Communications over Sparse Channels:

Fundamental limits and practical design

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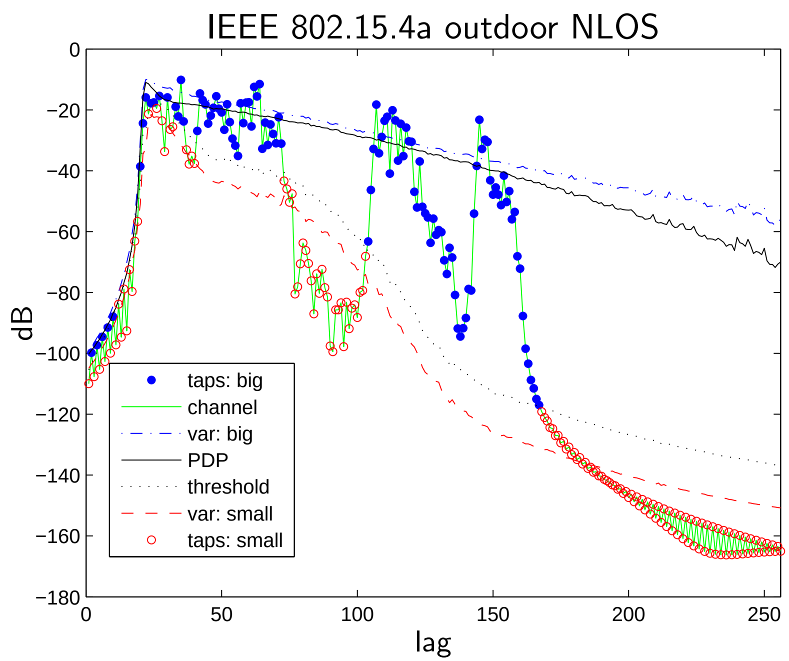
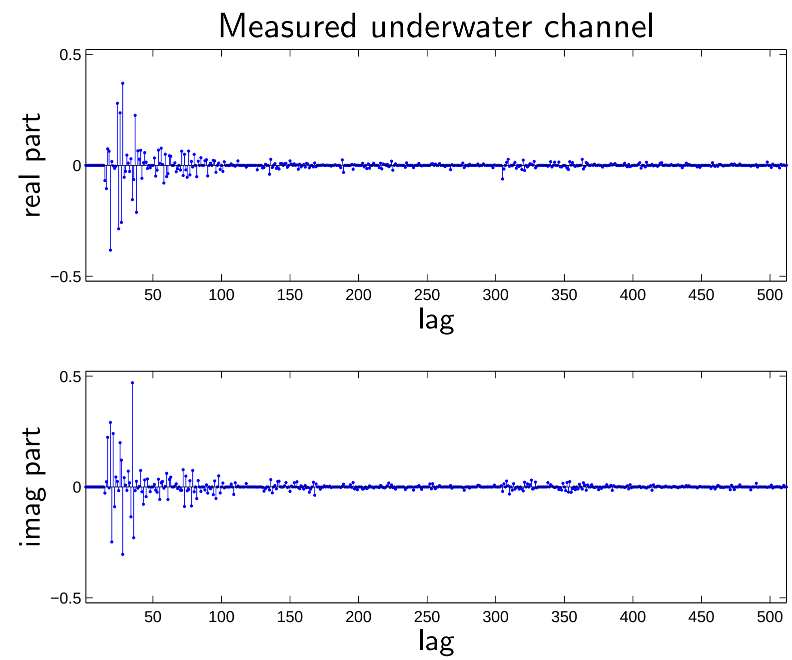
Sparse Channels:

- At large communication bandwidths, channel impulse responses are sparse.
- Below left shows channel taps $\boldsymbol{x} = [x_0, \ldots, x_{L-1}]$, where
  - $x_n = x(nT)$ for bandwidth $T^{-1} = 256$ MHz,
  - $x(t) = h(t) * p_{\text{RC}}(t)$, and
  - $h(t)$ is generated randomly using 802.15.4a outdoor NLOS specs.





Simplified Channel Model:

First, let's simplify things to talk concretely about sparse channels. . .

Consider a discrete-time channel that is

- block-fading with block size N,
- frequency-selective with impulse response length L (where $L < N$),
- sparse with S non-zero complex-Gaussian taps (where $0 < S \leq L$),

where *both the channel coefficients and support are unknown* to the receiver.

Important questions:

1. What is the capacity of this channel?
2. How can we build a practical comm system that operates near this capacity?

Noncoherent Capacity of the Sparse Channel:

For the unknown N-block-fading, L-length, S-sparse channel described earlier, we established [1] that

1. In the high-SNR regime, the ergodic capacity obeys

$$C_{\text{sparse}}(\mathsf{SNR}) = \frac{N-S}{N}\log(\mathsf{SNR}) + O(1).$$

2. To achieve the prelog factor $R_{\text{sparse}} = \frac{N-S}{N}$, it suffices to use
   - pilot-aided OFDM (with N subcarriers, of which S are pilots)
   - with *joint* channel estimation and data decoding.

Key points:

- The effect of *unknown channel support* manifests only in the $O(1)$ offset.
- Standard non-sparse-channel methods would use L pilots.
- "Compressed channel sensing" would use $S \operatorname{polylog} N$ pilots.

[1] A. Pachai-Kannu and P. Schniter, "On communication over unknown sparse frequency selective block-fading channels," *IEEE Trans. Info. Thy.*, Oct. 2011.

Practical Communication over the unknown Sparse Channel:

We now propose a communication scheme that. . .

- is practical, with decode complexity $O(N \log_2 N + N|\mathbb{S}|)$ per N-block,
- delivers outage rates matching the optimal prelog factor $R_{\text{sparse}} = \frac{N-S}{N}$,
- significantly outperforms "compressed channel sensing" (CCS) schemes.

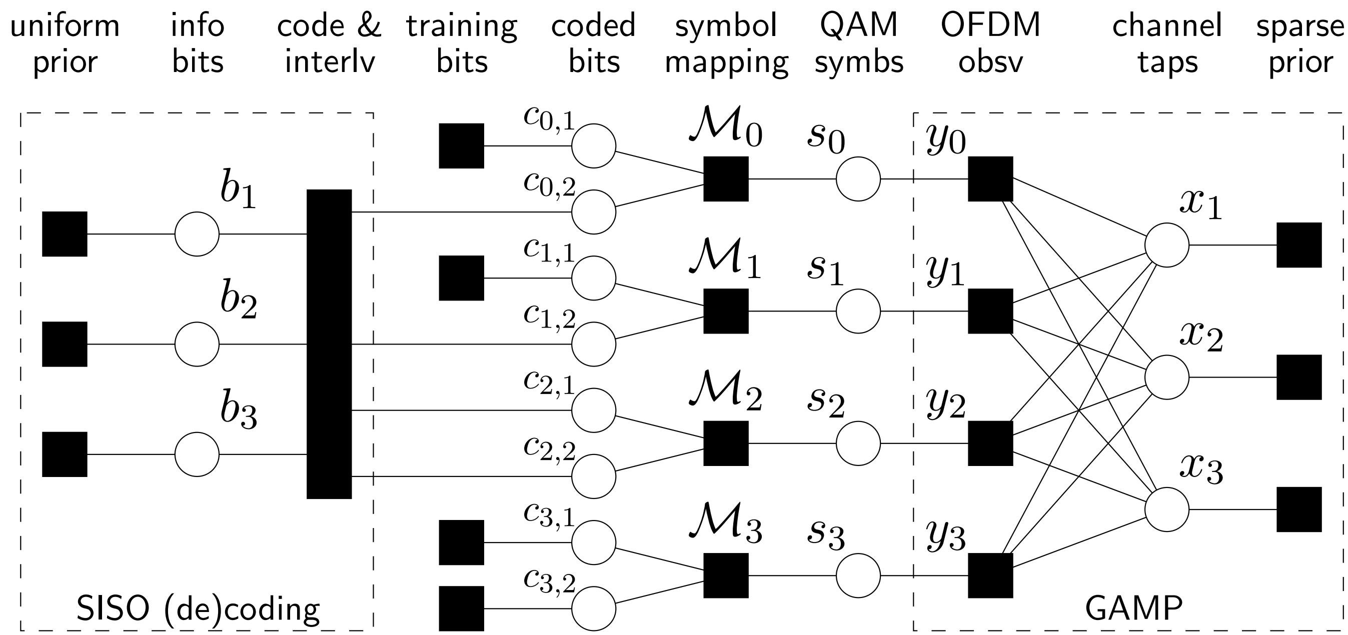
Our scheme uses. . .

- a conventional transmitter: pilot-aided BICM OFDM,
- a novel receiver: based on belief propagation with the generalized approximate message passing (GAMP) algorithm [3] used in a "turbo" configuration [2].

[2] P. Schniter, "'Turbo reconstruction of structured sparse signals," *CISS* 2010.

[3] S. Rangan, "Generalized approximate message passing for estimation with random linear mixing," *arXiv:1010.5141*, 2010.

Factor Graph for pilot-aided BICM-OFDM:



○ = random variable ■ = posterior factor

To jointly infer all random variables, we perform loopy-BP via the sum-product algorithm, using AMP approximations in the GAMP sub-graph.

Numerical Results — Perfectly Sparse Channel:

Transmitter:

- LDPC codewords with length ~ 10000 bits.
- 2^M-QAM with $2^M \in \{4, 16, 64, 256\}$ and multi-level Gray mapping.
- OFDM with $N = 1024$ subcarriers.
- P pilot subcarriers and/or T training MSBs.

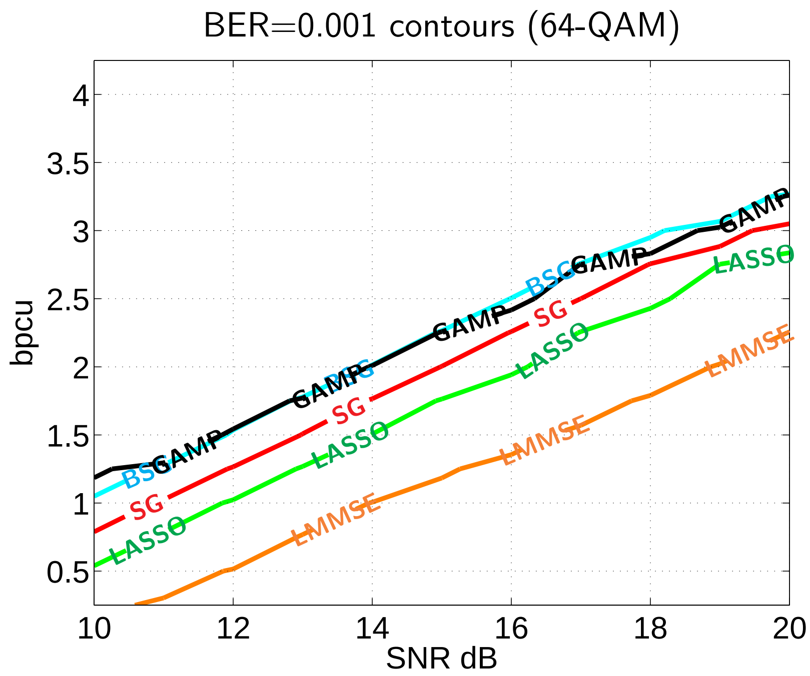
Channel:

- Length $L = 256 = N/4$.
- Sparsity $S = 64 = N/16$.

Reference Schemes:

- Pilot-aided LASSO (i.e., compressed channel sensing) with oracle tuning.
- Pilot-aided LMMSE, support-aware MMSE, and info-bit+support-aware MMSE channel estimates were also tested.

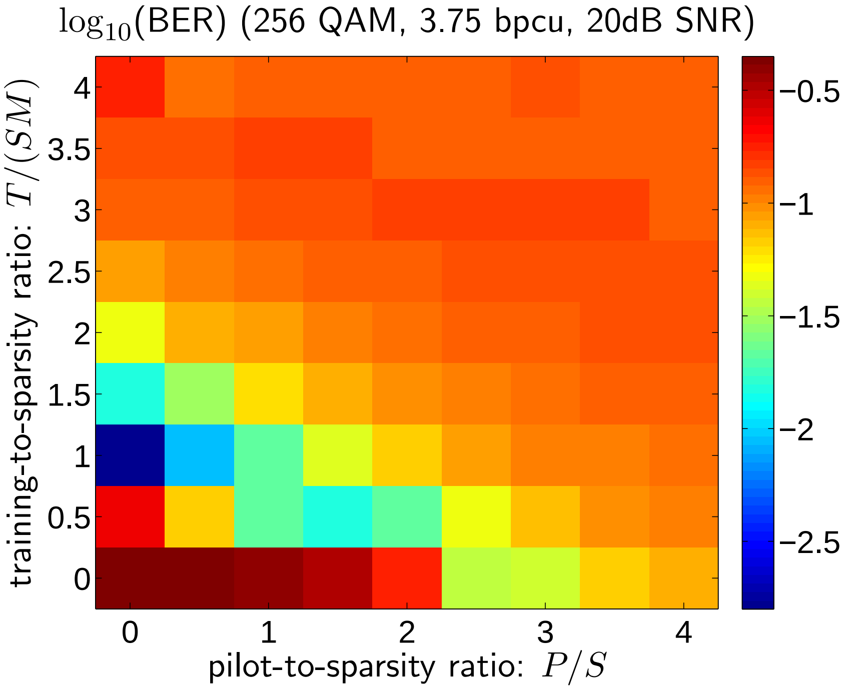
BER & Outage vs SNR (with $P = L$ pilots & $T = 0$ training MSBs):

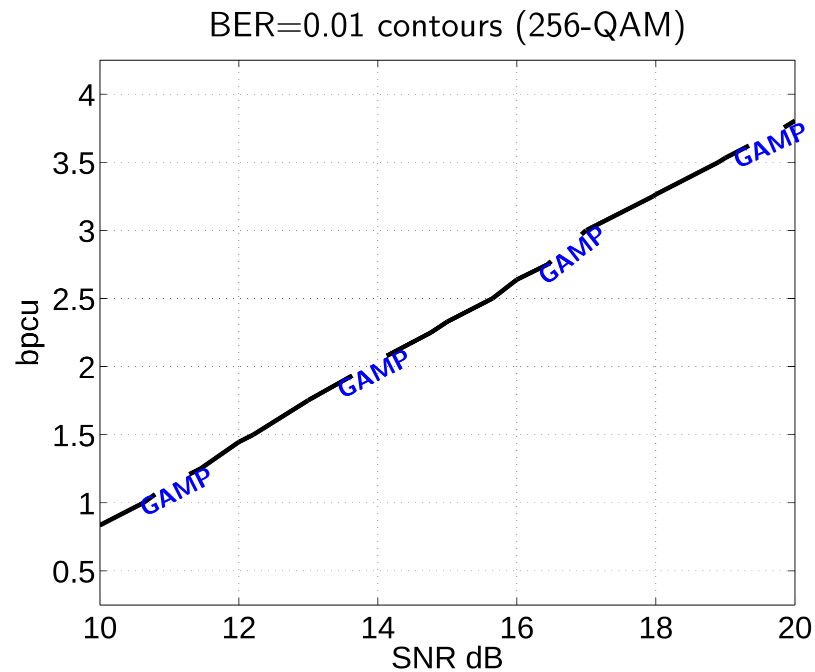


Key points:

- GAMP outperforms both LASSO and the support genie (SG).
- GAMP performs nearly as well as the info-bit+support-aware genie (BSG).
- With $P = L$, all approaches yield prelog factor $R = \frac{N-L}{N} = \frac{3}{4}$, which falls short of the optimal $R_{\text{sparse}} = \frac{N-S}{N} = \frac{15}{16}$.

BER & Outage vs SNR (with $P=0$ pilots & $T=SM$ training MSBs):



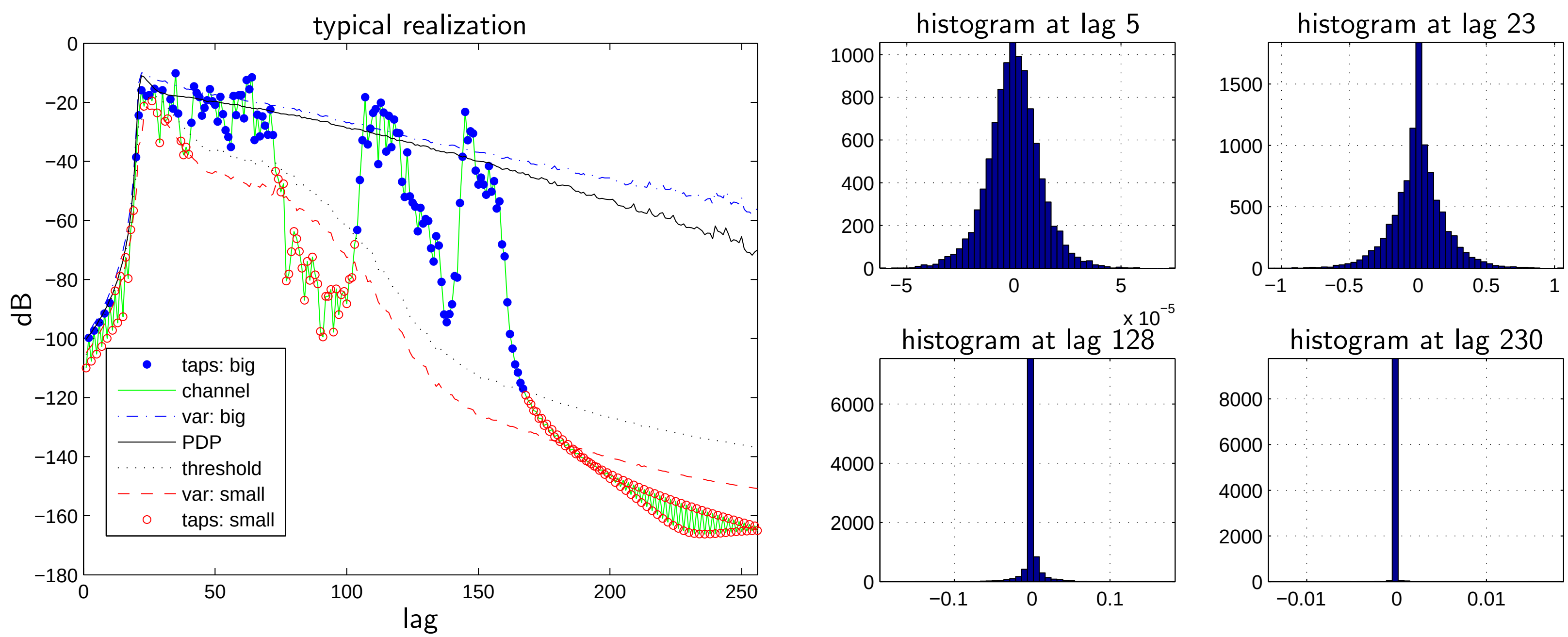


Key points:

- GAMP favors $P=0$ pilot subcarriers and $T=SM$ training MSBs.
  - Precisely the necc/suff redundancy of the capacity-maximizing system!
- GAMP achieves the sparse-channel's capacity-prelog factor, $R_{\text{sparse}} = \frac{N-S}{N}$.

In practice, channel taps are not perfectly sparse, nor i.i.d:

- For example, consider channel taps $\boldsymbol{x} = [x_0, \ldots, x_{L-1}]$, where
  - $x_n = x(nT)$ for bandwidth $T^{-1} = 256$ MHz,
  - $x(t) = h(t) * p_{\text{RC}}(t)$, and
  - $h(t)$ is generated randomly using 802.15.4a outdoor NLOS specs.



- The tap distribution *varies as the lag increases*, becoming more heavy-tailed.
- The big taps are *clustered together* in lag, as are the small ones.

Proposed channel model:

- Saleh-Valenzuela (e.g., 802.15.4a) models are accurate but difficult to exploit in receiver design.
- We propose a structured-sparse channel model based on a 2-state Gaussian Mixture model with discrete-Markov-chain structure on the state:

$$p(x_j \,|\, d_j) = \begin{cases} \mathcal{CN}(x_j; 0, \mu_j^0) & \text{if } d_j = 0 \quad \text{“small”} \\ \mathcal{CN}(x_j; 0, \mu_j^1) & \text{if } d_j = 1 \quad \text{“big”} \end{cases}$$

$$\Pr\{d_{j+1} = 1\} = p_j^{10} \Pr\{d_j = 0\} + (1 - p_j^{01}) \Pr\{d_j = 1\}$$

- Our model is parameterized by the lag-dependent quantities:

$\{\mu_j^1\}$: big-state power-delay profile

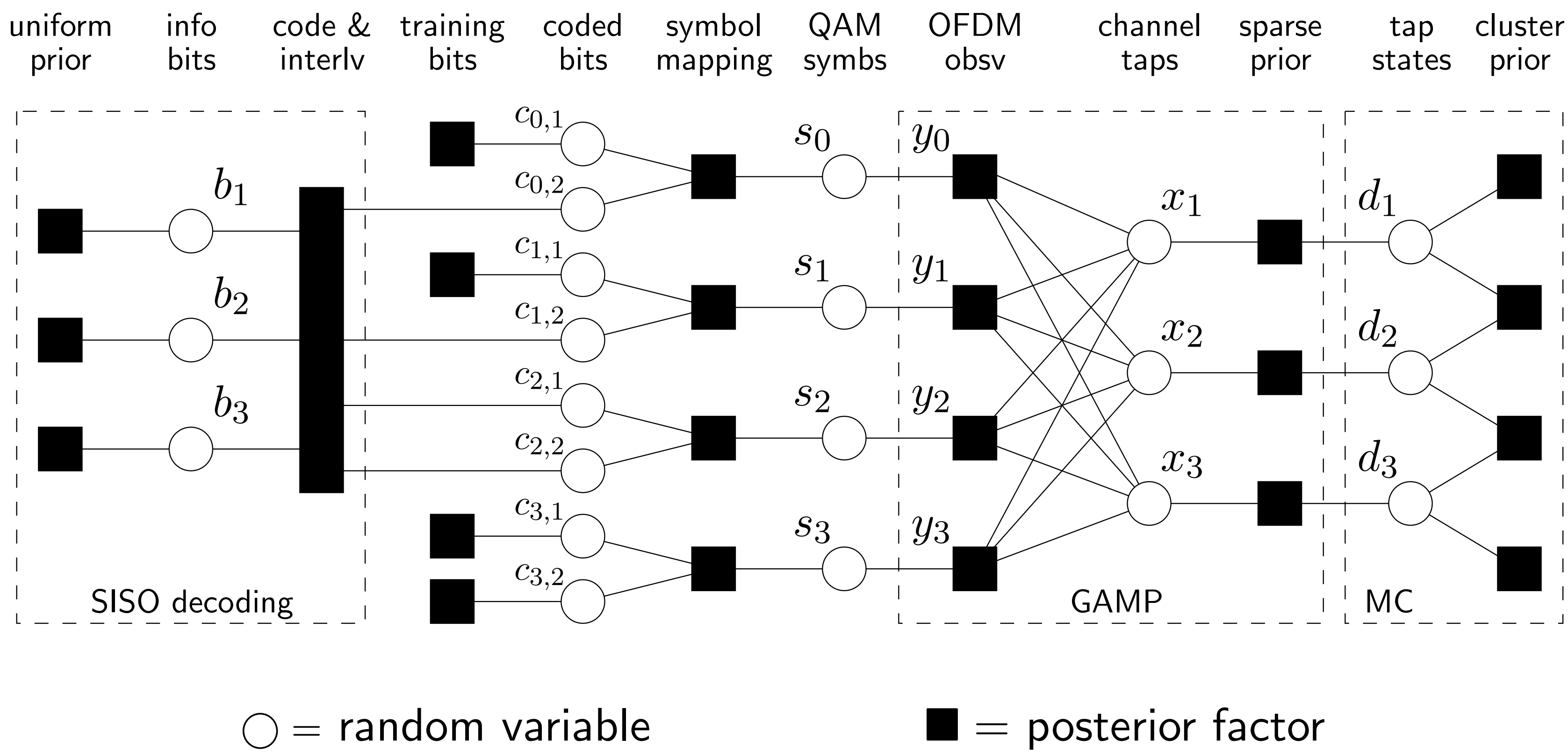
$\{\mu_j^0\}$: small-state power-delay profile

$\{p_j^{01}\}$: big-to-small transition probabilities

$\{p_j^{10}\}$: small-to-big transition probabilities

- Can learn these statistical params from observed realizations via the EM alg.

Factor graph for pilot-aided BICM-OFDM:



To jointly infer all random variables, we perform loopy-BP via the sum-product algorithm, using AMP approximations in the GAMP sub-graph.

Numerical results:

Transmitter:

- OFDM with $N = 1024$ subcarriers.
- 16-QAM with multi-level Gray mapping
- LDPC codewords with length ~ 10000 yielding spectral efficiency of 2 bpcu.
- P pilot subcarriers and T training MSBs.

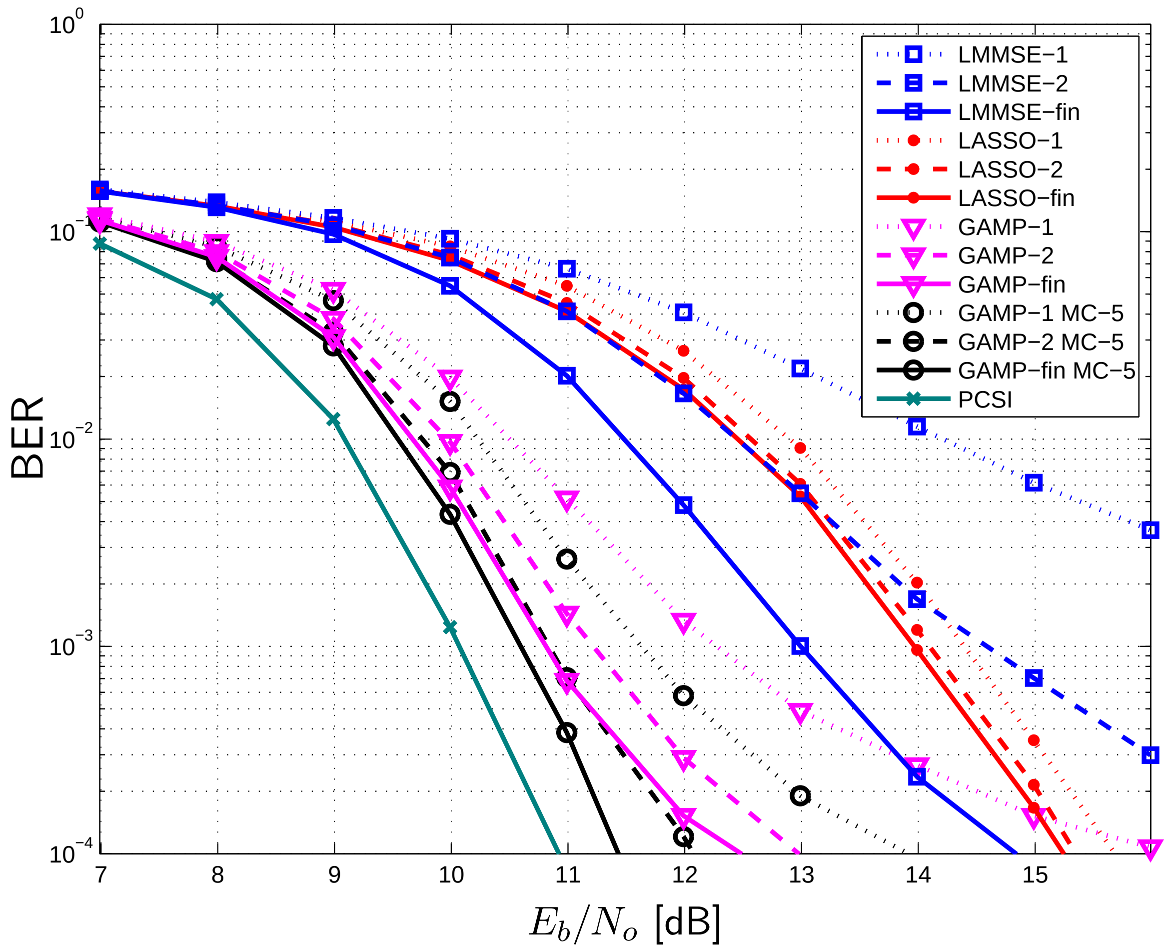
Channel:

- 802.15.4a outdoor-NLOS (not our Gaussian-mixture model!)
- Length $L = 256 = N/4$.

Reference Channel Estimation / Equalization Schemes:

- soft-input soft-output (SISO) versions of LMMSE and LASSO.
- perfect-CSI genie.

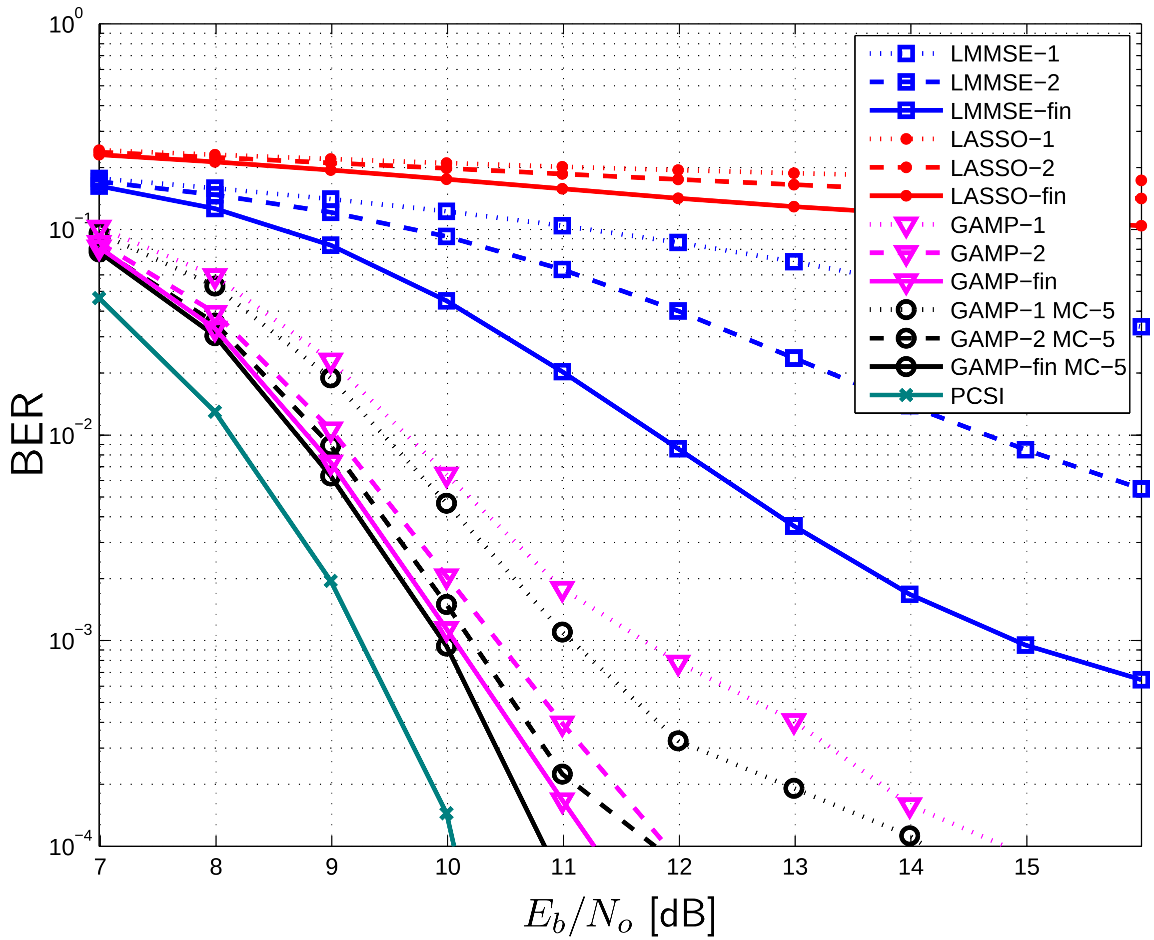
BER versus E_b/N_o for $P = 224$ pilots and $T = 0$ training MSBs:



Note 4dB improvement over (turbo) LASSO.

Only 0.5dB from perfect-CSI genie!

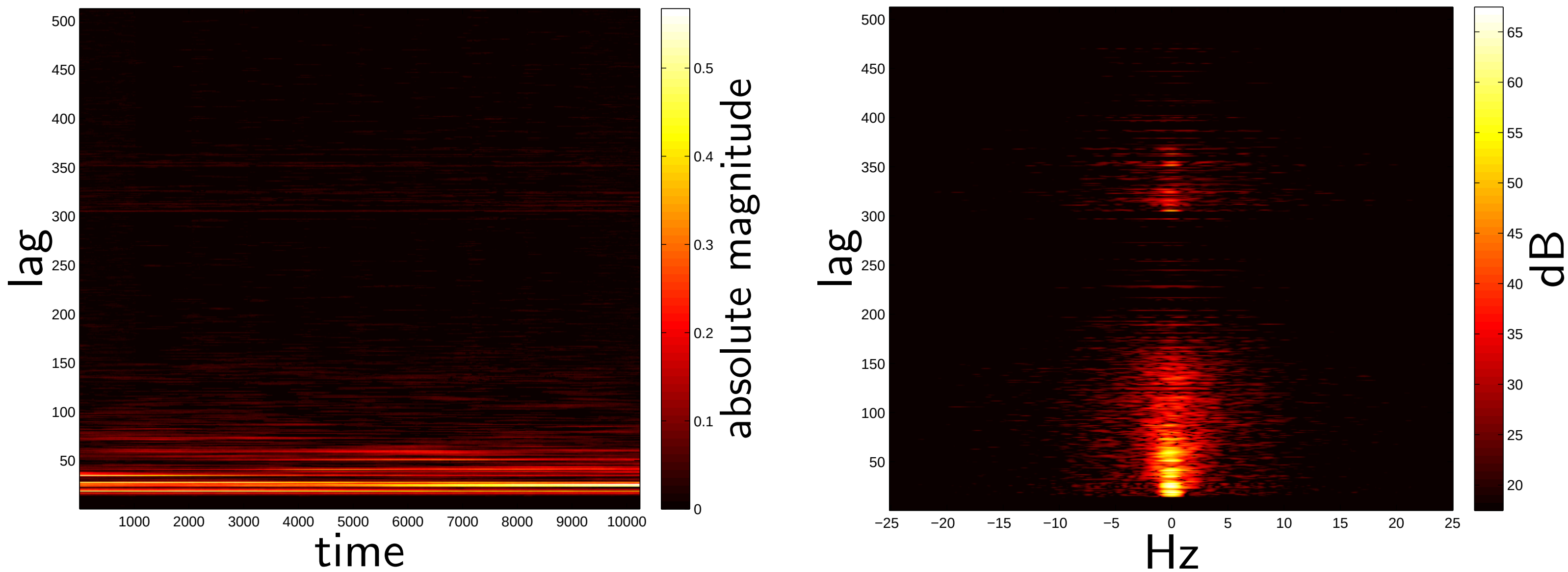
BER versus E_b/N_o for $P = 0$ pilots and $T = 448$ training MSBs:



Use of training MSBs gives 1dB improvement over use of pilot subcarriers!

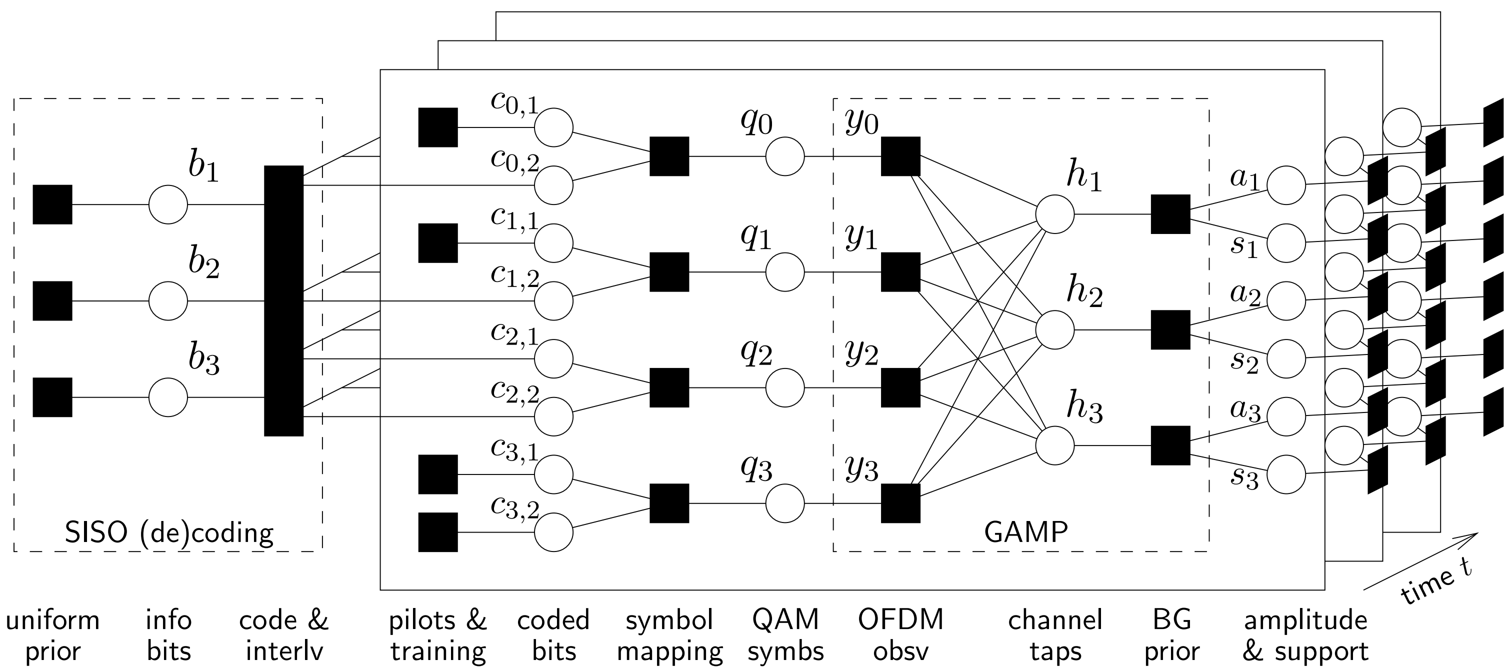
Communications over Underwater Channels:

- SPACE-08 Underwater Experiment `2920156F038_C0_S6`
- Time-varying channel response estimated using WHOI M-sequence:



- The channel is nearly over-spread: $f_d T_s L = 20 \times \frac{1}{10000} \times 400 = 0.8$!
- Can't afford to ignore structure of temporal variations!

BICM-OFDM Factor Graph with Temporal Channel Structure:



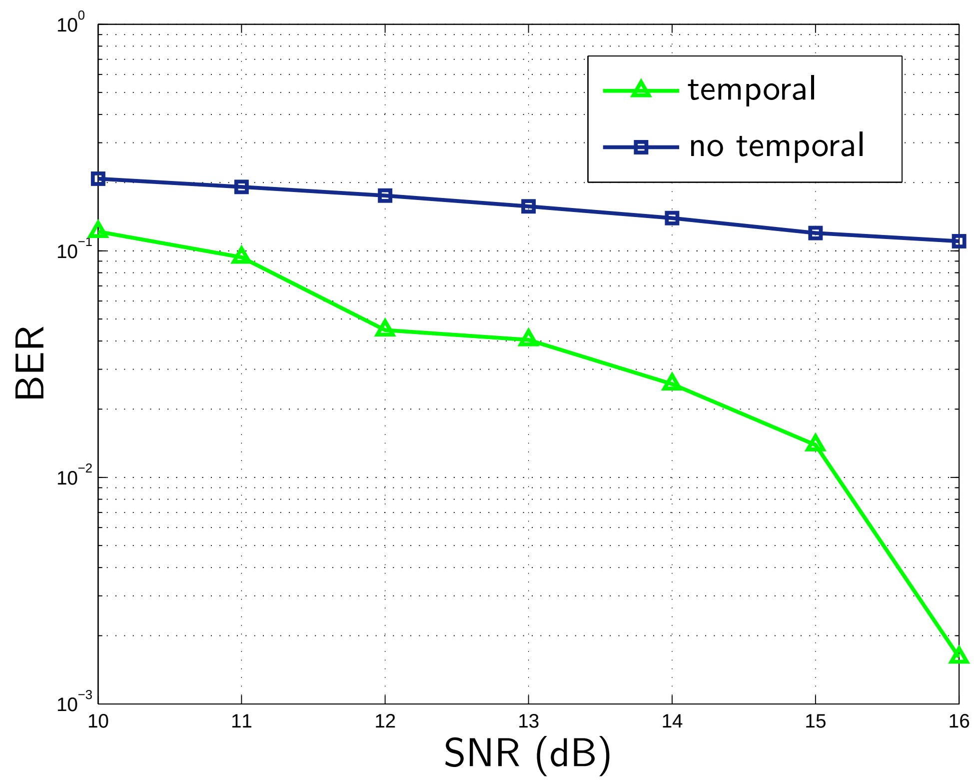
- Channel taps are modeled as independent Bernoulli-Gaussian processes:
  - each tap's amplitude follows a temporal Gauss-Markov chain
  - each tap's on/off state follows a temporal discrete-Markov chain

[4] P. Schniter and D. Meng, "A Message-Passing Receiver for BICM-OFDM over Unknown Time-Varying Sparse Channels," *Allerton* 2011.

Performance versus SNR:

Settings:

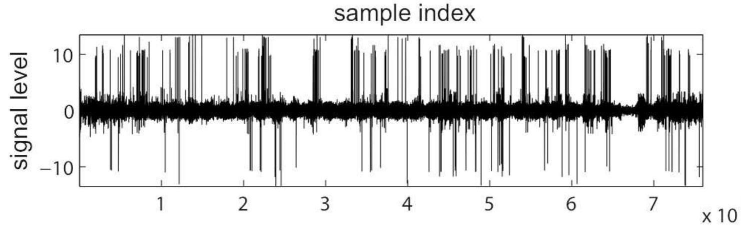
- experimentally measured underwater channel
- 16-QAM
- 1024 total tones
- 0 pilot tones
- 256 training MSBs
- LDPC length 10k
- LDPC rate 0.5

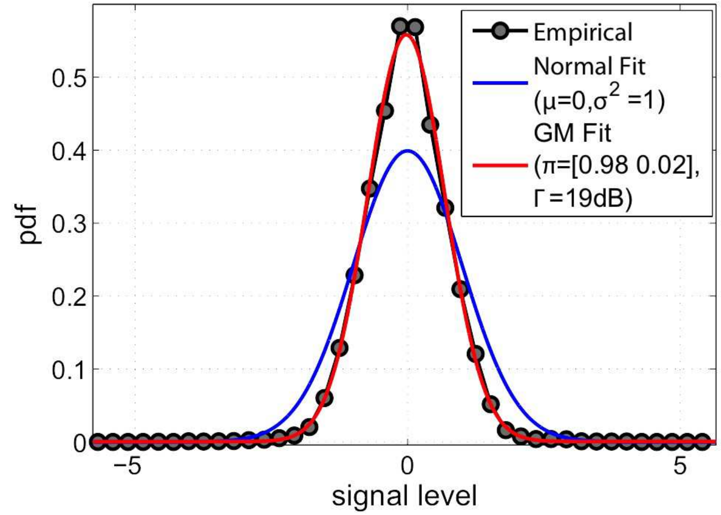


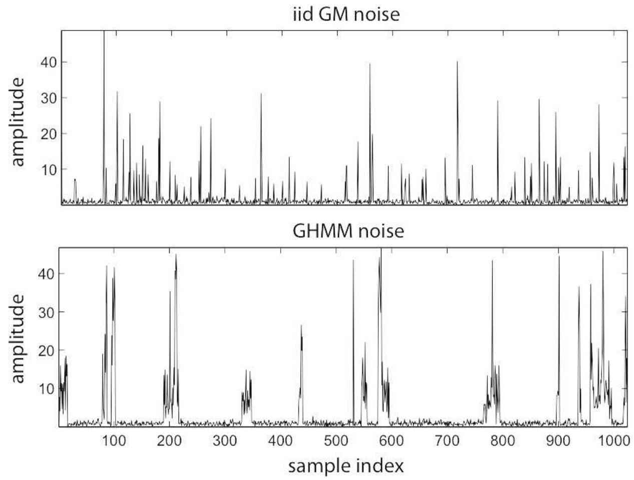
Exploiting the persistence in channel support and channel amplitudes was critical in this difficult underwater application.

Communications in Impulsive Noise:

- In many wireless and power-line communication systems, the (time-domain) noise is not Gaussian but impulsive.
- The marginal noise statistics are well captured by a 2-state Gaussian mixture (i.e., Middleton class-A) model.
- Noise burstiness is well captured by a discrete Markov chain on the noise state.







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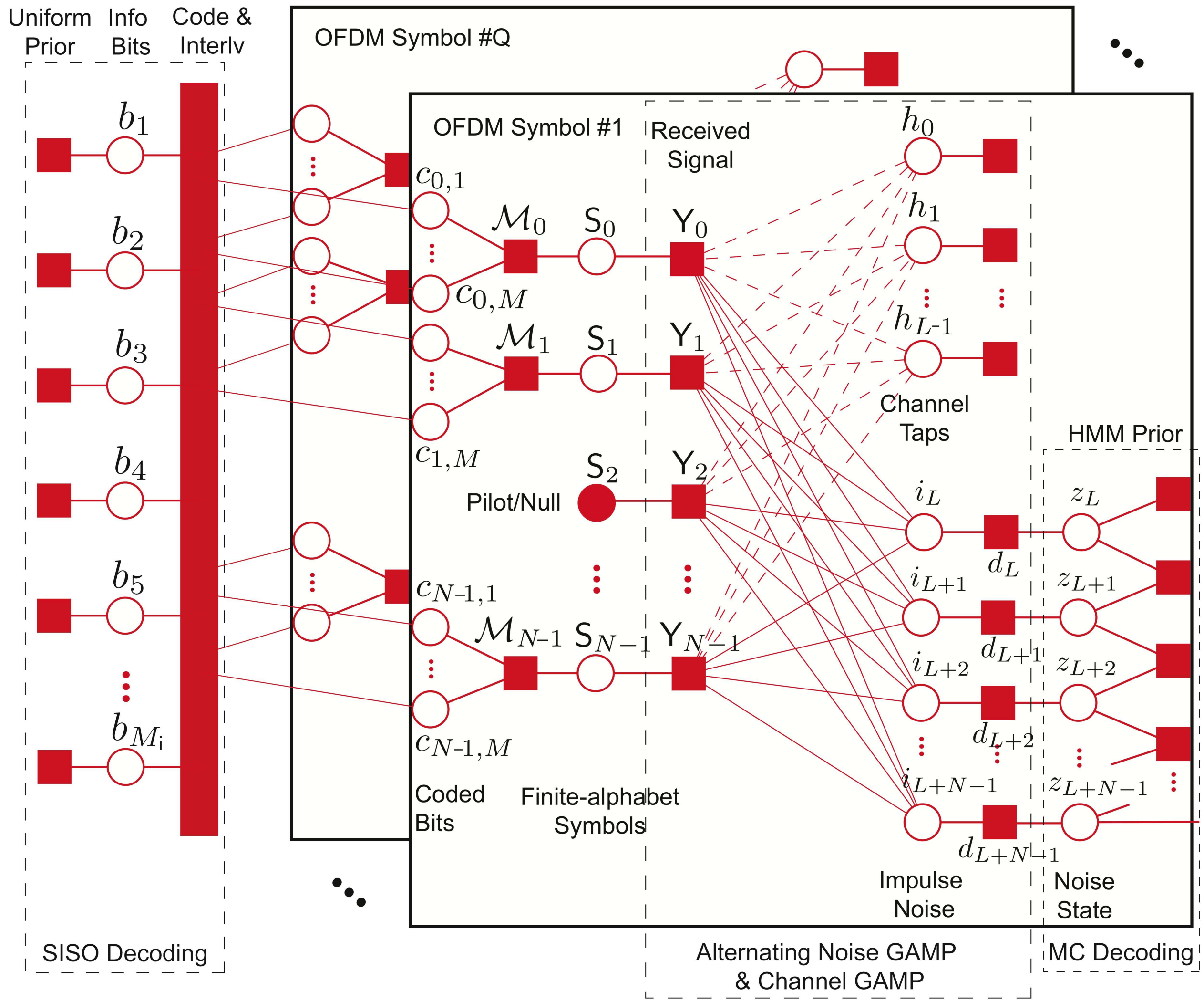
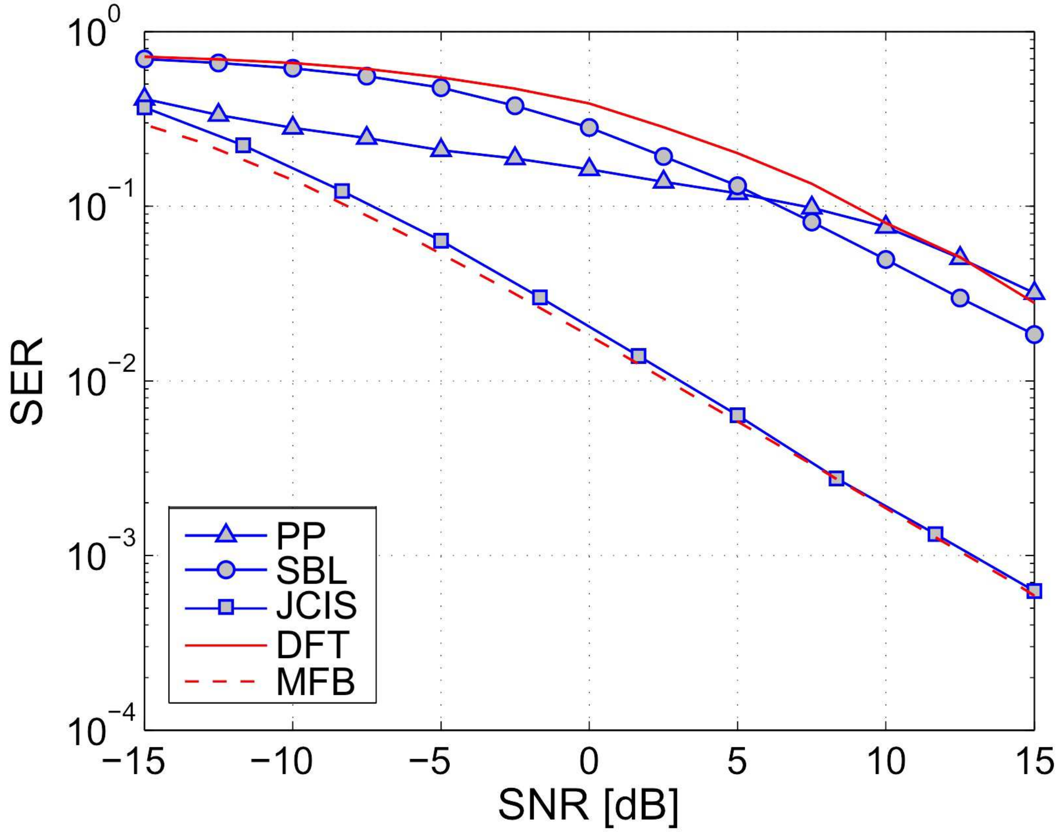


[5] M. Nassar, P. Schniter, and B. Evans, "A Factor-Graph Approach to Joint OFDM Channel Estimation and Decoding in Impulsive Noise Environments," *IEEE Trans. Signal Process.*, 2014.

Numerical Results — Uncoded Case:

Settings:

- 5 channel taps
- GM noise
- 256 total tones
- 15 pilot tones
- 80 null tones
- 4-QAM

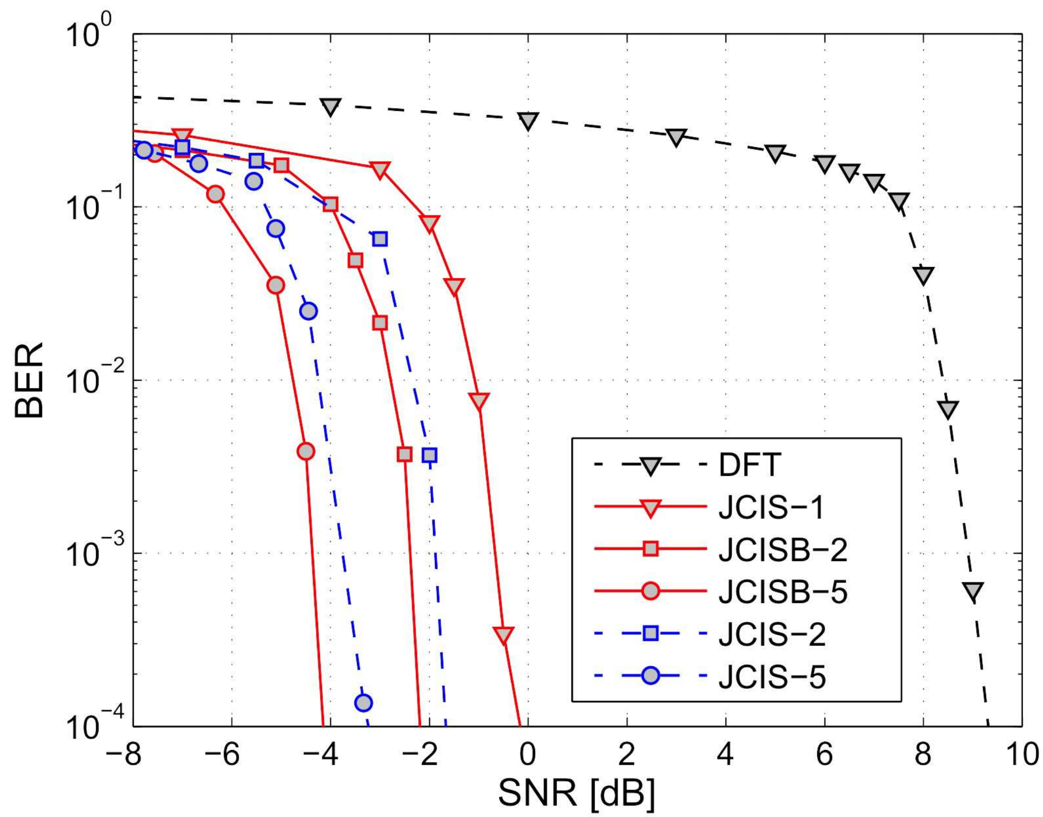


Proposed "joint channel/impulsive-noise/symbol" estimation (JCIS) scheme gives ∼15 dB gain over previous state-of-the-art and performs within 1 dB of MFB!

Numerical Results — Coded Case:

Settings:

- 10 channel taps
- GM noise
- 1024 total tones
- 150 pilot tones
- 0 null tones
- 16-QAM
- LDPC
- Rate 0.5
- Length 60k



Proposed "joint channel/impulsive-noise/symbol/bit" estimation (JCISB) scheme gives ∼15 dB gain over traditional DFT-based receiver!

Conclusions:

- At wide bandwidths, channel impulse responses are approximately sparse.
  - Sparsity increases the pre-log factor of high-SNR noncoherent ergodic capacity.
  - AMP-based joint channel-estimation/decoding delivers outage rates that empirically match the capacity pre-log factor.
  - Channels impulses are in fact structured-sparse, and exploiting this structure leads to additional performance gains.
  - Sparsity can also be exploited in time-varying channels.
- Impulsive noise is another source of sparsity in communications.
  - AMP-based joint channel-estimation/impulse-estimation/decoding delivers error-rates that approach the matched-filter bound.