HOMEWORK ASSIGNMENT #1

Due Fri. Apr. 11, 2008 (in class)

1. Say that complex-baseband signal $x_z(t)$ is up-converted using the standard quadrature modulator illustrated on page 3 of the review slides, and then down-converted using the standard quadrature demodulator illustrated on page 4, except that the phase of the demodulator oscillator is offset by $\theta(t)$ radians. In other words, at the demodulator, $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are replaced by $\cos(2\pi f_c t + \theta(t))$ and $\sin(2\pi f_c t + \theta(t))$, respectively.

   (a) Using $y_z(t)$ to denote the complex-valued demodulator output signal, show that $y_z(t) = x_z(t)\exp(-j\theta(t))$.

   (b) When $\theta(t) = 2\pi f_o t$, write the output energy spectrum $G_{Y_z}(f)$ in terms of the input energy spectrum $G_{X_z}(f)$.

2. Say that real-valued zero-mean white Gaussian noise $W(t)$ with PSD $S_W(f) = N_0/2$ is filtered by an LTI filter with Fourier transform

$$H(f) \;=\; \begin{cases} 2 & |f| \le 10 \\ 0 & \text{elsewhere} \end{cases}$$

   to produce the output $N(t)$.

   (a) What is $R_W(\tau)$, the autocorrelation of $W(t)$?

   (b) What is $\mathrm{E}\{N(t)\}$?

   (c) Give an expression for $S_N(f)$, the output PSD.

   (d) What is $\mathrm{E}\{N^2(t)\}$?

   (e) Give the pdf of one sample of $N(t)$, e.g., $f_{N(t_1)}(n)$.

   (f) Write $\Pr\{N(t_1) > 3\}$ in terms of $\mathrm{erf}(\cdot)$.

3. Say that a real-valued zero-mean white Gaussian noise $W(t)$ with PSD $S_W(f) = N_0/2$ is filtered by an ideal lowpass filter with bandwidth B_T to produce the output $N(t)$. Denote the impulse response of the lowpass filter by $h_R(t)$.

   (a) What is $R_N(\tau)$?

   (b) For what B_T does $\mathrm{E}\{N^2(t)\} = 1$?

   (c) Find $R_Z(\tau)$ for the random process $Z(t) = N(t) - N(t - t_1)$.

   (d) Assuming the value of B_T from part (b), which value of t_1 makes $\mathrm{E}\{Z^2(t)\} = 0.5$?

4. Say that real-valued zero-mean white Gaussian noise $W(t)$ with PSD $S_W(f) = N_0/2 = 0.005$ is filtered by LTI bandpass filter $H_R(f)$ to produce $N_c(t)$. Then, $N_c(t)$ is input to the standard quadrature demodulator (see on page 4 of review slides) to produce $N_z(t) = N_I(t) + jN_Q(t)$. Say that the power spectrum of the complex baseband output is known to be

$$S_{N_z}(f) \;=\; \begin{cases} 0.25 & |f| \le 100 \\ 0 & \text{elsewhere} \end{cases}.$$

   (a) Find $\mathrm{E}\{|N_z(t)|^2\}$.

   (b) Find $S_{N_I}(f)$ and $S_{N_I N_Q}(f)$.

   (c) What is $|H_R(f)|$?

   (d) Give the pdf of one sample of $N_I(t)$, e.g., $f_{N_I(t_1)}(n)$.

   (e) Write $\Pr\{N_I(t) > 4\}$ in terms of $\mathrm{erf}(\cdot)$.