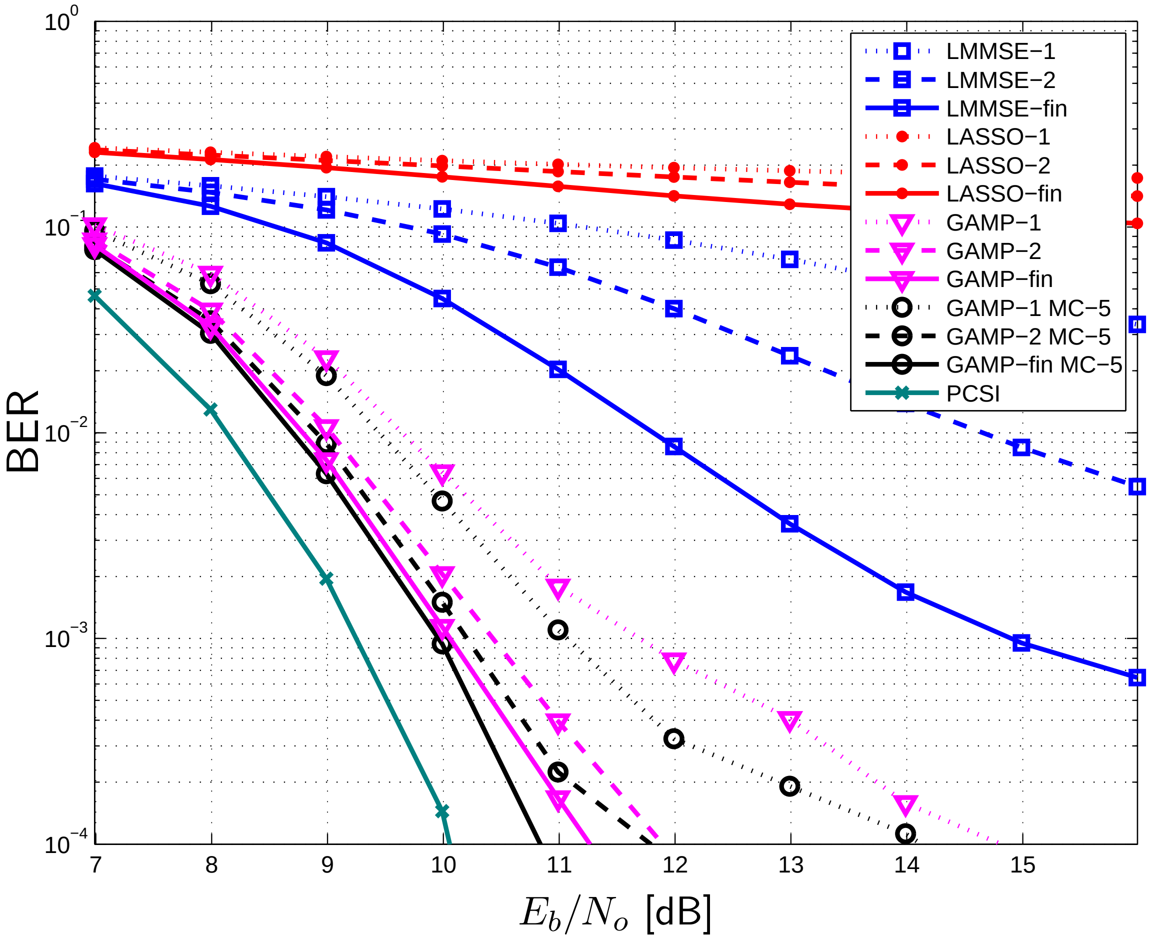
BER versus E_b/N_o for $P = 224$ pilots and $T = 0$ training bits:



Our scheme shows 4dB improvement over (turbo) LASSO.

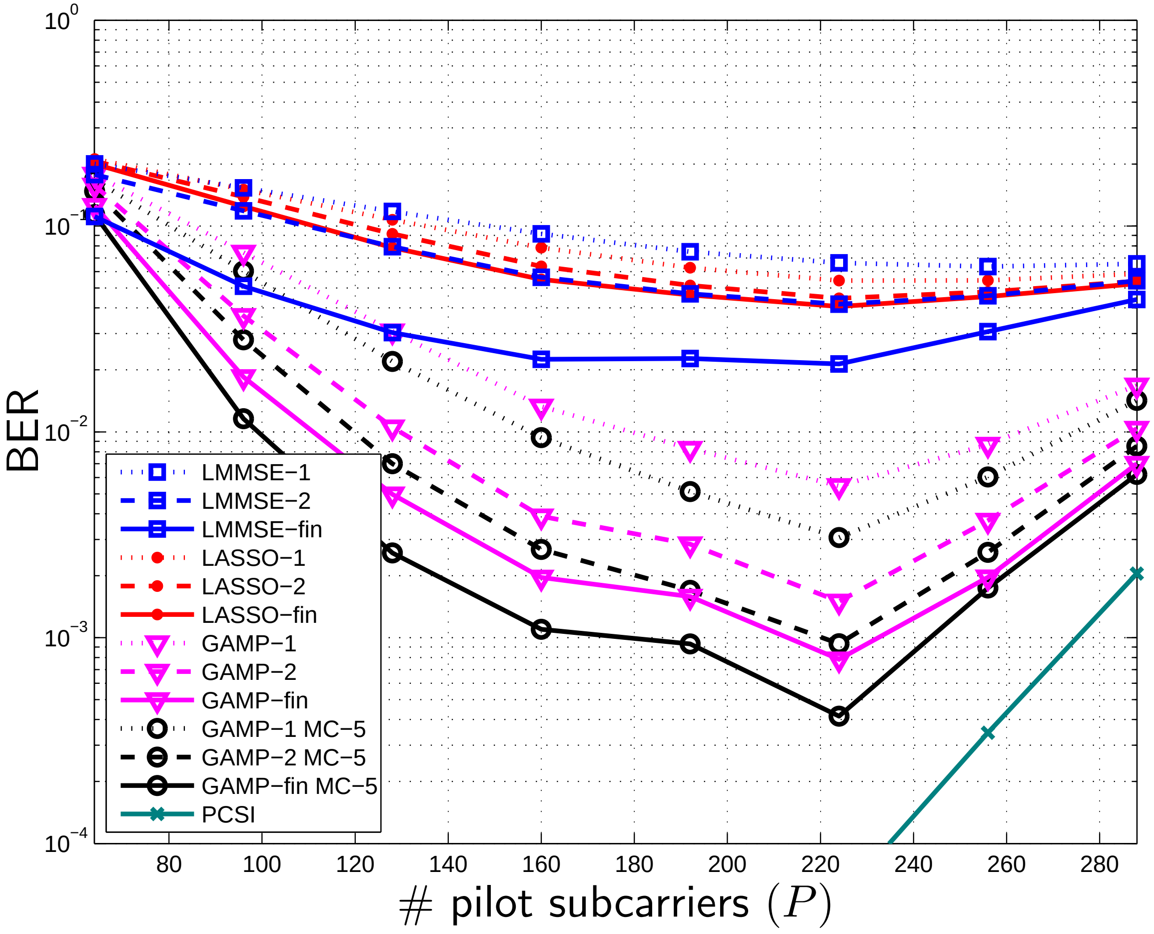
Our scheme only 0.5dB from perfect-CSI genie!

BER versus E_b/N_o for $P = 0$ pilots and $T = 448$ training bits:



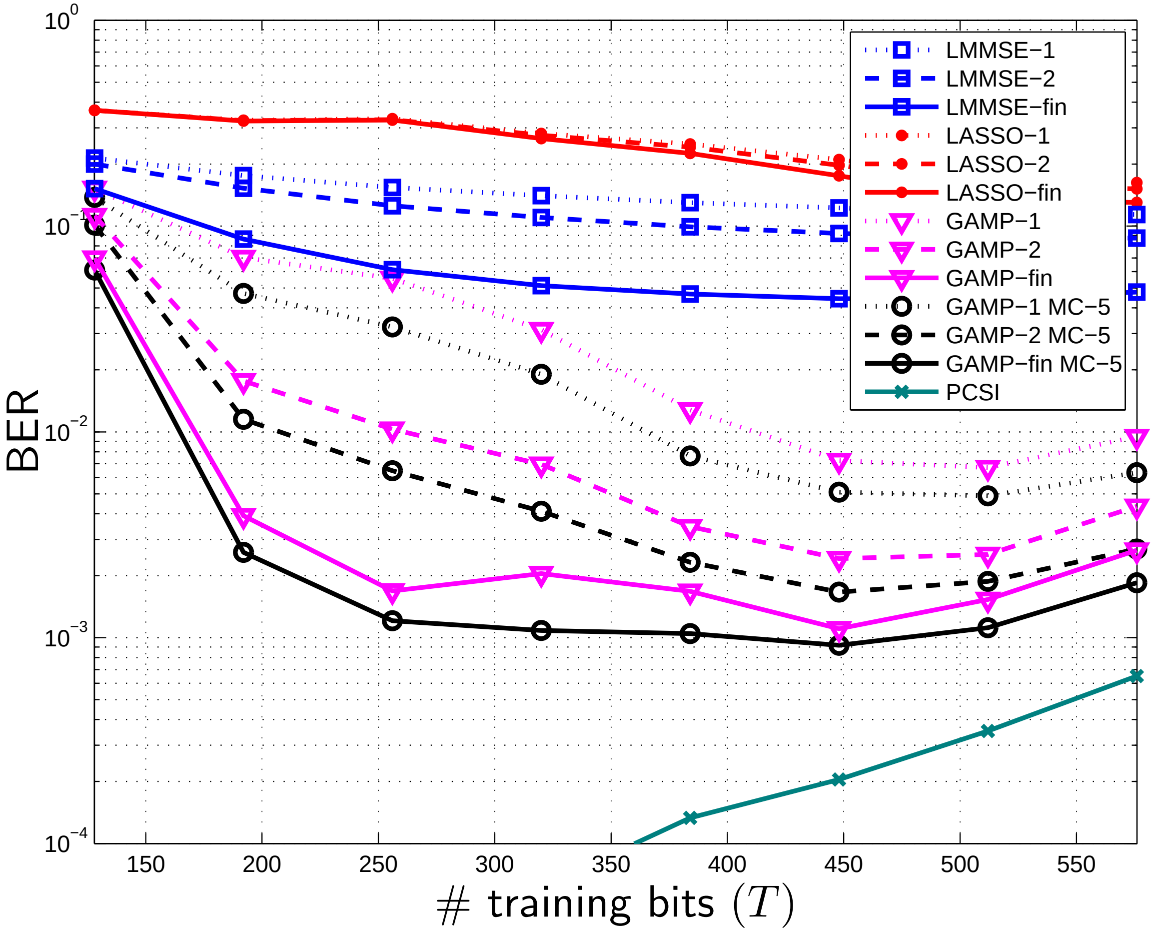
Use of training bits gives 1dB improvement over use of pilot subcarriers!

BER versus P pilot subcarriers for $T = 0$ training bits:



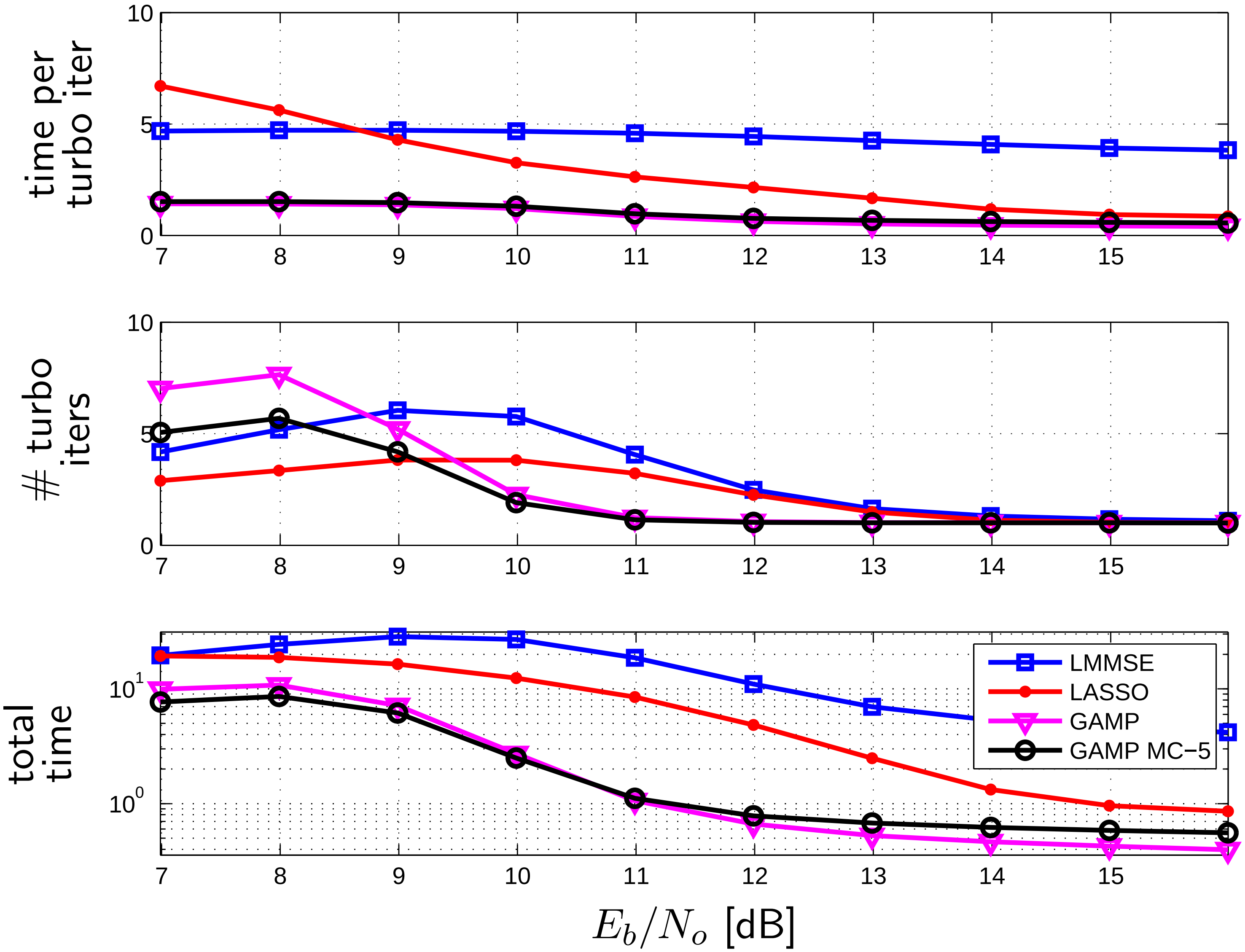
Too few pilots compromises channel estimation; too many compromises decoding.

BER versus T training bits for $P = 0$ pilot subcarriers:



Too few training bits compromises chan est; too many compromises decoding.

Convergence speed versus E_b/N_o for $P\!=\!224$ and $T=0$:



Conclusions:

- At larger communication bandwidths, channels become clustered-sparse, as seen by the 802.15.4a realizations.
- To exploit clustered-sparsity in receiver design, we proposed a channel model based on a 2-state Gaussian-mixture with Markov-chain state structure.
- Information theoretic analysis of sparse channels motivates OFDM transmission and joint channel-estimation/decoding.
- We proposed a factor-graph based OFDM receiver that accomplishes joint decoding / clustered-sparse channel estimation / cluster detection.
- Our receiver leverages Rangan's "generalized AMP" algorithm and our earlier work on "turbo sparse reconstruction"
- Our performance is 0.5dB from perfect-CSI bound 4dB beyond LASSO.
- Our complexity is only $\mathcal{O}(\log_2 N + |\mathbb{S}|)$ per symbol.