Non-Coherent Multi-User Detection Based on Expectation Propagation

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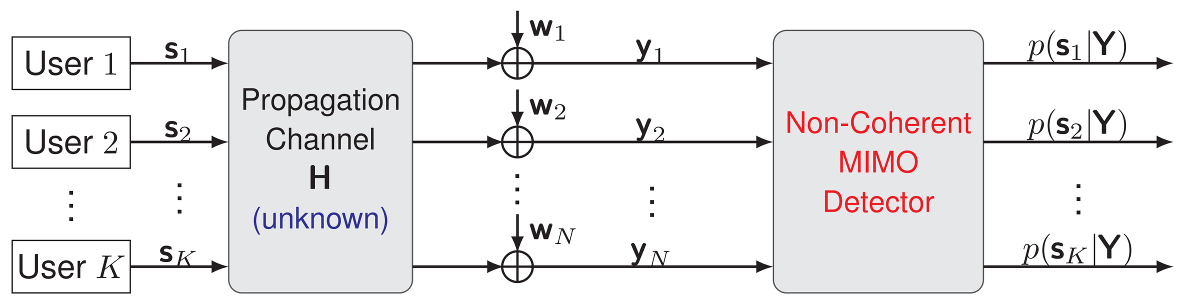
Background

- Coherent detection: the receiver knows channel state information (CSI) (e.g., from estimation)
- Non-coherent detection: the receiver knows only the channel statistics [1]
  → not so well investigated under general non-coherent codebook design assumption (no pilots)

In this work: Non-coherent soft multi-user detection with practical complexity.

System Model

- Multi-user SIMO: K single-antenna users, a N-antenna receiver



- Block fading channel, with coherence time T

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{s}_k \mathbf{h}_k^{\mathsf{T}} + \mathbf{W} = \mathbf{S}\mathbf{H}^{\mathsf{T}} + \mathbf{W}$$

  - Grassmannian modulation $\mathbf{S} = [\mathbf{s}_1 \dots \mathbf{s}_K] \in \mathbb{C}^{T\times K}$: $\mathbf{s}_k \in \mathcal{S}_k := \{\boldsymbol{s}_k^{(1)}, \dots, \boldsymbol{s}_k^{(|\mathcal{S}_k|)}\}$, $\|\mathbf{s}_k\| = 1$
  - Rayleigh fading $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_K] \in \mathbb{C}^{N\times K}$: $\mathbf{h}_k \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I}_N)$
  - Gaussian noise $\mathbf{W} \in \mathbb{C}^{T\times N}$: $\mathcal{N}(0,\sigma^2)$ independent entries

- Posterior marginalization: formidable for many users or large constellations

$$p(\mathbf{s}_k|\mathbf{Y}) = \sum_{\mathbf{s}_l \in \mathcal{S}_l, \forall l \neq k} p(\mathbf{S}|\mathbf{Y}), \quad \text{for } k \in [K]$$

$$p(\mathbf{S}|\mathbf{Y}) = \frac{p(\mathbf{Y}|\mathbf{S})p(\mathbf{S})}{p(\mathbf{Y})} \propto p(\mathbf{Y}|\mathbf{S})p(\mathbf{S})$$

Complexity: $O(K^3 2^{BK})$ with $T = O(K), N = O(K), |\mathcal{S}_k| = O(2^B), \forall k$.

- We seek a low-complexity approximation of the per-user marginals:

$$p(\mathbf{S}|\mathbf{Y}) \approx \hat{p}(\mathbf{S}|\mathbf{Y}) = \prod_{k=1}^{K} p(\mathbf{s}_k|\mathbf{Y}).$$

Expectation Propagation (EP) [2]

- $\mathbf{x}$: a set of variables with posterior $p(\mathbf{x}) \propto \prod_\alpha \psi_\alpha(\mathbf{x}_\alpha)$ for some partition $\{\mathbf{x}_\alpha\}$
- **Goal:** Estimate the marginalized posterior w.r.t. $\{\mathbf{x}_\beta\}$ as $p(\mathbf{x}) \approx \hat{p}(\mathbf{x}) = \prod_\beta \hat{p}_\beta(\mathbf{x}_\beta)$ where $\hat{p}_\beta(\mathbf{x}_\beta)$ belong to the exponential family

$$\prod_\alpha \psi_\alpha(\mathbf{x}_\alpha) \propto p(\mathbf{x}) \xleftarrow{\text{approximation in the exponential family}} \hat{p}(\mathbf{x})$$

$$\hat{p}(\mathbf{x}) \propto \prod_\alpha m_\alpha(\mathbf{x}_\alpha) = \prod_\alpha \prod_{\beta:\, \mathbf{x}_\beta \subset \mathbf{x}_\alpha} m_{\alpha,\beta}(\mathbf{x}_\beta) = \prod_\beta \prod_{\alpha:\, \mathbf{x}_\beta \subset \mathbf{x}_\alpha} m_{\alpha,\beta}(\mathbf{x}_\beta)$$

$$\hat{p}(\mathbf{x}) = \prod_\beta \hat{p}_\beta(\mathbf{x}_\beta) = \prod_\beta \prod_{\alpha:\, \mathbf{x}_\beta \subset \mathbf{x}_\alpha} \hat{p}_{\alpha,\beta}(\mathbf{x}_\beta)$$

moment matching

- EP algorithm:
  1. Initialize all $m_\alpha(\mathbf{x}_\alpha)$ and $\hat{p}_\beta(\mathbf{x}_\beta)$
  2. Iteratively update each m_α: $m_\alpha^{\text{new}}(\mathbf{x}_\alpha) = \frac{\hat{p}_\alpha^{\text{new}}(\mathbf{x}) m_\alpha(\mathbf{x}_\alpha)}{\hat{p}(\mathbf{x})}$, where

$$\hat{p}_\alpha^{\text{new}}(\mathbf{x}) = \arg\min_{\underline{p}\in\mathcal{P}} D(q_\alpha(\mathbf{x}) \,\|\, \underline{p}(\mathbf{x})) \quad \text{with} \quad q_\alpha(\mathbf{x}) := \frac{\hat{p}(\mathbf{x})\psi_\alpha(\mathbf{x}_\alpha)}{m_\alpha(\mathbf{x}_\alpha)}.$$

  $\hat{p}_\alpha^{\text{new}}(\mathbf{x})$ can be found by moment matching.

EP-Based Detection: Formulation

- Let $\mathbf{s}_k = \boldsymbol{s}_k^{(\mathrm{i}_k)}$ with $\mathrm{i}_k \in [|\mathcal{S}_k|]$, we estimate $p(\mathrm{i}_k|\mathbf{Y})$

$$\mathbf{y} := \mathrm{vec}(\mathbf{Y}^{\mathsf{T}}) = \sum_{k=1}^{K} \underbrace{(\boldsymbol{s}_k^{(\mathrm{i}_k)} \otimes \boldsymbol{I}_N)\, \mathbf{h}_k}_{=:\mathbf{z}_k} + \underbrace{\mathrm{vec}(\mathbf{W}^{\mathsf{T}})}_{=:\mathbf{w}}$$

- With $\mathbf{z} := [\mathbf{z}_1^{\mathsf{T}}, \dots, \mathbf{z}_K^{\mathsf{T}}]^{\mathsf{T}}$ and $\mathbf{i} := [\mathrm{i}_1, \dots, \mathrm{i}_k]^{\mathsf{T}}$,

$$p(\mathbf{i},\mathbf{z}|\mathbf{y}) \propto p(\mathbf{i},\mathbf{z},\mathbf{y}) = p(\mathbf{y}|\mathbf{z})p(\mathbf{z}|\mathbf{i})p(\mathbf{i}) = p(\mathbf{y}|\mathbf{z}) \left[\prod_{k=1}^{K} p(\mathbf{z}_k|\mathrm{i}_k)\right] \left[\prod_{k=1}^{K} p(\mathrm{i}_k)\right]$$

$$= \psi_0(\mathbf{z}_1,\dots,\mathbf{z}_K) \left[\prod_{k=1}^{K} \psi_{k1}(\mathbf{z}_k,\mathrm{i}_k)\right] \left[\prod_{k=1}^{K} \psi_{k2}(\mathrm{i}_k)\right]$$

approximation in the exponential family

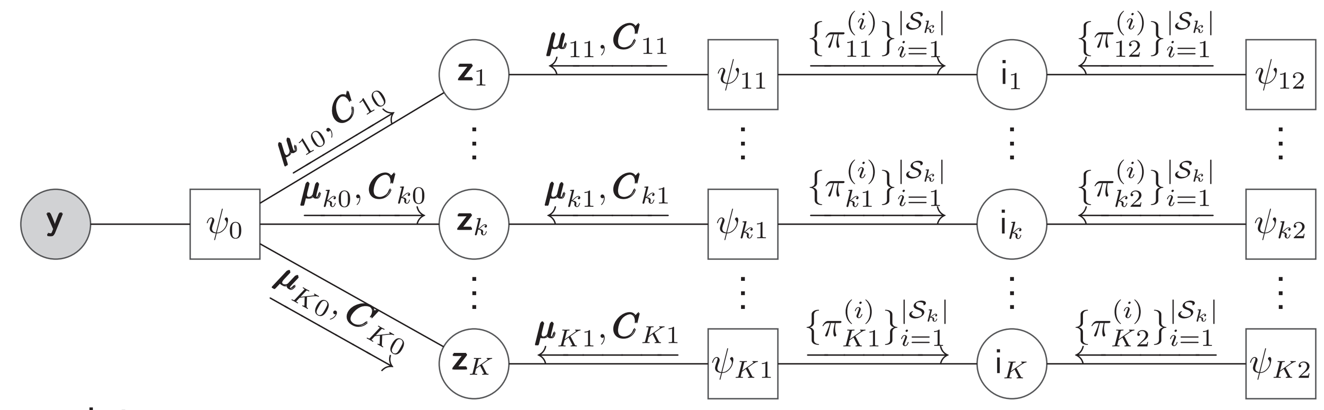
$$\hat{p}(\mathbf{i},\mathbf{z}|\mathbf{y}) \propto \underbrace{m_0(\mathbf{z}_1,\dots,\mathbf{z}_K)}_{\propto \prod_{k=1}^{K} \mathcal{N}(\mathbf{z}_k;\boldsymbol{\mu}_{k0},\boldsymbol{C}_{k0})} \left[\prod_{k=1}^{K} \underbrace{m_{k1}(\mathbf{z}_k,\mathrm{i}_k)}_{\propto \mathcal{N}(\mathbf{z}_k;\boldsymbol{\mu}_{k1},\boldsymbol{C}_{k1})\pi_{k1}^{(\mathrm{i}_k)}}\right] \left[\prod_{k=1}^{K} \underbrace{m_{k2}(\mathrm{i}_k)}_{=\pi_{k2}^{(\mathrm{i}_k)}}\right]$$

$$= \prod_{k=1}^{K} \mathcal{N}(\mathbf{z}_k;\boldsymbol{\mu}_{k0},\boldsymbol{C}_{k0})\mathcal{N}(\mathbf{z}_k;\boldsymbol{\mu}_{k1},\boldsymbol{C}_{k1})\pi_{k1}^{(\mathrm{i}_k)}\pi_{k2}^{(\mathrm{i}_k)}$$

moment matching

$$\hat{p}(\mathbf{i},\mathbf{z}|\mathbf{y}) = \prod_{k=1}^{K} \hat{p}_k(\mathbf{z}_k)\hat{p}_k(\mathrm{i}_k) = \prod_{k=1}^{K} \mathcal{N}(\mathbf{z}_k;\hat{\boldsymbol{z}}_k,\boldsymbol{\Sigma}_k)\, \hat{\pi}_k^{(\mathrm{i}_k)}$$

EP-Based Detection: Message Update



- Message update

$$\pi_{k2}^{(\mathrm{i}_k)} = \frac{1}{|\mathcal{S}_k|}, \quad \mathrm{i}_k \in [|\mathcal{S}_k|] \tag{1}$$

$$\pi_{k1}^{(\mathrm{i}_k)} = \frac{\mathcal{N}(\mathbf{0};\boldsymbol{\mu}_{k0}, (\boldsymbol{s}_k^{(\mathrm{i}_k)}\boldsymbol{s}_k^{(\mathrm{i}_k)\mathsf{H}}) \otimes \boldsymbol{I}_N + \boldsymbol{C}_{k0})}{\sum_{i=1}^{|\mathcal{S}_k|} \mathcal{N}(\mathbf{0};\boldsymbol{\mu}_{k0}, (\boldsymbol{s}_k^{(i)}\boldsymbol{s}_k^{(i)\mathsf{H}}) \otimes \boldsymbol{I}_N + \boldsymbol{C}_{k0})}, \quad \mathrm{i}_k \in [|\mathcal{S}_k|] \quad \triangleright \text{ sought posterior} \tag{2}$$

$$\boldsymbol{C}_{k1} = (\boldsymbol{\Sigma}_k^{-1} - \boldsymbol{C}_{k0}^{-1})^{-1}, \; \boldsymbol{\mu}_{k1} = \boldsymbol{C}_{k1}(\boldsymbol{\Sigma}_k^{-1}\hat{\boldsymbol{z}}_k - \boldsymbol{C}_{k0}^{-1}\boldsymbol{\mu}_{k0}), \tag{3}$$

where $\hat{\boldsymbol{z}}_k = \sum_{i=1}^{|\mathcal{S}_k|} \pi_{k1}^{(i)} \hat{\boldsymbol{z}}_{ki}$, $\boldsymbol{\Sigma}_k = \sum_{i=1}^{|\mathcal{S}_k|} \pi_{k1}^{(i)} (\hat{\boldsymbol{z}}_{ki}\hat{\boldsymbol{z}}_{ki}^{\mathsf{H}} + \boldsymbol{\Sigma}_{ki}) - \hat{\boldsymbol{z}}_k\hat{\boldsymbol{z}}_k^{\mathsf{H}}$

$$\boldsymbol{\Sigma}_{ki} = \left([(\boldsymbol{s}_k^{(i)}\boldsymbol{s}_k^{(i)\mathsf{H}}) \otimes \boldsymbol{I}_N]^{-1} + \boldsymbol{C}_{k0}^{-1}\right)^{-1}, \quad \hat{\boldsymbol{z}}_{ki} = \boldsymbol{\Sigma}_{ki}\boldsymbol{C}_{k0}^{-1}\boldsymbol{\mu}_{k0}$$

$$\boldsymbol{C}_{k0} = \sigma^2 \boldsymbol{I}_{NT} + \sum_{j\neq k} \boldsymbol{C}_{j1}, \quad \boldsymbol{\mu}_{k0} = \mathbf{y} - \sum_{j \neq k} \boldsymbol{\mu}_{j1} \tag{4}$$

- Message initialization with the prior: $\pi_{k1}^{(i)} = \frac{1}{|\mathcal{S}_k|}$, $i \in [|\mathcal{S}_k|]$; $\boldsymbol{C}_{k1} = \frac{1}{|\mathcal{S}_k|} \sum_{i=1}^{|\mathcal{S}_k|} (\boldsymbol{s}_k^{(i)}\boldsymbol{s}_k^{(i)\mathsf{H}}) \otimes \boldsymbol{I}_N$; $\boldsymbol{\mu}_{k1} = \mathbf{0}$; $\boldsymbol{C}_{k0} = \sigma^2 \boldsymbol{I}_{NT} + \sum_{j\neq k} \frac{1}{|\mathcal{S}_j|} \sum_{i=1}^{|\mathcal{S}_j|} (\boldsymbol{s}_j^{(i)}\boldsymbol{s}_j^{(i)\mathsf{H}}) \otimes \boldsymbol{I}_N$; $\boldsymbol{\mu}_{k0} = \mathbf{y}$.
- Complexity: $O(K^7 2^B n_{\text{iterations}})$

MMSE-SIC: A Simplification of EP

- Approximate (3) by $\boldsymbol{C}_{k1} = \sum_{i=1}^{|\mathcal{S}_k|} \pi_{k1}^{(i)} (\mathbf{s}_k^{(i)}\mathbf{s}_k^{(i)\mathsf{H}}) \otimes \boldsymbol{I}_N$, $\boldsymbol{\mu}_{k1} = \mathbf{0}$, then (2) becomes

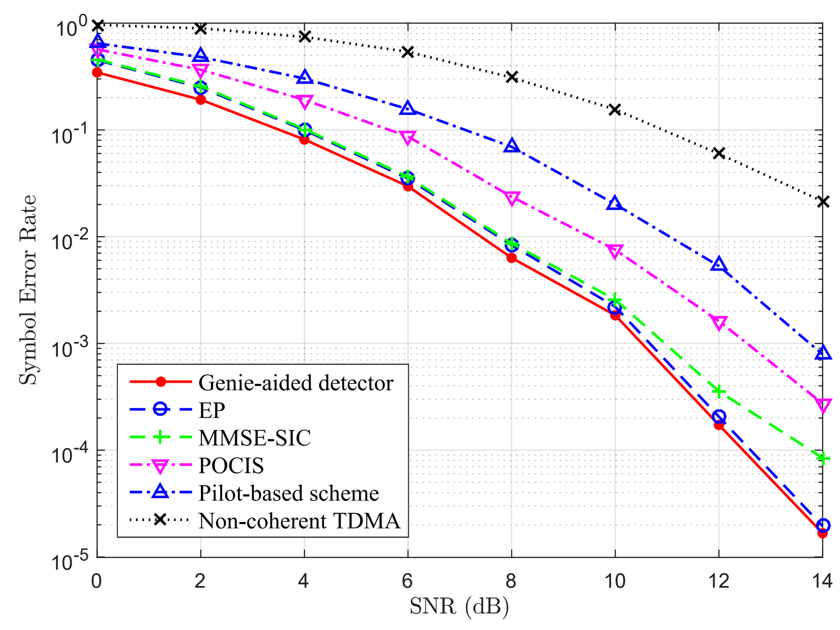
$$\pi_{k1}^{(\mathrm{i}_k)} = \frac{\mathcal{N}\left(\mathbf{0};\mathbf{y}, (\boldsymbol{s}_k^{(\mathrm{i}_k)}\boldsymbol{s}_k^{(\mathrm{i}_k)\mathsf{H}} + \boldsymbol{Q}_k) \otimes \boldsymbol{I}_N\right)}{\sum_{i=1}^{|\mathcal{S}_k|} \mathcal{N}\left(\mathbf{0};\mathbf{y}, (\boldsymbol{s}_k^{(i)}\boldsymbol{s}_k^{(i)\mathsf{H}} + \boldsymbol{Q}_k) \otimes \boldsymbol{I}_N\right)}, \quad \mathrm{i}_k \in [|\mathcal{S}_k|], \tag{5}$$

with $\boldsymbol{Q}_k := \sum_{l\neq k} \boldsymbol{R}_l + \sigma^2 \boldsymbol{I}_T$, $\quad \boldsymbol{R}_k := \sum_{i=1}^{|\mathcal{S}_k|} \pi_{k1}^{(i)} \mathbf{s}_k^{(i)}\mathbf{s}_k^{(i)\mathsf{H}}$

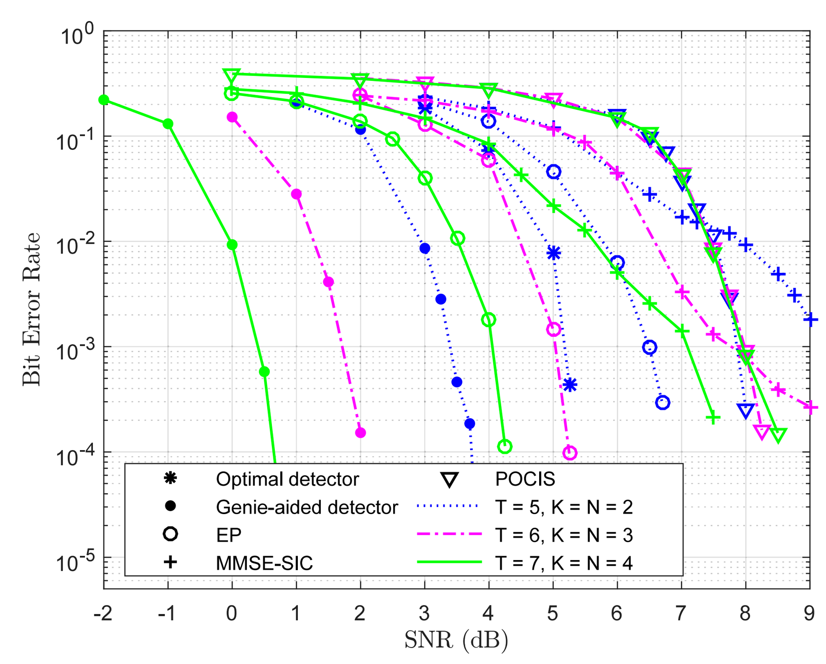
- Interpretation as MMSE-SIC: $\mathbf{y} = (\mathbf{s}_k \otimes \boldsymbol{I}_N)\mathbf{h}_k + \sum_{l\neq k} \mathbf{z}_l + \mathbf{w}$
  - Treat interference as Gaussian: $\sum_{l\neq k} \mathbf{z}_l + \mathbf{w} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}_k \otimes \boldsymbol{I}_N)$
  - MMSE estimate: $\hat{p}(\mathbf{y}|\mathbf{s}_k) = \mathcal{N}(\mathbf{y}; \mathbf{0}, (\mathbf{s}_k\mathbf{s}_k^{\mathsf{H}} + \boldsymbol{Q}_k) \otimes \boldsymbol{I}_N)$, then $\hat{p}(\mathbf{s}_k|\mathbf{y})$ coincides with (5)
  - Successively update $\boldsymbol{R}_k$ and $\boldsymbol{Q}_k$
- Complexity: $O((K^4 + K^3 2^B) n_{\text{iterations}})$

Performance

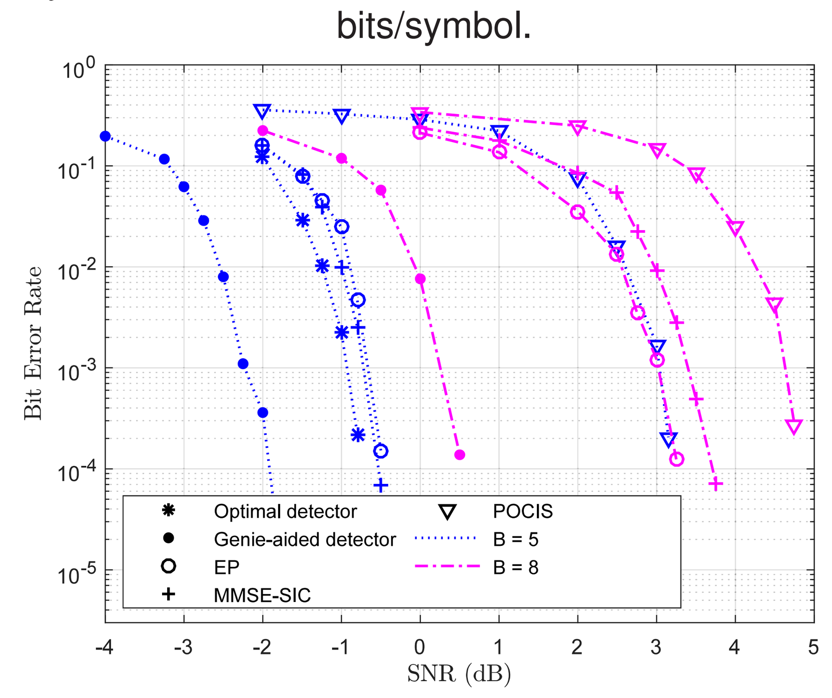
- Test codebook: Grassmannian codebook from [3], no pilots
- 20 iterations, with damping
- Baselines: POCIS detector [3], pilot-based scheme, non-coherent TDMA
- Benchmarks:
  - Optimal ML detector (for small K)
  - Genie-aided detector: give the interfering signals (but not the channel) to the receiver
- LTE turbo code: rate $1/3$, 1008 bits/packet, 10 decoding iterations



Symbol error rate for $T = 6$, $K = 3$, $N = 8$, $B = 8$ bits/symbol.



Coded bit error rate for $B = 8$ bits/symbol, $K = N$.



Coded bit error rate for $T = 6$, $K = 3$, and $N = 4$.

References

Long version: K.-H. Ngo, M. Guillaud, A. Decurninge, S. Yang, and P. Schniter, "Multi-user detection based on expectation propagation for the non-coherent SIMO multiple access channel," *sumitted to IEEE Trans. Wireless Commun.*, 2019, (preprint: https://arxiv.org/pdf/1905.11152.pdf).

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