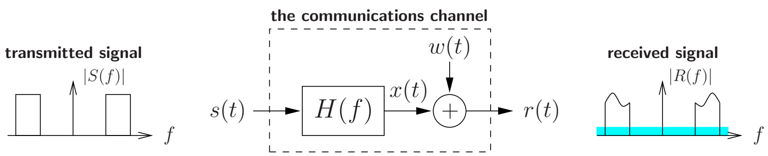
The Communications Channel (Ch.11):

The effects of signal propagation are usually modeled as:



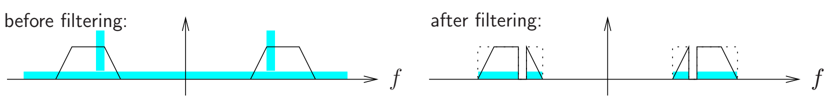
where $H(f)$: linear filtering due to multipath propagation
$w(t)$: additive noise/interference.

Noise/interference sources include:

- electronic circuitry ("thermal/shot noise" or "quantization noise"); usually broadband,
- other comm systems ("multi-access interference" or "co-channel interference"); broadband or narrowband.

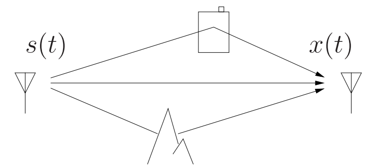
SNR can be improved with appropriate filtering at receiver:

- signal-to-noise ratio (SNR) $= \frac{\text{signal energy}}{\text{noise energy}} = \frac{\mathcal{E}_s}{\mathcal{E}_n}$.
- roughly speaking, average SNR can be improved by filtering out the frequencies dominated by noise:



Filtering due to Multipath Propagation:

The signal may propagate along paths with different lengths:



Since different path lengths imply different path delays:

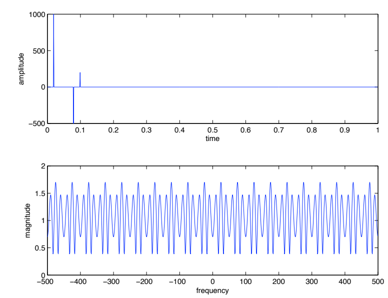
$$x(t) = c_1 s(t - \tau_1) + c_2 s(t - \tau_2) + \cdots + c_N s(t - \tau_N),$$

which can be written as $x(t) = s(t) * h(t)$ for

$$h(\tau) = c_1 \delta(\tau - \tau_1) + c_2 \delta(\tau - \tau_2) + \cdots + c_N \delta(\tau - \tau_N).$$

The result is an $|H(f)|$ that varies with f, implying frequency-dependent signal attenuation:

```
t_max = 1;
Ts = 1/1000;
c1 = 1; tau1 = 0.02;
c2 = -0.5; tau2 = 0.08;
c3 = 0.2; tau3 = 0.10;
h = zeros(1,t_max/Ts);
h(tau1/Ts) = c1/Ts;
h(tau2/Ts) = c2/Ts;
h(tau3/Ts) = c3/Ts;
plottf(h,Ts);
```

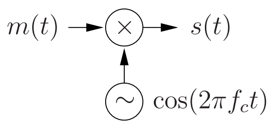


Analog Communication (Ch.3-4):

1. Amplitude modulation (AM)
2. Quadrature amplitude modulation (QAM)
3. Vestigial sideband modulation (VSB)
4. Frequency modulation (FM)

AM with "suppressed carrier":

AM of real-valued message $m(t)$ (e.g., music) is

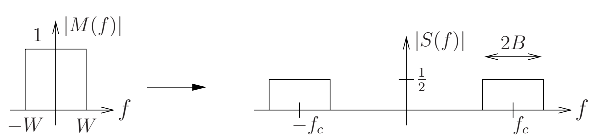


$s(t) = m(t)\cos(2\pi f_c t)$,

f_c = carrier freq.

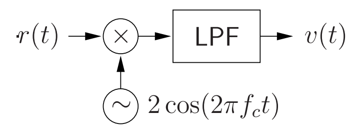
Euler's $\cos(2\pi f_c t) = \frac{1}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$ then implies

$$\begin{aligned} S(f) &= \int_{-\infty}^{\infty} m(t)\cos(2\pi f_c t)e^{-j2\pi f t}dt \\ &= \frac{1}{2}\int_{-\infty}^{\infty} m(t)e^{-j2\pi(f-f_c)t}dt + \frac{1}{2}\int_{-\infty}^{\infty} m(t)e^{-j2\pi(f+f_c)t}dt \\ &= \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c). \end{aligned}$$



Because $m(t) \in \mathbb{R}$, know $|M(f)|$ symmetric around $f = 0$, implying the AM transmitted spectrum below f_c is redundant! This motivates the QAM and VSB modulation schemes...

With f_c known, AM demodulation can be accomplished by:

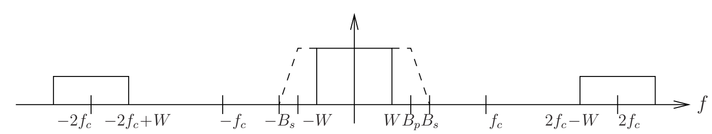


$v(t) = \text{LPF}\{r(t)\cdot 2\cos(2\pi f_c t)\}$.

For a trivial noiseless channel, we have $r(t) = s(t)$, so that

$$\begin{aligned} v(t) &= \text{LPF}\{s(t)\cdot 2\cos(2\pi f_c t)\} \\ &= \text{LPF}\{m(t)\cdot \underbrace{2\cos^2(2\pi f_c t)}_{1+\cos(2\pi\cdot 2f_c t)}\} \\ &= \text{LPF}\{m(t) + m(t)\cos(2\pi\cdot 2f_c t)\} \\ &= m(t), \end{aligned}$$

assuming a LPF with passband cutoff $B_p \geq W$ Hz and stopband cutoff $B_s \leq 2f_c - W$ Hz:



Note that we've assumed perfectly synchronized oscillators!

When the receiver oscillator has {freq,phase} offset $\{\gamma, \phi\}$:

$$v(t) = \text{LPF}\Big\{m(t)\underbrace{\cos(2\pi f_c t)\cdot 2\cos(2\pi(f_c+\gamma)t+\phi)}_{\cos(2\pi\gamma t+\phi)\;+\;\cos(2\pi(2f_c+\gamma)t+\phi)}\Big\}$$

$$= m(t)\underbrace{\cos(2\pi\gamma t+\phi)}_{\text{time-varying attenuation!}}\ .$$

Note: a freq offset of $\lambda = \frac{\nu f_c}{c}$ Hz can occur when there is relative velocity of ν m/s between transmitter and receiver.

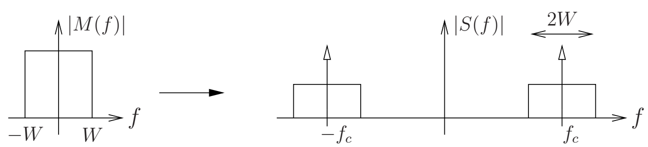
AM with "pilot tone" or "carrier tone":

It's common to include a pilot/carrier tone with frequency f_c:

$$s(t) = m(t)\cos(2\pi f_c t) + \underbrace{A\cos(2\pi f_c t)}_{\text{pilot/carrier tone}}$$

$$= [m(t)+A]\cos(2\pi f_c t)$$

$$S(f) = \tfrac{1}{2}\Big[M(f-f_c)+M(f+f_c)+A\delta(f-f_c)+A\delta(f+f_c)\Big]$$



Advantage: aids receiver with carrier synchronization.

Disadvantage: consumes transmission power.

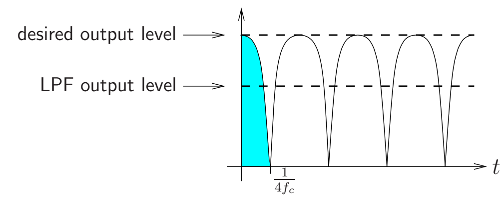
While modern systems choose $A \ll \max|m(t)|$, many older systems use $A > \max|m(t)|$, known as "large carrier AM," allowing reception based on *envelope detection*:

$$v(t) = \frac{\pi}{2}\text{LPF}\{\,|r(t)|\,\} - A$$

$$\approx m(t) \quad \text{(with a trivial channel)}$$

where $|\cdot|$ can be easily implemented using a diode.

The gain $\frac{\pi}{2}$ above makes up for the loss incurred when LPFing the rectified signal:



$$\frac{\int_0^{\frac{1}{4f_c}}\cos(2\pi f_c t)dt}{\frac{1}{4f_c}} = \frac{2}{\pi}.$$

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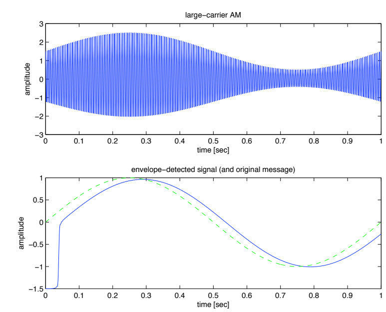
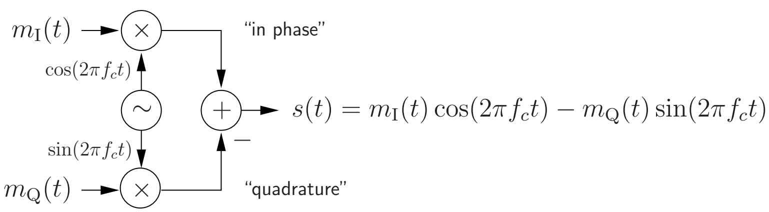


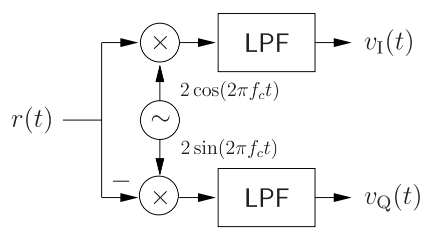
Quadrature Amplitude Modulation (QAM):

QAM is motivated by unwanted redundancy in the AM spectrum, which was symmetric around f_c.

QAM sends two real-valued signals $\{m_{\mathrm{I}}(t), m_{\mathrm{Q}}(t)\}$ simultaneously, resulting in a non-symmetric spectrum.



QAM demodulation is accomplished by:



where the LPF specs are the same as in AM, i.e., passband edge $B_p \geq W$ Hz and stopband edge $B_s \leq 2f_c - W$ Hz.

For a trivial channel, we have $r(t) = s(t)$, so that

$$\begin{aligned} v_{\mathrm{I}}(t) &= \mathsf{LPF}\{r(t)\cdot 2\cos(2\pi f_c t)\} \\ &= \mathsf{LPF}\Big\{m_{\mathrm{I}}(t)\underbrace{2\cos^2(2\pi f_c t)}_{1+\cos(4\pi f_c t)} \\ &\qquad - m_{\mathrm{Q}}(t)\underbrace{2\sin(2\pi f_c t)\cos(2\pi f_c t)}_{\sin(4\pi f_c t)}\Big\} \\ &= m_I(t) \\ v_{\mathrm{Q}}(t) &= \mathsf{LPF}\{-r(t)\cdot 2\sin(2\pi f_c t)\} \\ &= \mathsf{LPF}\Big\{-m_{\mathrm{I}}(t)\underbrace{2\cos(2\pi f_c t)\sin(2\pi f_c t)}_{\sin(4\pi f_c t)} \\ &\qquad + m_{\mathrm{Q}}(t)\underbrace{2\sin^2(2\pi f_c t)}_{1-\cos(4\pi f_c t)}\Big\} \\ &= m_Q(t), \end{aligned}$$

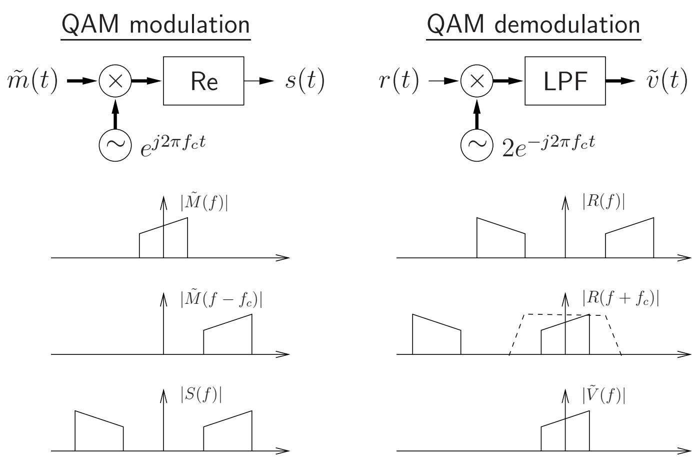
assuming synchronized oscillators.

When the oscillators are not synchronized, one gets coupling between the I&Q components as well as attenuation of each.

Writing the I&Q signals in the "complex-baseband" form

$$\tilde{m}(t) = m_{\mathrm{I}}(t) + j m_{\mathrm{Q}}(t)$$
$$\tilde{v}(t) = v_{\mathrm{I}}(t) + j v_{\mathrm{Q}}(t)$$

yields a much simpler description of QAM:



Note: $\mathrm{Re}\{u(t)\} = \frac{1}{2}\big[u(t) + u^*(t)\big] \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\big[U(f) + U^*(-f)\big]$.

We now verify the complex-baseband model for modulation:

$$\begin{aligned}
&\mathrm{Re}\{\tilde{m}(t)e^{j2\pi f_c t}\} \\
&= \mathrm{Re}\Big\{\big(m_{\mathrm{I}}(t) + j m_{\mathrm{Q}}(t)\big)\big(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\big)\Big\} \\
&= m_{\mathrm{I}}(t)\cos(2\pi f_c t) - m_{\mathrm{Q}}(t)\sin(2\pi f_c t) \;=\; s(t),
\end{aligned}$$

as well as for demodulation (assuming $r(t) = s(t)$):

$$\begin{aligned}
\tilde{v}(t) &= \mathsf{LPF}\{s(t)\cdot 2e^{-j2\pi f_c t}\} \\
&= \mathsf{LPF}\{\big(m_{\mathrm{I}}(t)\cos(2\pi f_c t) - m_{\mathrm{Q}}(t)\sin(2\pi f_c t)\big)\cdot 2e^{-j2\pi f_c t}\} \\
&= \mathsf{LPF}\Big\{m_{\mathrm{I}}(t)\big(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\big)e^{-j2\pi f_c t} \\
&\qquad - m_{\mathrm{Q}}(t)\big(je^{-j2\pi f_c t} - je^{j2\pi f_c t}\big)e^{-j2\pi f_c t}\Big\} \\
&= \mathsf{LPF}\Big\{m_{\mathrm{I}}(t)\big(1 + e^{-j4\pi f_c t}\big) - m_{\mathrm{Q}}(t)\big(je^{-j4\pi f_c t} - j\big)\Big\} \\
&= m_{\mathrm{I}}(t) + j m_{\mathrm{Q}}(t).
\end{aligned}$$

The convenience of complex-baseband results in widespread use of complex-valued signals for comm systems!

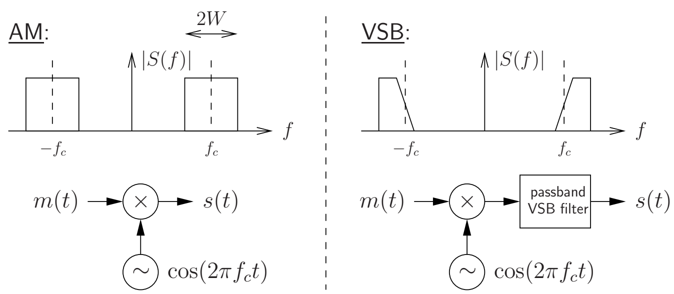
Note: To get the complex baseband formulation for AM, we simply set $m_{\mathrm{Q}}(t) = 0$ and $m_{\mathrm{I}}(t) = m(t)$.

Vestigial Sideband Modulation (VSB):

VSB is another way to restore regain the spectral efficiency lost in AM. It's used to transmit North American terrestrial TV, both analog (NTSC) and digital (ATSC) formats.

Like AM, it can operate with or without a carrier tone.

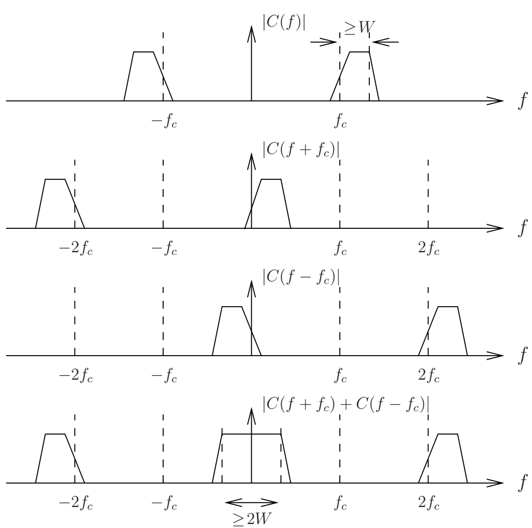
Basically, VSB suppresses most of the redundant AM spectrum by filtering it:



The passband VSB filter is a BPF $C(f)$ where

$$C(f-f_c)+C(f+f_c)=2 \quad \text{for } |f|\le W,$$

which implies its inside rolloff is symmetric around $f=f_c$:



For VSB modulation, we have

$$\begin{aligned} s(t) &= m(t)\cos(2\pi f_c t) * c(t) \\ S(f) &= \frac{1}{2}\big[M(f+f_c)+M(f-f_c)\big]C(f). \end{aligned}$$

It turns out that VSB demod is *identical* to AM demod:

$$\begin{aligned} v(t) &= \text{LPF}\big\{r(t)\cdot 2\cos(2\pi f_c t)\big\} \\ &= \text{LPF}\big\{s(t)\cdot 2\cos(2\pi f_c t)\big\} \quad \text{(trivial channel)} \\ V(f) &= \text{LPF}\big\{S(f-f_c)+S(f+f_c)\big\} \\ &= \frac{1}{2}\text{LPF}\Big\{\big[M(f)+M(f-2f_c)\big]C(f-f_c) \\ &\qquad + \big[M(f+2f_c)+M(f)\big]C(f+f_c)\Big\} \\ &= M(f)\underbrace{\frac{1}{2}\big[C(f-f_c)+C(f+f_c)\big]}_{=1 \text{ for } f\in[-W,W]} \\ &= M(f). \end{aligned}$$

We note that the property

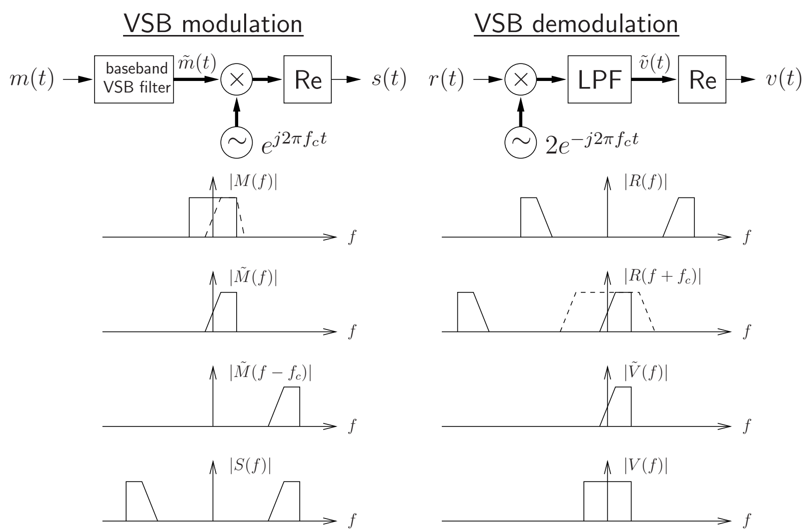
$$\mathcal{F}\Big\{\cos(2\pi f_c t)c(t)\Big\} = \frac{1}{2}\Big[C(f-f_c)+C(f+f_c)\Big]$$

may be convenient, e.g., for testing whether a given filter $c(t)$ satisfies the passband VSB criterion.

VSB filtering can also be implemented at baseband using a complex-valued filter response $\tilde{c}(t)$ which satisfies

$$\tilde{C}(f) + \tilde{C}^*(-f) = 2 \quad \text{for } |f| \leq W,$$

generating the complex-baseband message signal $\tilde{m}(t)$. The message can be recovered by simplying ignoring the imaginary part of the complex-baseband output $\tilde{v}(t)$.



Motivation: filtering at baseband is usually much cheaper than filtering at passband.

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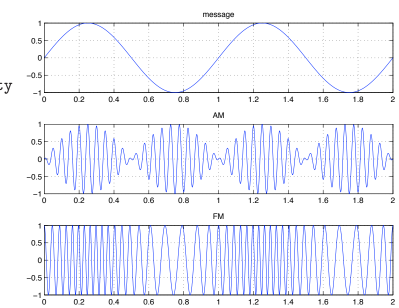
Frequency Modulation (FM):

While AM modulated the carrier amplitude, FM modulates the carrier frequency.

```
t_max = 2.0; W = 1;          % message params
Ts = 1/1000; t = 0:Ts:t_max;
m = sin(2*pi*W*t);           % message signal
fc = 20;                     % carrier freq
D = 15;                      % FM mod index
kf = D*W/max(abs(m));        % freq sensitivity
s_am = m.*cos(2*pi*fc*t);
s_fm = cos(2*pi*fc*t+2*pi*kf*cumsum(m)*Ts);
subplot(3,1,1)
 plot(t,m);
 grid on; title('message');
subplot(3,1,2)
 plot(t,s_am);
 grid on; title('AM');
subplot(3,1,3)
 plot(t,s_fm);
 grid on; title('FM');
```



In particular, FM modulates the real-valued message $m(t)$ via

$$s(t) = \cos\Big(2\pi f_c t + \underbrace{2\pi k_{\mathsf{f}} \int_0^t m(\tau) d\tau}_{\varphi(t) \text{ "instantaneous modulation phase"}}\Big).$$

where k_{f} is called the "frequency-sensitivity factor." Since the instantaneous modulation frequency

$$\frac{d\varphi(t)}{dt} = 2\pi k_{\mathsf{f}}\, m(t)$$

is a scaled version of the message $m(t)$, it is fitting to call this scheme "frequency modulation."

Using the peak frequency deviation $\Delta_{\mathsf{f}} = k_{\mathsf{f}} \max|m(t)|$, the "modulation index" D is defined as

$$D = \frac{\Delta_{\mathsf{f}}}{W} \leftarrow \text{single-sided BW of } m(t).$$

Increasing D decreases spectral efficiency but increases robustness to noise/interference.

$$D \ll 1 : \text{"narrowband FM"},$$
$$D \gg 1 : \text{"wideband FM"}.$$

Carson's Rule approximates the FM passband signal-BW as

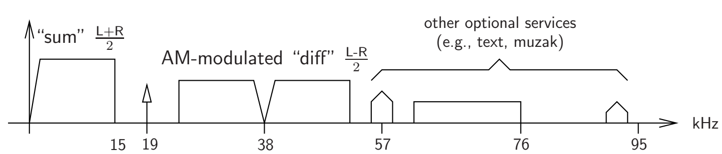
$$\mathsf{BW}_{99} \approx 2(\Delta_{\mathsf{f}} + W) = 2(D+1)W.$$

Example: Mono FM radio:

- Message signal filtered to freq interval [30,15k] Hz.
- FCC limits $\Delta_{\mathsf{f}} \le 75$ kHz (channels 200 kHz apart).

  $\Rightarrow D = \frac{75}{15} = 5$

FM stereo uses smaller D due to message spectrum:



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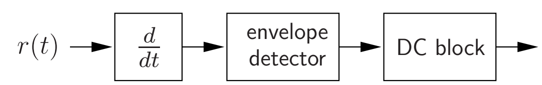
There are various FM demodulators, but the "discriminator" is one of the best known. Recalling that

$$\frac{d}{dt}\cos\big(\varphi(t)\big) = -\frac{d\varphi(t)}{dt}\sin\big(\varphi(t)\big),$$

we see that

$$\begin{aligned}\frac{d}{dt}s(t) &= \frac{d}{dt}\cos\big(2\pi f_c t + 2\pi k_{\mathsf{f}} \textstyle\int_0^t m(\tau)d\tau\big) \\ &= -\big[2\pi f_c + 2\pi k_{\mathsf{f}} m(t)\big]\sin\big(2\pi f_c t + 2\pi \textstyle\int_0^t m(\tau)d\tau\big)\end{aligned}$$

is a form of large-carrier AM (assuming $f_c > k_{\mathsf{f}} m(t)$), which can be demodulated using an envelope detector as follows:



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