HOMEWORK ASSIGNMENT #2

Due Thurs. Sept. 30, 1999 (in class)

1. *Optimal Predictor Design:*
   In this problem you will experiment with linear predictor design.

   (a) Consider input $x(n)$ characterized by the model $x(n) = \sum_{i=0}^{N_b} b_i v(n-i)$, where $v(n)$ is zero-mean white Gaussian noise with variance σ_v^2. Derive an expression for the autocorrelation $r_x(k)$ in terms of b_i and σ_v^2.

   (b) Prove that the power spectrum $S_x(e^{j\omega}) = \sum_{k=-\infty}^{\infty} r_x(k)e^{-j\omega k}$ can be written

$$S_x(e^{j\omega}) = -r_x(0) + 2\sum_{k=0}^{\infty} r_x(k)\cos(\omega k). \qquad (1)$$

   (From (1), it should be evident that $S_x(e^{j\omega})$ is real-valued and symmetric.) Assuming $\sigma_v^2 = 1$ and $\{b_i\}$ in the table below, plot $r_x(k)$ and $S_x(e^{j\omega})$ calculated using (1).

b_0	b_1	b_2	b_3
1	0.9	0.8	0.7

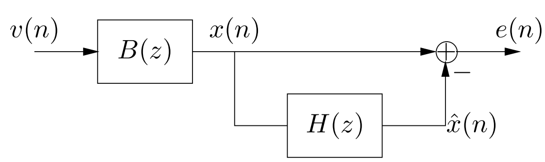
   (c) Calculate the spectral flatness measure

$$\text{SFM}_x = \frac{\exp\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln S_x(e^{j\omega})d\omega\right)}{\frac{1}{2\pi}\int_{-\pi}^{\pi} S_x(e^{j\omega})d\omega}$$

   and $\sigma_e^2\big|_{\min}$ as predictor length N goes to infinity. (Hint: use a Riemann approximation of the integral—do *not* use the symbolic `int` command.) How does $\sigma_e^2\big|_{\min}$ compare to σ_v^2?

   (d) Calculate prediction coefficients and $\sigma_e^2\big|_{\min,N}$ for predictor lengths $N = 3$ and $N = 20$. (You may find the Matlab command `toeplitz` useful for construction of the autocorrelation matrix.)

   (e) From the diagram below, it can be seen that $E(z) = B(z)\big(1-H(z)\big)V(z)$ where $B(z) = \sum_{i=0}^{N_b} b_i z^{-i}$ and $H(z) = \sum_{i=1}^{N_h} h_i z^{-i}$. Using convolution, it is possible to calculate $\{q_i\}$, the impulse response of $Q(z) = B(z)\big(1-H(z)\big)$, and hence $r_e(k)$ by the method of part (a).



   Plot $r_e(k)$ and $S_e(e^{j\omega})$ for the two predictors. Is the prediction error white? How "flat" is the error spectrum? How could one quantify "flatness"?

   (f) Using a $M = 10000$-length version of $x(n)$ in Matlab and the two predictors designed in 1(c), compute the prediction error sequences $e(n)$ and measure their variance. How do they compare to the theoretical $\sigma_e^2\big|_{\min,N}$?

2. *DPCM Structures:*
Here you will investigate the four PCM/DPCM structures discussed in the notes.

(a) Using $x(n)$ from 1(d) and the length-3 predictor from 1(c), implement the coder/decoder in Fig. 1 and compute the mean-squared reconstruction error $\mathcal{E} = \frac{1}{M}\sum_{n=0}^{M-1}|y(n) - x(n)|^2$.

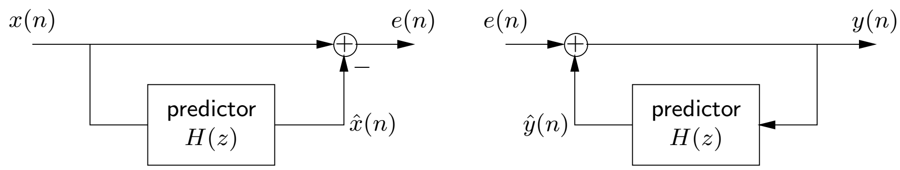


Figure 1: Prediction Error Transmission System.

(b) Repeat for the PCM system in Fig. 2. Assume uniform quantization with $L = 32$, and choose Δ to minimize $\mathcal{E}$ based on the experiments you did in Homework 1.

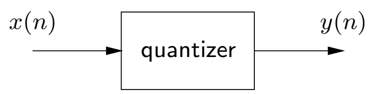


Figure 2: PCM System.

(c) Repeat for the coder/decoder in Fig. 3 (but make sure to redesign Δ appropriately). Discuss how $\mathcal{E}$ compares with $\text{var}\big(\tilde{e}(n) - e(n)\big)$.

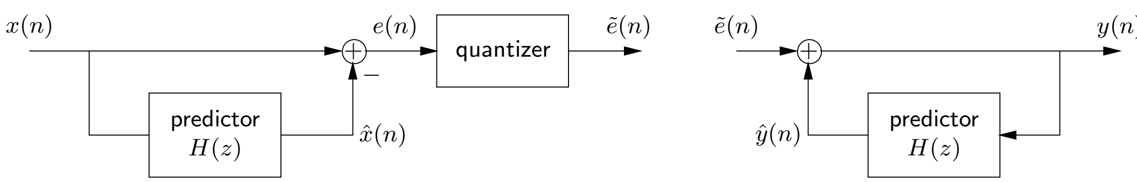


Figure 3: Predictive Coding System.

(d) Repeat for the coder/decoder in Fig. 4 using the same quantizer as in (c). Discuss how $\mathcal{E}$ compares with $\text{var}\big(\tilde{e}(n) - e(n)\big)$.

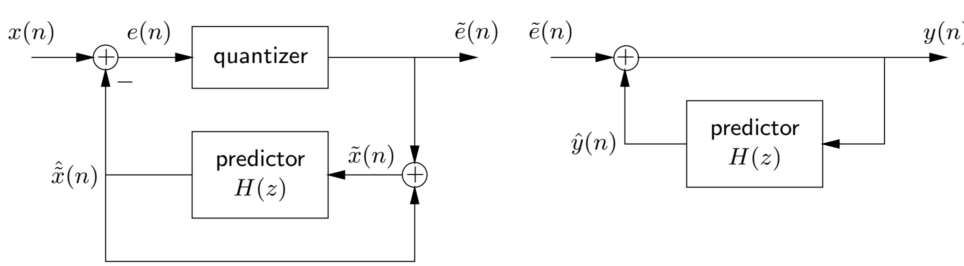


Figure 4: Differential-PCM System.

(e) Since $\mathcal{E}$ was calculated using the same input signal in parts (a)-(d), a direct comparison of the 4 systems is possible. Discuss the advantages and disadvantages of each system.

3. *Optimal Coding:*

Now that you are having fun, it's time to investigate minimum bit rate versus SNR for coding of $\tilde{e}(n)$ and $x(n)$.

(a) Using $x(n)$ from 1(a), the structure in Fig. 4, and a predictor length of $N = 20$, calculate the SNR ($= 10\log_{10}(\sigma_x^2/\mathcal{E})$) for uniform quantizers with $L = 2, 4, 8, 16, 32, 64, 128$. Use quantization stepsize $\Delta = 2e_{\max}/L$ based on the table below, assuming $\sigma_e^2 \approx \sigma_e^2\big|_{\min,N}$.

L	2	4	8	16	32	64	128
$e_{\max}/\sigma_e$	1.6	2	2.3	2.7	3.1	3.4	3.7

(b) For the same input and DPCM systems, calculate the entropy rate of $\tilde{e}(n)$. (You may assume that $e(n)$ is zero-mean Gaussian and use `erfc` as in Homework 1. σ_e^2 can be obtained experimentally for each L.)

(c) Using the results from (a) and (b), plot the bit rate anticipated from optimal entropy coding of $\tilde{e}(n)$ (see Fig. 5) versus SNR. Superimpose, on the same plot, the bit rate versus SNR curve for the optimal coder of $x(n)$. Discuss.

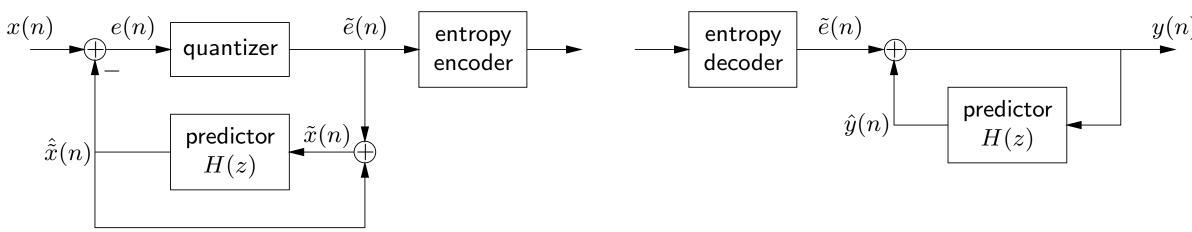


Figure 5: Entropy-Encoded DPCM System.