

# Stable Cooperative Surveillance With Information Flow Constraints

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**Abstract**—We consider a cooperative surveillance problem for a group of autonomous air vehicles (AAVs) that periodically receives information on suspected locations of targets from a satellite and then must cooperate to decide which AAV should search for each target. This cooperation must be performed in spite of imperfect intervehicle communications (e.g., messages with random but bounded delays), less than full communication connectivity between vehicles, uncertainty in target locations, and imperfect vehicle search sensors. We represent the state of the search progress with a “search map,” and use an invariant set to model the set of states where there is no useful information on target locations. Arrivals of new suspected target location information from the satellite corresponds to perturbations of the search map from this invariant set. A cooperation strategy that pursues a type of “persistent area denial” will try to force trajectories of the system into the invariant set by exploiting initial target information and search progress by the AAVs. We show that the invariant set is exponentially stable for a class of cooperative surveillance strategies. We provide a comparative analysis of cooperative and noncooperative strategies. Next, we show via simulations the impact of imperfect communications, imperfect vehicle search sensors, uncertainty in search locations, and pop-up suspected locations on performance.

**Index Terms**—Autonomous vehicles, cooperative control, cooperative search, networked control systems, stability.

## I. INTRODUCTION

GROUPS of possibly many autonomous air vehicles (AAVs) of different types, connected via a communication network to implement a “vehicle network,” are technologically feasible and hold the potential to greatly expand operational capabilities at a lower cost (e.g., due to the economies of scale gained by manufacturing many simpler vehicles). Cooperative control for navigation of such vehicle groups involves coordinating the activities of several agents so they work together to complete tasks in order to achieve a common goal. Here, we study how to perform a cooperative AAV surveillance task that involves two components: 1) search for stationary targets located in a region and 2) reacting to “pop up” suspected target locations that are provided by a satellite (or other high-flying platform).

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There is a significant amount of current research activity focused on cooperative control of AAVs, and some research directions in this field are identified in [1]. Solutions to general cooperative control problems can be obtained via solutions to vehicle route planning (VRP) problems [2]. While VRP methods can be used to allocate AAVs to tasks in order to minimize the mission completion time, generally the methods are only applicable when uncertainties are not present in the environment. Additional VRP-related work focusing on cooperative search and coordinated sequencing of tasks is in [3]–[6]. Other cooperative control methods include gradient algorithms [7]–[9], multisensor fusion [10], surrogate optimization [11], and receding horizon control [3], [4], [12]. Using such approaches, significant mission performance benefits can be realized via cooperation in some situations, most notably when there is not a high level of uncertainty.

One challenge in cooperative control problems is to overcome the effects of uncertainty so that benefits of cooperation can still be realized. When uncertainty dominates the system, cooperative strategies will not be able to achieve the high level of coordination achieved in many of the above-mentioned studies that assume perfect communications. The most that can be hoped for is to achieve *some* benefit from cooperation. Along these lines, recent work considering imperfect communications is found in [13]–[19]. Here, we continue along the lines of such work, but with a focus on cooperative surveillance when there are a variety of information flow constraints. Of all the current work in cooperative control, the most closely related to this study is the “persistent area denial (PAD)” problem studied in [20] and [21], search theory [22], [23], the application of the  $m$ -person Dynamic Traveling Repairperson Problem ( $m$ -DTRP) to cooperative control [24], and the “map-based approaches” in [25]–[30].

The main contributions of this paper are: 1) the use of search-theoretic ROR maps [23] for the coordination of the search efforts of multiple AAVs for finding targets in a limited area with uncertainties from sources like communication delays and partial knowledge of target locations; 2) a novel characterization of cooperation objectives as stability properties of an invariant set; and 3) the use of standard Lyapunov stability-theoretic methods for verification of cooperative control systems. Our stability analysis provides design guidelines, analogous to how it does for conventional control. Our simulation studies show the value of the design guidelines. In Section II the cooperative surveillance problem is formulated and modeled. The stability analysis is in Section III. An analytical study is used to compare the cooperative surveillance approach to a noncooperative case in Section IV. Simulations are in Section V and conclusions and future directions are provided in Section VI.

## II. COOPERATIVE SURVEILLANCE PROBLEM FORMULATION

Suppose that there is a group of AAVs that performs surveillance of a given area and can use information from a satellite to pursue suspected locations in that area. Assume that the set of AAVs is given a number of targets to search for with their respective likely locations. AAVs must work together autonomously in order to try to maximize the probability of finding targets in the environment with minimal AAV effort. A table with a summary of all the variables can be found in the Appendix.

### A. Discrete Event System Model

Here we represent the cooperative surveillance problem as a nonlinear discrete time asynchronous dynamic system [31] with the model  $G = (\mathcal{X}, \mathcal{E}, f_e, g, E_v)$ . Here,  $\mathcal{X}$  is the set of states. The set of events is denoted by  $\mathcal{E}$ . State transitions are defined by  $f_e : \mathcal{X} \rightarrow \mathcal{X}, e \in \mathcal{E}$ . The enable function,  $g$ , defines the occurrence of an event  $e$  by  $g : \mathcal{X} \rightarrow \mathcal{P}(\mathcal{E}) - \{\emptyset\}$ , being  $\mathcal{P}(\mathcal{E})$  the power set of  $\mathcal{E}$ . Note that  $f_e$  is defined when  $e \in g$ . Let  $k \geq 0$  be the time indices for the states  $x(k) \in \mathcal{X}$  and the enabled events  $e(k) \in g$ . Let  $E_v$  denote a set of “valid” event trajectories. Next, we define these for the cooperative surveillance problem.

### B. Vehicle Model

Suppose that the  $m$ th AAV flies at a constant altitude and obeys a continuous time kinematic model given by the Dubin’s car [32],  $\dot{x}_1^m(t) = v \cos \theta^m(t)$ ,  $\dot{x}_2^m(t) = v \sin \theta^m(t)$ ,  $\dot{\theta}^m(t) = \omega_{\max} u^m(t)$  where  $x_1^m$  is its horizontal position,  $x_2^m$  is its vertical position,  $v$  is its forward (constant) velocity,  $\theta^m$  is its heading direction,  $\omega_{\max}$  is its maximum angular velocity, and  $-1 \leq u^m \leq 1$  is the steering input. Hence,  $u^m = +1(-1)$  stands for the sharpest possible turn to the right (respectively, left). The minimum turn radius for the vehicle is  $R = (v)/(\omega_{\max})$ . Rather than allow  $u^m(t)$  to be arbitrary, vehicles will either travel on the minimum turn radius or on straight lines that are the optimal paths from a vehicle’s current location and orientation to a desired location and orientation [33].

### C. Search Environment and Targets

The search environment is assumed to be rectangular with length  $L$  and width  $W$ . Divide the edge with length  $L$  into  $r \in \mathbb{Z}^+$  segments and the edge with length  $W$  into  $s \in \mathbb{Z}^+$  segments ( $\mathbb{Z}^+$  is the set of the positive integers). This decomposes the search area into  $rs$  cells, each with a size of  $lw = (LW)/(rs)$ . Thus, there are  $N_Q = rs$  discrete cells to search, and these cells are numbered so that the discretized search space is  $Q = \{1, 2, \dots, N_Q\}$ . It is assumed that all the AAVs know how the cells are numbered and hence know  $N_Q$ . It is also assumed that there are  $N_V$  AAVs that search for targets and let  $V = \{1, 2, \dots, N_V\}$  denote the set of vehicles.

Assume that there are  $N_D$  distinct valid stationary targets that the AAVs are searching for and let  $D = \{1, 2, \dots, N_D\}$  denote the set of targets. Suppose that the size of each target is considerably smaller than the size of each cell. Also, there could be more than one target located in one cell; however, it is assumed that all targets inside the same cell are separated in such a way that they can be detected by the sensors of the AAVs if, for example, they take enough “looks” at the cell.

AAV sensors return information about the targets. Assume that the sensor of AAV  $m$  has a rectangular “footprint” with depth  $d_{f_1}^m = N_\ell^m(L)/(r)$ , width  $d_{f_2}^m = N_w^m(W)/(s)$ ,  $N_\ell^m, N_w^m \in \mathbb{Z}^+$  with  $N_\ell^m < r, N_w^m < s$ , and the distance from AAV  $m$  to the center of the footprint is denoted by  $d_s^m$  for all  $m \in V$ . For convenience, here it is assumed that  $d_{f_1}^m = d_{f_1} = d_{f_2}^m = d_{f_2}$  and  $d_s^m = d_s$  for all  $m \in V$  so the footprint can be perfectly aligned with the grid of search cells. Whenever an AAV is going to search a cell, it will approach this cell at an angle in the set  $\{0, (\pi)/(2), \pi, (3\pi)/(2)\}$ . Due to sensor inaccuracies, it is assumed that searching a cell once may not result in finding a target even if it is there. Multiple searches of the same cell may be needed to find a target. In order to avoid detection by an adversary, AAVs turn their sensors on only when they have reached the desired orientation and location with respect to the center of the cell where the target is suspected to be located. Once an AAV takes a snapshot of the cell to be searched, the AAV will turn its sensor off until a new cell of interest is reached again. Thus, suppose that AAV  $m$  takes a snapshot of the cell of interest and that this covers  $N_\ell^m N_w^m$  cells. It is assumed that AAV  $m$  is able to analyze all the cells covered by the sensor and can determine, possibly after a number of snapshots, which targets are located in various cells. When targets are placed on the boundaries of the cells, it is assumed that when a cell is searched the search includes the left and lower boundaries of the cell, but it does not include the two endpoints lying opposite to where the boundaries join. This guarantees that the same object cannot be found by searching two different cells.

### D. Rate of Return (ROR) Map

Let  $p_j^m(q)$  be the AAV  $m \in V$  a priori probability that target  $j \in D$  is in cell  $q \in Q$ . These distributions are assumed to be mutually independent and the targets are assumed to be distinguishable when they are detected; that is, an AAV can identify which target has been detected. These a priori probabilities must be specified by information gathering sources (e.g., the satellite or other high-flying platform). Ideally, the sum  $\sum_{q \in Q} p_j^m(q) = 1$  for any  $j \in D$  and  $m \in V$ , but this may not be true if there is a chance the target is not in any of the cells to be searched. For simplicity, we assume here that  $p_j^m(q) = p_j(q)$  for all  $m \in V$  so all AAVs get the same information.

Let  $\ell^m(q, k) \in \{0, 1, 2, \dots\}$  be the number of looks (passes when a “snapshot” is taken) performed in cell  $q \in Q$  by AAV  $m \in V$  by time index  $k$ , which is a measure of the amount of search effort dedicated to cell  $q \in Q$  [23]. The AAVs share  $\ell^m(q, k)$  for all  $m \in V$ , via intervehicle send/receive communications that may have arbitrary but finite delays. These delays should not be thought of as arising only from delays on, for instance, communication network links, but also from occlusions and sensing/communication range constraints, or temporary loss of a communication link between vehicles. The maximum delay between AAV  $m'$  and AAV  $m$  is known and equal to  $B^{m'm} \in \mathbb{Z}^+$ . The information transmitted by the AAVs is assumed to take at least one time index to arrive at any AAV. The unknown but bounded delay between AAV  $m'$  and AAV  $m$  is modeled by a random choice of  $\tau_k^{m'm}, 1 \leq \tau_k^{m'm} \leq B^{m'm}$  (with no assumption on the underlying statistics). If  $m = m'$ ,

then  $\tau_k^{m'm} = 0$  for all  $k$  since a vehicle knows its own information. Each AAV  $m \in V$  stores the values of  $\ell^{m'}(q, k - \tau_k^{m'm})$  (i.e., the most recent information update that it has received from AAV  $m' \in V$ ) if the received  $\ell^{m'}(q, k - \tau_k^{m'm})$  is greater than the current value held by AAV  $m$ . Otherwise, AAV  $m$  discards  $\ell^{m'}(q, k - \tau_k^{m'm})$  and keeps the old one (this event could arise due to the random nature of the intervehicle communication that could result in message misordering of the  $\ell^{m'}(q, k - \tau_k^{m'm})$  values sent by AAV  $m'$ ). Each AAV  $m \in V$  in general only has up to date information on  $\ell^m(q, k)$  (its own number of looks), not on the number of looks of other AAVs  $m' \neq m$  that are out of date by  $k - \tau_k^{m'm}$ . AAV  $m \in V$  uses the latest information it has to form  $\hat{\ell}^m(q, k)$  its estimate of the total number of looks taken by all  $N_V$  AAVs. Here, we use  $\hat{\ell}^m(q, k) = \sum_{m'=1}^{N_V} \ell^{m'}(q, k - \tau_k^{m'm})$  as the value for this estimate for each  $m \in V, q \in Q$ , and  $k \geq 0$ . Clearly if there were no communication delays, then  $\hat{\ell}^m(q, k)$  would be the total number of looks that the group of AAVs have taken on cell  $q \in Q$  by time index  $k \geq 0$ .

Let  $L^m \subset V$  be a set that contains the AAVs  $m', m' \neq m$  that have looked at cells  $q \in Q$ , possibly multiple times denoted by  $n^m(\ell^{m'}(q, k)) \geq 1$ , between the time when AAV  $m$  decides to perform an additional look at target  $j$  in cell  $q$  and the time when AAV  $m$  actually receives the new looks. Let  $Q^m \subset Q$  be the set that contains the cells that have been visited by AAV  $m' \in L^m \subset V$  during the time when AAV  $m$  decides to perform an additional look at target  $j$  in cell  $q$  and the time when AAV  $m$  receives new look values. Let  $C_m$  denote the set of cells covered by the sensor footprint of AAV  $m$ . Define the set of "local" events for each AAV  $m$  as  $\{e_1^{m, L^m, Q^m}, e_2^{m, C_m}, e_3^{m, L^m, Q^m, C_m}\}$ , where  $e_1^{m, L^m, Q^m} =$  "Reception of looks at  $q \in Q^m$  from  $m' \in L^m$ ",  $e_2^{m, C_m} =$  "AAV  $m$  sensor on for cells  $q \in C_m$ ",  $e_3^{m, L^m, Q^m, C_m} =$  "AAV  $m$  sensor on for cells  $q \in C_m$  and  $e_2^{m, C_m}$  occur simultaneously" so that the set of events of the system such that  $e(k) \in g$  is any subset of

$$\mathcal{E} = \mathcal{P} \left( \bigcup_{m=1}^{N_V} \left\{ e_1^{m, L^m, Q^m}, e_2^{m, C_m}, e_3^{m, L^m, Q^m, C_m} \right\} \right) - \{\emptyset\} \quad (1)$$

that contains all the elements of the right side of (1) except that only one of the three types of local events can be in any event  $e \in \mathcal{E}$ . Thus, the enable function  $g$  allows the occurrence of a set of possible events at the same time.

Let  $a_{j,q}^m > 0$  be a parameter that is proportional to the sensor capabilities of AAV  $m$  for target  $j$  in cell  $q$ . Let  $\pi_{j,q}^m \in (0, 1]$  be the probability that AAV  $m$  detects target  $j$  in cell  $q$  on a single look, given that target  $j$  is in cell  $q$ . Assume that AAV  $m$  knows each  $\pi_{j,q}^{m'}$  and  $a_{j,q}^{m'}$  from AAV  $m', m' \neq m, m \in V$ . It is necessary to relate the search effort made by any AAV when it looks for a target to the probability of detecting a target. This is done through a "detection function" [22], [23]. Let the detection function,  $b_j^m(\hat{\ell}^m(q, k))$ , be the conditional probability of detecting target  $j$  in cell  $q$  by time index  $k$  with all the  $\ell^{m'}(q, k - \tau_k^{m'm}), m' \in V$  search values of cell  $q$  held by AAV  $m$ , given that target  $j$  is in cell  $q$ . If each cell look

has an independent probability of finding the target, then the detection function is

$$b_j^m(\hat{\ell}^m(q, k)) = 1 - \prod_{m'=1}^{N_V} \left( 1 - \pi_{j,q}^{m'} \right)^{a_{j,q}^{m'} \ell^{m'}(q, k - \tau_k^{m'm})}. \quad (2)$$

Since  $\hat{\ell}^m(q, k)$  is an estimate,  $b_j^m(\hat{\ell}^m(q, k))$  should be thought of as an estimate of the probability  $j \in D$  was found. AAV  $m \in V$  will use this estimate to decide where to search.

To clarify the meaning of (2), note that if  $\ell^{m'}(q, k - \tau_k^{m'm}) = 0$  in (2) for all  $m' \neq m, m' \in V$ , then the detection function is given by

$$b_j^m(\hat{\ell}^m(q, k)) = b_j^m(\ell^m(q, k)) = 1 - (1 - \pi_{j,q}^m)^{a_{j,q}^m \ell^m(q, k)}. \quad (3)$$

Use of probability theory to specify (3) gives  $a_{j,q}^m = 1$ , but this parameter is sometimes included to model sensor characteristics. For example,  $a_{j,q}^m > 1 (< 1)$  corresponds to a more (less, respectively) effective sensor. Equation (3) models the common situation with sensors where the probability of detecting a target increases as the number of searches of a cell increases. Higher values of  $\pi_{j,q}^{m'}$  and  $a_{j,q}^{m'}$  mean that the sensor is better at finding a target in a cell since then fewer searches of  $q$  are needed to achieve target detection. Some cells can be more difficult to search. Some targets are more difficult to detect. Some AAVs are better suited to finding certain targets in certain cells. Each of these situations can be quantified with the  $\pi_{j,q}^{m'}$  and  $a_{j,q}^{m'}$  parameters.

The payoff to AAV  $m$  for searching for target  $j$  is defined as  $p_j(q) b_j^m(\hat{\ell}^m(q, k))$  which is the probability that target  $j$  is in cell  $q$  and will be detected by AAV  $m$  in  $\hat{\ell}^m(q, k)$  looks. The cost for searching for target  $j$  with  $\ell^m(q, k)$  looks is simply defined as  $c_j^m(\ell^m(q, k)) = c \ell^m(q, k)$ , where  $c > 0$  is the cost of a single search. Note that the cost of one look at target  $j$  in cell  $q$  is given by  $\gamma_j^m(\ell^m(q, k) + 1) = c_j^m(\ell^m(q, k) + 1) - c_j^m(\ell^m(q, k)) = c$ . In our simulations we will incorporate the cost of moving to a location as part of the decision making; however, in our theory  $c$  is independent of distance. It is a future direction to incorporate distance into  $c$  for the theory.

Let  $\hat{\beta}_j^m(\hat{\ell}^m(q, k) + 1)$  be the  $m$ th AAV's probability of failing to detect target  $j$  on the first  $\hat{\ell}^m(q, k)$  looks in cell  $q$  and succeeding on the  $\hat{\ell}^m(q, k) + 1$  look (where 1 in the expression  $\hat{\ell}^m(q, k) + 1$  is the look that AAV  $m$  is considering whether to take), given that target  $j$  is in cell  $q$  and assuming that no other AAVs look at cell  $q$  before AAV  $m$  does (i.e., it represents the increase in probability that AAV  $m$  finds target  $j$  in cell  $q$  if it takes one more look at it). The variable  $\hat{\beta}_j^m(\hat{\ell}^m(q, k) + 1)$  is

$$\hat{\beta}_j^m(\hat{\ell}^m(q, k) + 1) = \hat{b}_j^m(\hat{\ell}^m(q, k) + 1) - b_j^m(\hat{\ell}^m(q, k)) \quad (4)$$

where  $\hat{b}_j^m(\hat{\ell}^m(q, k) + 1)$  is the estimated detection function computed by time index  $k$  when AAV  $m$  will perform an additional look in cell  $q$ . It is an estimated value since there exists the possibility that new number of looks by AAVs  $m' \neq m$  could be taken before AAV  $m$  takes the look at cell  $q$ , which would change the expression shown in (4).

Next, the rate of return (ROR) [23] in cell  $q$  for target  $j$  and AAV  $m$  can be defined in order to determine the benefit that AAV  $m$  obtains at time index  $k$  when an additional look will be performed to find target  $j$  in cell  $q$ . Mathematically, this is described by the payoff divided by the cost, or

$$\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1) = \frac{p_j(q)\hat{\beta}_j^m(\hat{\ell}^m(q, k) + 1)}{\gamma_j^m(\ell^m(q, k) + 1)} \quad (5)$$

which gives the ratio of the increase in probability to the increase in cost when  $\hat{\ell}^m(q, k)$  looks for target  $j$  have been performed in cell  $q$  and an additional look will be executed by AAV  $m$  in cell  $q$ . Clearly AAV  $m$  would like to place search effort in cells where this quantity is the largest. When the number of looks increases, the value of  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$  will decrease. Also, note that each AAV has an ROR map for each target in any cell. Thus, each AAV has  $N_Q N_D$  ROR maps in total.

### E. Detection of Targets

When AAV  $m$  finds target  $j$ , this AAV makes  $p_j(q) = 0$  and communicates this information to the satellite, which broadcasts to all the AAVs the target that has been found. Hence, all the AAVs make their respective  $p_j(q) = 0$  as well. By making  $p_j(q) = 0$ , the AAV that found target  $j$  along with the ones that received this information are forcing their ROR maps to be equal to zero. This means that this target is no longer of interest to these AAVs. Below, in our stability analysis we will assume either that targets are not found, or that if one is then this corresponds to a state perturbation and hence is viewed as restarting the process.

### F. Percentage Rate of Return Map

Next, we introduce what we call the “percentage rate of return” (PROR) map  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$ , which is the ratio of the percentage change in probability to the percentage change in cost when  $\hat{\ell}^m(q, k)$  looks for target  $j$  have been performed by AAV  $m$  in cell  $q$  and an additional look will be executed by AAV  $m$  in cell  $q$ . Hence we have the equation shown at the bottom of the page. Note that there are differences between the PROR and the ROR map cases. However, as in the ROR case, AAV  $m$  would like to place search effort in cells where  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$  is the largest. The PROR maps are defined for all  $\ell^m(q, k) \geq 1$  so that there is a difference in the way in which the PROR maps are computed compared to the way in which ROR maps are (i.e., there is a divide by zero problem in the PROR case). Recall that in the ROR scenario, AAVs receive the *a priori* probabilities  $p_j(q)$  from the satellite and then compute the ROR maps according to (5) for any  $\ell^m(q, k) \geq 0$ . For the PROR case, the AAVs also require the  $p_j(q)$  values from the satellite, but these values are assumed to be as if the AAVs have not taken the first look, so  $\hat{\rho}_j^m(0) = p_j(q)$  for  $\ell^m(q, k) = 0$  and for all  $m \in V, j \in D, q \in Q$ .

### G. Cooperative Surveillance Strategies

Key challenges in cooperative surveillance include how to use the information on the number of looks from other AAVs and which strategy to use to guide vehicles using that information. Here, the ROR maps will be used in the strategies; however, the PROR maps can also be used by replacing each variable  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$  by  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$ . Next, one choice for search strategies is discussed. This strategy commands an AAV to search for the target in the cell with the highest ROR map that is obtained using both its own number of looks and the possibly outdated number of looks received from other AAVs through communications. In particular, the search decision for AAV  $m \in V$  at time index  $k$  is  $q^*(m, k) \in Q$  which defines the cell it will search in next. Here, the strategy “go to the cell where a single target is most likely to be present” is chosen as

$$q^*(m, k) = \arg \max_{j \in D, q \in Q} \left\{ \hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1) \right\} \quad (6)$$

and once AAV  $m$  reaches the location where cell  $q^*(m, k)$  is, it searches for all target types in it (break ties arbitrarily or via going to the closest cell). To implement this policy AAV  $m$  only needs the possibly outdated number of searches  $\ell^{m'}(q, k - \tau_k^{m'})$ ,  $m' \neq m, m' \in V$ , not the values of maps  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$ . If AAV  $m$  receives a new look value for cell  $q$  from any AAV  $m' \neq m$  while AAV  $m$  is heading to cell  $q$ , this could lead AAV  $m$  to cell  $q' \neq q$  next if the ROR map of cell  $q$  is not the maximizer at the reception time of the new information. Thus, AAV decision reconsiderations about the current maximum benefit are allowed in this framework.

A policy “go look in any cell such that *any* target is most likely to be present” can be defined as follows:

$$q^*(m, k) = \arg \max_{q \in Q} \left\{ \sum_{j \in D} \hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1) \right\} \quad (7)$$

where ties are broken arbitrarily or by going to the closest cell.

Note that both policies only rely on information at time index  $k$  that is locally available at AAV  $m$ ; hence, they can be implemented in a distributed fashion. Also, both policies are computationally simple. Once the ROR values are computed via (5) and (6) only requires the computation of the maximum element of  $N_D N_Q$  values and (7) requires a sum of  $N_D$  values  $N_Q$  times then a choice of the maximum. Equation (6) is a policy biased toward looking for the single most likely target to find. For some patterns of suspected information on targets it could be less effective than the policy in (7) at finding the most targets in the shortest time. In other cases, (6) could be more effective.

Other strategies could also be defined. For instance, assume that  $N_\ell^m = N_\ell, N_w^m = N_w$  for all  $m \in V$  and that the remainders,  $(r)/(N_\ell)$  and  $(s)/(N_w)$ , are both zero. Divide the edge with length  $L$  into  $(r)/(N_\ell)$  segments and the edge with length  $W$

$$\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1) = \frac{p_j(q) \left( \hat{b}_j^m(\hat{\ell}^m(q, k) + 1) - b_j^m(\hat{\ell}^m(q, k)) \right) c_j^m(\ell^m(q, k))}{b_j^m(\hat{\ell}^m(q, k)) \left( c_j^m(\ell^m(q, k) + 1) - c_j^m(\ell^m(q, k)) \right)}$$

into  $(s)/(N_w)$  segments. This decomposes the search area into  $(rs)/(N_\ell N_w)$  “supercells,” each with a size of  $N_\ell N_w$  cells. We designate as a supercell all the cells grouped within the sensor footprint. Also, assume that all the AAVs know how the supercells are numbered so that the discretized search space is  $Q_S = \{1, 2, \dots, rs/(N_\ell N_w)\}$ . Thus, a policy “go look in any supercell such that any target is most likely to be present” can be defined as  $S_q^*(m, k) = \arg \max_{S_q \in Q_S} \{\sum_{j \in D} \hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)\}$  where  $S_q^*(m, k)$  represents the supercell covered by any sensor footprint. A strategy like this but analogous to (6) can also be used.

#### H. Distributed ROR Map Updates

The cooperative AAV surveillance problem can be viewed as being composed of  $N_V$  subsystems and AAV  $m \in V$  is associated with the  $m$ th subsystem. Let the state of AAV  $m$  be  $x^m(k) \in \mathbb{R}^{+N_D N_Q}$  where

$$x^m(k) = [\rho_1^m(\hat{\ell}^m(1, k)), \dots, \rho_1^m(\hat{\ell}^m(N_Q, k)); \dots; \rho_{N_D}^m(\hat{\ell}^m(1, k)), \dots, \rho_{N_D}^m(\hat{\ell}^m(N_Q, k))]^\top$$

is the vector that contains all the ROR map values that AAV  $m$  holds by time index  $k$ . Let

$$x(k) = [x^1(k)^\top, \dots, x^{N_V}(k)^\top]^\top = \mathcal{X} \in \mathbb{R}^{+N_V N_D N_Q} \quad (8)$$

be the states of the system at time index  $k$ . The values of  $\rho_j^m(\hat{\ell}^m(q, k))$ , which will be defined below, are not the same as the  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$  in (5) even though they are both computed at time index  $k$ . The  $\rho_j^m(\hat{\ell}^m(q, k))$  value is the updated ROR map when an event  $e_i^{m,q}(k) \in e(k) \in g(x(k))$  occurs at time index  $k$ . Later, AAV  $m$  computes at  $k$  the estimated rate of return,  $\hat{\rho}_j^m(\hat{\ell}^m(q, k) + 1)$ , of all its maps when an additional look will be performed to find target  $j$  in cell  $q$  based on  $\rho_j^m(\hat{\ell}^m(q, k))$ . Finally, any cooperative surveillance strategy defined in Section II-G could be used to decide which cell AAV  $m$  will likely visit next.

Let the sequence  $\{x(k-1)\} \in \mathcal{X}^{\mathbb{N}}$  denote the state trajectory such that  $x(k) = f_{e(k)}(x(k-1))$  for some  $e(k) \in g(x(k))$  for all  $k \geq 0$ . Define the sequence  $\{e(k)\} \in \mathcal{E}^{\mathbb{N}}$  as an event trajectory such that there exists a state trajectory,  $\{x(k)\} \in \mathcal{X}^{\mathbb{N}}$ , where  $e(k) \in g(x(k))$  for every  $k$ . The set of all event trajectories is denoted by  $E \subset \mathcal{E}^{\mathbb{N}}$ . Since each AAV  $m$  stores  $\ell^{m'}(q, k)$  from any AAV  $m' \neq m$  when it is greater than the current value that AAV  $m$  holds, the set  $E_v \subset E$  can account for these events and it prunes the set  $E$ .

An ROR map will be updated at anytime when an event  $e(k) \in g(x(k))$  occurs using the state transition function  $f_e$ . Thus, there are three ways of updating the ROR maps for each AAV corresponding to the three event types. An ROR map update is performed by AAV  $m$  when it is not visiting a cell of interest and receives one or more number of looks from AAV  $m' \neq m$ . For event  $e_1^{m, L^m(q, k), Q^m(k)} \in e(k) \in g(x(k))$ , components of  $x^m(k)$  with  $q \in Q^m(k)$  have

$$\rho_j^m(\hat{\ell}^m(q, k)) = \rho_j^m(\hat{\ell}^m(q, k-1)) \times \prod_{m'' \in L^m(q, k)} \left(1 - \pi_{j,q}^{m''}\right)^{a_{j,q}^{m''} n^m(\ell^{m''}(q, k))}. \quad (9)$$

The occurrence of event  $e_2^{m, C_m(k)} \in e(k) \in g(x(k))$  forces AAV  $m$  to update  $\rho_j^m(\hat{\ell}^m(q, k))$  so that the components of  $x^m(k)$  with  $q \in C_m(k)$  have

$$\rho_j^m(\hat{\ell}^m(q, k)) = \rho_j^m(\hat{\ell}^m(q, k-1)) (1 - \pi_{j,q}^m)^{a_{j,q}^m}. \quad (10)$$

When  $e_3^{m, L^m(q, k), Q^m(k), C_m(k)} \in e(k) \in g(x(k))$  occurs, components of  $x^m(k)$  with  $q \in C_m(k) \cap Q^m(k)$  have

$$\rho_j^m(\hat{\ell}^m(q, k)) = \rho_j^m(\hat{\ell}^m(q, k-1)) \times (1 - \pi_{j,q}^m)^{a_{j,q}^m} \prod_{m'' \in L^m(q, k)} \left(1 - \pi_{j,q}^{m''}\right)^{a_{j,q}^{m''} n^m(\ell^{m''}(q, k))} \quad (11)$$

and the components with  $q \in (C_m(k) - Q^m(k))$  have

$$\rho_j^m(\hat{\ell}^m(q, k)) = \rho_j^m(\hat{\ell}^m(q, k-1)) (1 - \pi_{j,q}^m)^{a_{j,q}^m}$$

while the components of  $x^m(k)$  with  $q \in (Q^m(k) - C_m(k))$  have

$$\rho_j^m(\hat{\ell}^m(q, k)) = \prod_{m'' \in L^m(q, k)} (1 - \pi_{j,q}^{m''})^{a_{j,q}^{m''} n^m(\ell^{m''}(q, k))} \times \rho_j^m(\hat{\ell}^m(q, k-1)). \quad (12)$$

If the PROR approach is used, then map updates can also be derived as above.

### III. STABILITY ANALYSIS

Let  $H = \{1, 2, \dots, N_V N_D N_Q\}$ . If  $z$  is a vector, then  $(z)_i$  denotes the  $i$ th component of this vector. Let  $\mathcal{X}_\varepsilon = \{x(k) \in \mathcal{X} : 0 \leq (x(k))_i \leq \varepsilon, \text{ for all } i \in H\}$ ,  $\varepsilon > 0$  be the set that holds the AAVs' ROR map values that are less than or equal to  $\varepsilon$  at time index  $k$ . Typically,  $\varepsilon$  is chosen to be small so that, since the search cost  $c$  is fixed, the set  $\mathcal{X}_\varepsilon$  characterizes when the probability that further looks of any cell are unlikely to find a target. Hence, when the state is in  $\mathcal{X}_\varepsilon$  the group of AAVs has “used up” the information about likely target locations that was provided initially by the satellite. Initially, the AAVs by themselves are assumed to have no target location information (so the state is in  $\mathcal{X}_\varepsilon$ ) and then at  $k = 0$  the satellite provides  $p_j(q)$  and this corresponds to a perturbation from the  $\mathcal{X}_\varepsilon$  set. Let  $d(x(k), \mathcal{X}_\varepsilon) : \mathcal{X} \rightarrow \mathbb{R}^+$  denote a metric on  $\mathbb{R}^+$

$$d(x(k), \mathcal{X}_\varepsilon) = \inf \left\{ \max_i \{ |(x(k))_i - (x')_i| \} : \text{for all } i \in H, x' \in \mathcal{X}_\varepsilon \right\}. \quad (13)$$

We are interested in showing that the cooperative surveillance strategy will drive perturbations from  $\mathcal{X}_\varepsilon$  back into  $\mathcal{X}_\varepsilon$ . To do that, we first introduce a lemma that shows that every component  $(x(k))_i$  will get closer to the set  $\mathcal{X}_\varepsilon$  several time indices after  $k$ . The derivation of these time indices after  $k$  depends on temporal (i.e., delays) and spatial (i.e., the position of all the AAVs at time index  $k$ , and the density of the suspected target locations) characteristics of the cooperative surveillance problem.

Finally, we use the result obtained in the lemma to show that  $\mathcal{X}_\varepsilon$  is exponentially stable. The proof for every lemma and theorem can be found in the Appendix.

*Lemma III.1:* Let  $N_q^{m'}$  be the number of times that AAV  $m'$  visits cell  $q$  between time indices  $k$  and  $k + C(N_q^{m'})$ . Let  $q^\ell$  be the last cell visited by AAV  $m$  at time index  $k + C(N_q^{m'})$ . If we define a Lyapunov function

$$V(k) = \max_i \{(x(k))_i - \varepsilon\} \quad (14)$$

and  $N_V$  AAVs use any cooperative surveillance strategy that satisfies (6), then there exists a function

$$C(N_q^{m'}) = N_Q + (N_Q - 1)(N_V - 1) + \sum_{m' \neq m} \left[ \sum_{q=1}^{N_Q} N_q^{m'} + \sum_{q \neq q^\ell} N_q^{m'} (N_V - 1) + N_{q^\ell}^{m'} (N_V - 2) \right] \quad (15)$$

that guarantees that  $V(k + C(N_q^{m'})) < V(k)$ .

*Remark III.1:* A lower bound for  $C(N_q^{m'})$  could be obtained in (15) for the special case where every AAV visits every cell once (i.e.,  $N_q^{m'} = 1$  for all  $m' \in V, q \in Q$ ) and the delays are so large that there is no sharing of the looks performed by any AAV between the time indices  $k$  and  $k + C(1)$ . Thus, (15) is

$$C(1) = N_Q + \sum_{m' \neq m} \sum_{q=1}^{N_Q} 1 = N_Q + (N_V - 1)N_Q = N_Q N_V. \quad (16)$$

*Remark III.2:* A similar result to (16) can be obtained for the case when the delays are small and when every AAV  $m$  visits a group of cells  $q \in Q_c^m \subset Q$  between  $k$  and  $k + C(N_q^{m'})$ . Assume that every AAV  $m$  visits every cell  $q \in Q_c^m$  once,  $N_q^m = 1$ , and that every AAV  $m' \neq m$  receives this information. Also, assume that every AAV  $m$  visits different cells between  $k$  and  $k + N_q^{m'}$  such that  $Q_c^m \cap Q_c^{m'} = \{\emptyset\}$  for all  $m \neq m'$  and  $\bigcup_{m=1}^{N_V} Q_c^m = Q$ . Using (17) for this particular case, we obtain

$$C(1) = N_V \sum_{q \in Q_c^m} 1 + \sum_{m' \neq m} \left[ N_V \sum_{q \in Q_c^{m'}} 1 \right] = N_Q N_V \quad (17)$$

where the fact that  $\sum_{q \in Q_c^m} 1 + \sum_{m' \neq m} \sum_{q \in Q_c^{m'}} 1 = N_Q$  was used in (17). Note that the time indices  $C(1)$  here are completed with the occurrence of the reception of a number of look and not with a visit the last cell; otherwise, there would be no time to communicate the result of the look taken by any AAV at the last cell  $q^\ell$ . Furthermore, the benefit of sharing information is shown in this particular scenario since every AAV does not have to visit every cell, which minimizes AAV's fuel expenditure.

Next, the result of the above lemma is used to show that  $\mathcal{X}_\varepsilon$  is exponentially stable.

*Theorem III.2:* If  $N_V$  AAVs use any cooperative surveillance strategy that satisfies (6), then the set  $\mathcal{X}_\varepsilon$  is invariant and exponentially stable in the large w.r.t.  $E_v$ .

*Remark III.3:* If  $c_3 = \bar{\pi}^{\underline{a}}$  in (35), ROR map updates are driven by the AAV with the most inaccurate sensor and best

probability of detecting targets on a single look. On the other hand, if  $c_3 = 1 - \max\{(1 - \underline{\pi})^{\underline{a}}, \sigma\}$ , it is desirable to have the expression  $\max\{(1 - \underline{\pi})^{\underline{a}}, \sigma\}$  close to 0. This could be achieved, ignoring  $\sigma$ , when  $\underline{\pi} \approx 1$  and/or  $\underline{a} \gg 1$  so ROR maps are flattened out by the AAV with the most inaccurate sensor and smallest probability of detecting targets on a single look. Hence, the final result is related to the fact that the ROR map values will enter faster into the invariant set when the probability of detecting targets in cells on one single look is close to 1 (i.e.,  $\underline{\pi} = \bar{\pi} \approx 1$ ) and the AAVs' group possesses high-quality sensors (i.e.,  $\underline{a} \gg 1$ ) for all  $m \in V, j \in D, q \in Q$ .

*Remark III.4:* An analogous stability analysis can be carried out for the strategy shown in (7). For instance, let  $x^m(k) = [\sum_{j \in D} \rho_j^m(\hat{\ell}^m(1, k)), \dots, \sum_{j \in D} \rho_j^m(\hat{\ell}^m(N_Q, k))]^\top \in \mathbb{R}^{+N_Q}$  be the vector that contains all the ROR map values that AAV  $m$  holds by time index  $k$ . Also, let  $H = \{1, 2, \dots, N_V N_Q\}$  and  $x(k) = [x^1(k)^\top, \dots, x^{N_V}(k)^\top]^\top \in \mathbb{R}^{+N_V N_Q}$  be the states of the system at time index  $k$ . Assume that the definitions for the set  $\mathcal{X}_\varepsilon$ , the metric  $d(x(k), \mathcal{X}_\varepsilon)$ , and the Lyapunov function  $V(k)$  are the same as the ones defined in Section III. It can be shown that the set  $\mathcal{X}_\varepsilon$  is invariant and exponentially stable in the large w.r.t.  $E_v$ .

#### IV. NONCOOPERATIVE SURVEILLANCE PROBLEM

Consider the case where the AAVs do not communicate with each other so they do not share the number of looks. We call this the "noncooperative" problem. In this case, let  $\ell^m(q, k)$  be the number of looks by AAV  $m$ . The rate of return in cell  $q$  for target  $j$  and AAV  $m$  is defined as in classical search theory by  $\rho_j^m(\ell^m(q, k) + 1) = (1)/(c) p_j^m(q) (1 - \pi_{j,q}^m)^{a_{j,q}^m} \ell^m(q, k) (1 - (1 - \pi_{j,q}^m)^{a_{j,q}^m})$ . The strategy "go to the cell where a single target is most likely to be present" defined in Section II-G is

$$q^*(m, k) = \arg \max_{j \in D, q \in Q} \{\rho_j^m(\ell^m(q, k) + 1)\} \quad (18)$$

and once AAV  $m$  reaches the location where cell  $q^*(m, k)$  is, it searches for all target types in it (break ties arbitrarily or by going to the closest cell). Here, the decisions are made at time  $k$  based on real values of the ROR maps when an additional look will be performed at target  $j$  in cell  $q$  and not on estimated values like in the cooperative case. It is the "real value" since there is no information shared by any other AAV and decision reconsiderations are not allowed on this case. The other strategies of Section II-G can also be defined similarly.

Let  $x^m(k)$  and  $x(k)$  be defined as in Section II-H. Furthermore, the ROR map updates need to be defined for this problem. Thus, the components of  $x^m(k)$  with  $q \notin C_m(k)$  remain the same and components with  $q \in C_m(k)$  have  $\rho_j^m(\ell^m(q, k)) = \rho_j^m(\ell^m(q, k - 1))(1 - \pi_{j,q}^m)^{a_{j,q}^m}$ . Next, a comparison between the cooperative surveillance problem and the noncooperative one is established to determine when the cooperative strategies could be superior. To do that, the same definitions introduced in Section III for  $\mathcal{X}_\varepsilon$ ,  $d(x(k), \mathcal{X}_\varepsilon)$ , and  $V(k)$  are used here. Also, we show that the noncooperative strategy will drive perturbations from  $\mathcal{X}_\varepsilon$  back into  $\mathcal{X}_\varepsilon$ .

*Lemma IV.1:* Let  $\bar{N}_q^{m'}$  be the number of times that AAV  $m'$  visits cell  $q$  during time indices  $k$  and  $k + C(\bar{N}_q^{m'})$ . Denote  $q^\ell$

as the last cell visited by AAV  $m$  at time index  $k + C(\bar{N}_q^{m'})$ . If we use (14) and  $N_V$  AAVs use any noncooperative surveillance strategy that satisfies (18), then there exists a function

$$C(\bar{N}_q^{m'}) = N_Q + \sum_{m' \neq m} \sum_{q=1}^{N_Q} \bar{N}_q^{m'} \quad (19)$$

that guarantees that  $V(k + C(\bar{N}_q^{m'})) < V(k)$ .

*Theorem IV.2:* If  $N_V$  AAVs use any noncooperative surveillance strategy that satisfies (18), then the set  $\mathcal{X}_\varepsilon$  is invariant and exponentially stable in the large.

#### A. Comparative Analysis Between Cooperative and Noncooperative Strategies

First of all, we use (13) as the performance metric for comparisons. This metric indicates how fast the components of  $x(k)$  approach the set  $\mathcal{X}_\varepsilon$ . The faster the components approach  $\mathcal{X}_\varepsilon$ , the quicker the targets could likely be found with minimal effort. We start the comparison using the results obtained in Lemmas III.1 and IV.1. Recall that the time indices are driven by the occurrence of events and not by step sizes, so the above lemmas can only be compared under special cases. Thus, we assume in this analysis that the number of time indices generated by both strategies between  $k$  and  $k + C(N_q^{m'})$  ( $k + C(\bar{N}_q^{m'})$  respectively) takes place during an equal time range.

Note that although we consider that the time indices generated by both strategies elapse during the same time, it is still difficult to see which strategy is superior since the sequence of cells visited for each AAV might evolve differently in each problem resulting in different  $N_q^{m'}$  and  $\bar{N}_q^{m'}$  values. Thus, we further narrow the comparison for the special case described in Remark III.1 and also assume that  $\bar{N}_q^m = 1$  in (19) for all  $m \in V, q \in Q$ . For this particular case,  $C(\cdot) = N_Q N_V$  in (15) and (19). This means that when the delays in the cooperative case are larger than the time between indices  $k$  and  $k + C(N_q^{m'})$ , the performance of the cooperative strategy is equal to the noncooperative one since the components, ROR maps of  $x(k)$ , decrease the same amount in both problems during the same time interval. Furthermore, if at least one value of the number of looks transmitted by AAV  $m'$  is received by at least one AAV  $m \neq m'$  between  $k$  and  $k + C(N_q^{m'})$ , then one component of the state of the cooperative strategy is closer to  $\mathcal{X}_\varepsilon$  than one component of the state of the noncooperative problem. Therefore, these two facts show that the cooperative strategy can perform no worse than the noncooperative one for the special case studied here. Moreover, notice that if Remark III.2 is used for the cooperative problem, then the performance of the cooperative strategy is equal to the noncooperative one provided that  $\pi_{j,q}^m = \pi$  and  $a_{j,q}^m = a$  for all  $m \in V, q \in Q, j \in D$ ; however, the former minimizes fuel expenditure by visiting less suspected cells than the noncooperative case.

Next, we provide conditions to evaluate when the cooperative approach is superior to the noncooperative one. For this, we consider again that the time indices generated by both strategies elapse during the same time. Using the conditions of Lemma III.1 defined for the cooperative case, the conditions stated in Lemma IV.1 for the noncooperative case, and the ROR map update formulas, it can be shown that the ROR map values of AAV

$m$  for the cooperative case are less than the ones for the noncooperative case. Moreover, if the condition

$$\left(1 - \pi_{j,q}^{m'}\right)^{a_{j,q}^{m'} N_q^{m'}} \prod_{m'' \neq \{m', m\}} \left(1 - \pi_{j,q}^{m''}\right)^{a_{j,q}^{m''} N_q^{m''}} < \left(1 - \pi_{j,q}^{m'}\right)^{a_{j,q}^{m'} \bar{N}_q^{m'}}$$

is satisfied for the ROR map update formulas of each AAV  $m' \neq m$ , then the performance of the cooperative strategies is superior to the noncooperative strategies.

One important fact to highlight is the result obtained in (35) and (36). Although the upper bounds for these equations are equal, this is a consequence of the conservative nature of the analysis in the cooperative problem. For instance, notice how the final result would change in (35) if the above special case is considered (i.e., AAV  $m$  always receives at least one value of the number of looks between  $k$  and  $k + C(N_q^{m'})$ ). The update of the ROR map in the cooperative case would be made by means of either (27) or (30). It is easy to see, under the considerations assumed above, that the components of the state for the cooperative case would enter faster into the set  $\mathcal{X}_\varepsilon$  than in the noncooperative case. This is precisely the benefit obtained by the AAVs sharing information.

## V. SIMULATIONS

Two simulations are shown in this section. The first one shows how the cooperative surveillance strategies react, during the ongoing mission, to the arrivals of new suspected target location information from the satellite. The second simulation illustrates the performance of nonplanning and planning cooperative strategies, with different horizon lengths, in the presence of communication delays in different ranges.

Travel distances between cells are considered for each AAV at each decision time. Let  $x^q = [x_1^q, x_2^q]^T$  denote the coordinates in the  $(x_1^q, x_2^q)$  plane of the center of the  $q$ th cell. Let  $d_s(x_m(k), x^q) > 0$  be the minimum distance that AAV  $m$  must travel at index  $k$  from its current location and orientation  $x_m(k) = [x_1^m(k), x_2^m(k), \theta^m(k)]^T$  to take a snapshot at cell  $q$ . Notice that since all AAVs are moving all the time, it is not possible to have  $d_s(x_m(k), x^q) = 0$  due to if an AAV needs to take two consecutive snapshots of the same cell, then this particular AAV will travel a distance, different from zero, determined by the path generated by Dubin's car. The cooperative strategy defined in (6) is modified for the nonplanning case as follows:

$$q^*(m, k) = \arg \max_{j \in D, q \in Q} \left\{ \hat{\rho}_j^m \left( \hat{\ell}^m(q, k) + 1 \right) + \frac{1}{d_s(x_m(k), x^q)} \right\}$$

and the update of the ROR maps is performed as defined in Section II-H.

Consider, for the cooperative strategy with planning, that AAV  $m$  plans to visit a sequence of cells next at index  $k$ . The planning cooperative surveillance strategy for AAV  $m$  is made over the ROR maps, where the plan of length  $P$  at index  $k$  is denoted by  $q^*[m, k, P] = q^*(m, k, 0), q^*(m, k, 1), \dots, q^*(m, k, P - 1)$ . Let  $C_m(k, 0)$  denote all the cells covered by the sensor footprint of the  $m$ th

AAV's first plan. Suppose now that the AAVs share the cells  $q^*[m, k, P]$  along with the number of looks for all  $m \in V$ , via intervehicle communications with arbitrary but finite delays. Let  $Q^{m', m}[k, P] \subset Q$  be a set held by AAV  $m$  at index  $k$  that contains the possibly out-of-date sequence of all the cells planned to be visited by each AAV  $m'$ ,  $m' \neq m$  such that  $Q^{m', m}[k, P] = C_{m'}(k - \tau_k^{m'm}, 0), C_{m'}(k - \tau_k^{m'm}, 1), \dots, C_{m'}(k - \tau_k^{m'm}, P - 1)$  while the set  $Q^m[k, P] \subset Q$  denotes the sequence of all the cells planned to be visited by AAV  $m$  at index  $k$  such that  $Q^m[k, P] = C_m(k, 0), C_m(k, 1), \dots, C_m(k, P - 1)$ . Let  $H_\ell = \{0, 1, \dots, P\}$  denote the set of horizon lengths. Thus, the cooperative strategy defined in (6) for  $i \in H_\ell$  is modified for this case to

$$q^*(m, k, i) = \arg \max_{j \in D, q \in Q} \left\{ \hat{\rho}_j^m \left( \hat{\ell}^m(q, k, i) + 1 \right) + \frac{1}{d_s(x_m(k, i), x^q)} \right\}$$

where  $d_s(x_m(k, i), x^q)$  is the distance between AAV  $m$  and cell  $q$  for the  $i$ th plan. The maps  $\hat{\rho}_j^m(\hat{\ell}^m(q, k, i) + 1)$  for  $i \geq 1, i \in H_\ell$  with any cell in both  $q \in Q^{m', m}[k, P]$  and  $q \in Q^m[k, i - 1]$  (note that when  $i = 0$  the updates are performed as described in Section II-H) have

$$\hat{\rho}_j^m \left( \hat{\ell}^m(q, k, i) + 1 \right) = \hat{\rho}_j^m \left( \hat{\ell}^m(q, k, 0) \right) (1 - \pi_{j,q}^m)^{a_{j,q}^m} \times \prod_{q \in Q^{m', m}[k, P]} (1 - \pi_{j,q}^m)^{a_{j,q}^m} \prod_{q \in Q^m[k, i]} (1 - \pi_{j,q}^m)^{a_{j,q}^m}.$$

The set  $Q^m[k, i]$  takes into account all the cells covered by the  $m$ th AAV's sensor footprint up to the  $i$ th projection horizon. On the other hand, the set  $Q^{m', m}[k, P]$  includes all the cells, possibly out of date by  $\tau_k^{m'm}$ , in the projection length  $P$  of each AAV  $m' \in V, m' \neq m$ . Therefore, each AAV  $m$  first flattens out each ROR map associated with each cell contained in the projection length  $P$  of each AAV  $m' \neq m$ , and then it further flattens out, except in the first projection, each ROR map covered by the sensor footprint in its previous projection length before making any decision about the cell to be visited next.

Let the length and width of the search region be  $L = 10000$  m and  $W = 10000$  m, respectively, and let  $r = s = 100$  so that the search environment is divided into  $rs = 10000$  cells and the size of each cell is  $10000$  m<sup>2</sup>. Suppose that a group of three AAVs is considered. AAVs are assumed to move at a constant speed of  $v = 150$  m/s and have a minimum turning radius of 800 m. Each AAV sensor footprint is  $500$  m  $\times$   $500$  m and the distance from each AAV to the center of the footprint is 400 m. There are four suspected areas in the environment (see Fig. 1). Let  $\pi_{j,q}^m = 0.8, a_{j,q}^m = 0.9$  (e.g., imperfect sensors), for all  $m \in V, j \in D, q \in Q$ , the sampling time  $T_s = 0.1$  s, and the simulation length of 600 s. All these above parameters are used in the simulations unless otherwise stated.

#### A. Performance in the Presence of Pop-Up Suspected Locations

The behavior of the cooperative strategies with planning is shown in this section when some suspected locations pop-up

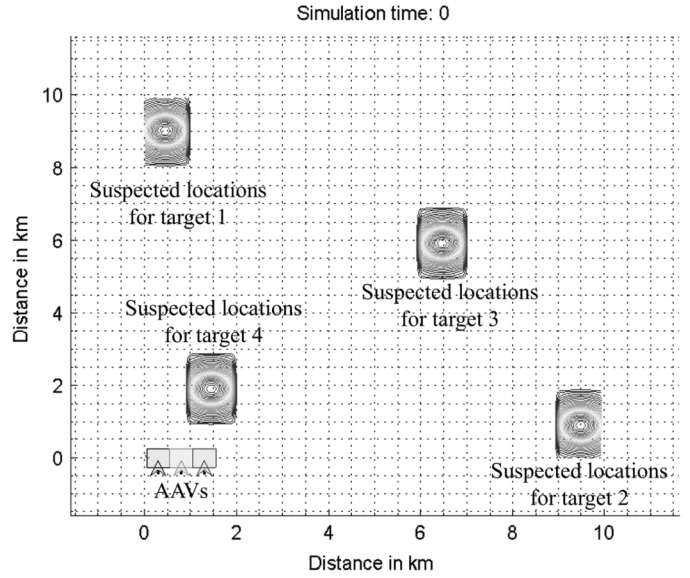


Fig. 1. Initial positions and orientations of AAVs and suspected locations of targets. AAVs are labeled 1 up to 3 from left to right.

when the group of AAVs have already begun the mission. The same parameters mentioned above are used in this simulation except the following. The suspected area of targets 1 and 2 are available to all the AAVs at the beginning of the simulation, while the suspected areas of targets 3 and 4 pop up at time 230 s. Moreover, the horizon length for each AAV is  $P = 3$  and the communication delays,  $1 \leq \tau_k^{m'm} \leq 3$  s, are fixed randomly between AAVs during the mission. The trajectories of the AAVs are shown in Fig. 2. Note that the group of AAVs focus their effort in the suspected locations of targets 1 and 2 since the beginning of the mission and up to the 230 s [see Fig. 2(a)]. Thus, the suspected locations of targets 3 and 4 pop-up 230 s after the mission has begun and the group of AAVs focus on those new areas and finally make some visits to the location where targets 1 and 2 might be [see Fig. 2(b)].

#### B. Influences of Communication Delays and Plan Horizons

We focus here on the AAVs' performance when both non-planning and planning strategies are used for the cooperative surveillance in a mission. Specifically, the impact of both communication delays and the planning horizon length on the planning strategies is investigated. We seek to determine if planning cooperative strategies are always superior to nonplanning cooperative ones. We would like to obtain conditions under which it is best to plan over the maps and when it is best not to do it in the presence of communications delays. We run a Monte Carlo simulation with the following values: the maximum delay in a range  $B^{m'm} \in \{0.2, 1, 3, 5, 10, 20\}$  s for each  $m, m', m \neq m'$ , and a projection lengths range of  $P \in \{1, 2, 3, 4, 5, 6\}$ . Each delay-projection length case consists of 35 simulations where the communication delays are randomly generated in each of these simulations such that  $1 \leq \tau_k^{m'm} \leq B^{m'm}$  for all  $m \neq m'$ . The number of simulations for each delay-projection length combination was chosen such that the standard deviation of the



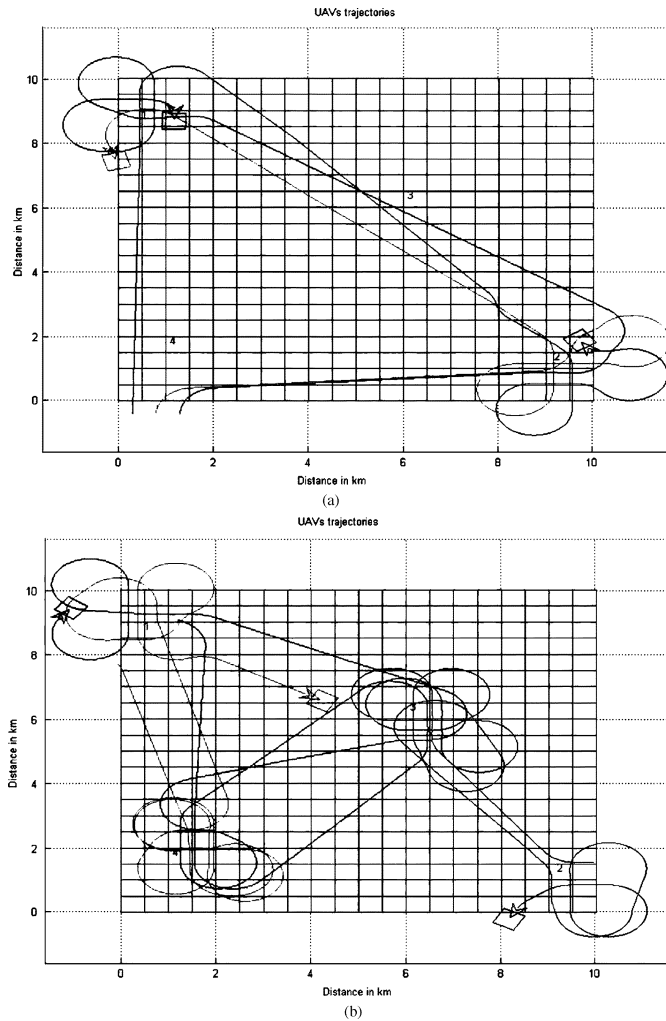


Fig. 2. Performance of cooperative surveillance strategies in the presence of pop-up suspected locations. (a) Mission performance without targets 3 and 4. (b) Mission performance with targets 3 and 4.

performance measures (introduced below) did not change significantly and settled to a relatively small constant value beyond 35 simulations.

To establish a comparison between the performance of nonplanning cooperative and planning cooperative strategies, we use the metric defined in (13) as a way to evaluate the performance of the AAVs. We will compute the average of the average of the metric at each delay-projection length case with  $\varepsilon = 10^{-4}$ . We will also compute the maximum of the average of this quantity over the entire simulation run. The results of the Monte Carlo simulation are shown in Fig. 3. Fig. 3(a) shows the performance of both the nonplanning and planning cases for the average of the average of the metric. Notice that the performance of the nonplanning case stays almost constant for the range of communication delays considered in the simulation. For the planning case, the performance decreases for small delays, except for the “one look ahead” case, due to the fact that most of the time the AAVs make decisions at the same time, which results in a worst performance for the bounded delay equals to 0.2 s. Then, all the performances increase almost steadily for larger delays than 1 s. It is important to highlight that it is

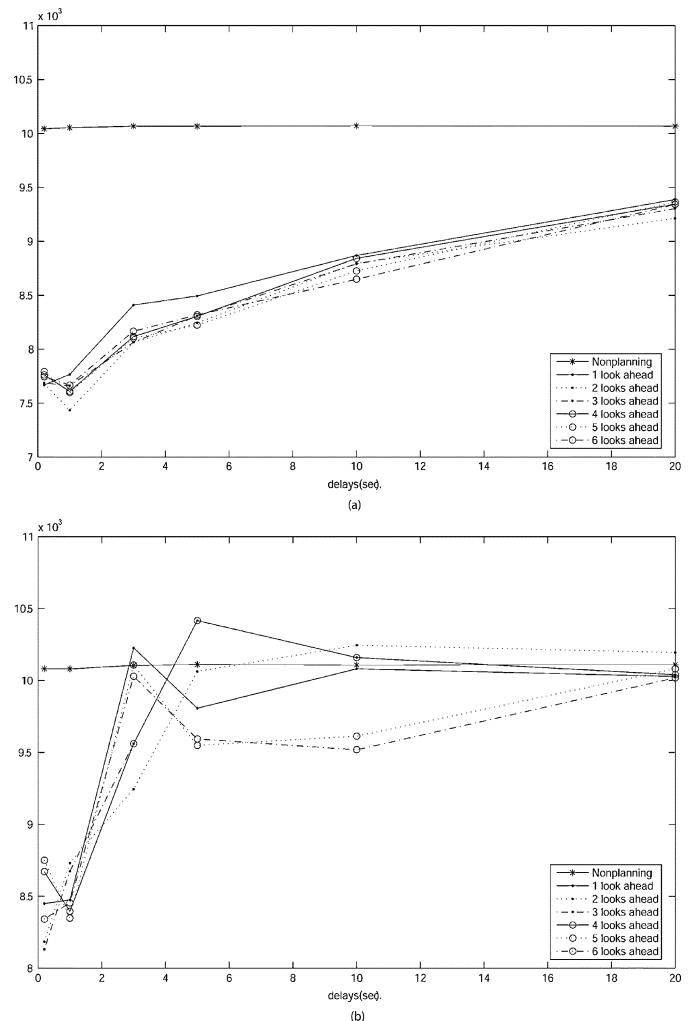


Fig. 3. Performance measures of the nonplanning and planning case. (a) Average of average of metric. (b) Maximum of average of metric.

worthwhile to consider large projection lengths for small communication delays. However, when the communication delays are large then the value of planning ahead over the maps is not useful (e.g., note, for instance, how the performance for all the projection lengths have almost the same value for a delay of 20 s). Moreover, if we run the simulation for larger delays, then the performance of the planning case will eventually be equal to the nonplanning case. Fig. 3(b) shows the performance for both the nonplanning and planning case when the maximum of the average of the metric is used as a performance measure. Notice that it is better to use a nonplanning strategy for the AAVs instead of some planning strategies for delays greater than 3 s. However, all the planning strategies are superior to the nonplanning one for delays less than 3 s. Note that when the delay is equal to 20 s all the performances are about the same so it is better to use a nonplanning strategy for large delays. The results obtained in this section could be used as design guidelines in order to decide whether it is worthwhile to plan ahead or not to do it in the presence of communication delays. In the case of deciding to plan ahead, the proper horizon length could also be chosen depending on the known bounded communication delay present in the network.

## VI. CONCLUDING REMARKS

We have used classical search theory for the coordination of multiple AAVs for finding targets in a limited area with uncertainties from several sources. Also, we have characterized cooperation objectives as stability properties of an invariant set and have used standard Lyapunov stability-theoretic methods for the verification of cooperative control systems. We introduced a noncooperative problem and derived conditions under which its performance is inferior to the cooperative problem. Moreover, design guidelines for tuning the cooperative controller are provided when there are several sources of uncertainties in the system.

Future work should investigate the incorporation of travel distance into the cost  $c$ , the accuracy and speed trade-off in the surveillance mission (e.g., integration of “whereabouts search” [23]), the possibility of adding the predicted information [20] in the current framework in order to enhance team performance, and the incorporation of the optimal search and stop problem [23] into the cooperative surveillance framework when there are rewards for finding the targets in the environment and costs associated with searching. A summary of all the variables defined in this study is given in Table I.

### APPENDIX

1) *Proof of Lemma III.1:* Here, we derive the number of time indices,  $C(N_q^{m'})$ , that elapse after the current time index  $k$  for which we are guaranteed that  $V(k + C(N_q^{m'})) < V(k)$ . We assume the worst case scenario for the temporal-spatial factors, which are when: 1) the delays are so large so that every AAV must visit every cell (i.e., AAV  $m$  visits each cell  $q$  and some time later AAV  $m$  receives new look values from some AAV  $m'$  taken at cell  $q$ ); 2) AAV  $m$  is located in the search environment in such a way that it takes the longest time to visit all the  $N_Q$  cells; and 3) the ROR maps of all the AAVs have the same value in (8) by time index  $k$  (i.e., there are  $N_V N_D N_Q$  maximizers). Note that since all the positions of the AAVs  $m' \neq m$  are closer to the suspected location of the targets than the  $m$ th AAV's position, AAVs  $m'$  could take  $N_q^{m'} \geq 1$  looks in cell  $q$  before AAV  $m$  visits all the  $N_Q$  cells.

We first describe the time indices of  $C(N_q^{m'})$  associated with AAV  $m$ . Note first that AAV  $m$  visits  $N_Q$  cells. Assume that the looks taken in  $N_Q - 1$  cells by AAV  $m$  are received by  $N_V - 1$  AAVs some time later (i.e.,  $(N_Q - 1)(N_V - 1)$  time indices in total). There are  $N_Q - 1$  look values that are received by AAVs  $m' \neq m$  since AAV  $m$  visits the last cell  $q^\ell$  at time index  $k + C(N_q^{m'})$  so there is no time for AAVs  $m' \neq m$  to receive this new information. Thus, the total time indices associated with AAV  $m$  are

$$N_Q + (N_Q - 1)(N_V - 1). \quad (20)$$

Now we account for the time indices of  $C(N_q^{m'})$  associated with each AAV  $m'$ . AAV  $m'$  visits each cell  $q$   $N_q^{m'}$  times (i.e., a total of  $\sum_{q=1}^{N_Q} N_q^{m'}$  time indices). Each visit performed by AAV  $m'$  in any cell  $q \neq q^\ell$  will be communicated and received by  $N_V - 1$  AAVs after some time (i.e., a total of  $\sum_{q \neq q^\ell} N_q^{m'}(N_V - 1)$  time indices). Similarly, each visit performed by AAV  $m'$  in cell  $q^\ell$  will be received by all the AAVs except for AAV  $m$  (i.e., a total

of  $N_q^{m'}(N_V - 2)$  time indices). Recall that, by assumption, a new look performed in cell  $q^\ell$  by AAV  $m'$  is not received by AAV  $m$  until the latter visits cell  $q^\ell$  for the first time. The total time indices associated with AAV  $m'$  are then

$$\sum_{q=1}^{N_Q} N_q^{m'} + \sum_{q \neq q^\ell} N_q^{m'}(N_V - 1) + N_q^{m'}(N_V - 2). \quad (21)$$

Since the time indices in (21) hold for each AAV  $m' \neq m$ , and adding in (20) the total is obtained. ■

2) *Proof of Theorem III.2:* This proof is shown in several steps. First we show that  $\mathcal{X}_\varepsilon$  is an invariant set. Next we show that the ROR maps of the system decrease when they are outside  $\mathcal{X}_\varepsilon$  between time indices  $k$  and  $k + C(N_q^{m'})$ . Later, we show how ROR maps decrease until they enter the invariant set. Finally, we combine all the results to show that the set  $\mathcal{X}_\varepsilon$  is invariant and exponentially stable in the large w.r.t.  $E_v$ . Note that the use of the above lemma guarantees that: 1) any ROR map will decrease in at least  $C(N_q^{m'})$  time indices even if any AAV holds out of date number of looks from other AAVs, and 2) any AAV  $m$  makes definite progress after  $C(N_q^{m'})$  time indices since all the ROR maps of AAV  $m$  have decreased either because AAV  $m$  has visited all the cells  $q \in Q$  or because AAV  $m$  has received a new number of looks performed by any AAV  $m'$  in some cells  $q \in Q$  (i.e., old information from the system has been purged).

$\mathcal{X}_\varepsilon$  is an invariant set. For any  $x(0) \in \mathcal{X}_\varepsilon$ , it is known that  $(x(0))_i \leq \varepsilon$ . An upper bound for the ROR map updates can be derived from (9)–(12) to have

$$\rho_j^m(\hat{\ell}^m(q, k)) \leq \rho_j^m(\hat{\ell}^m(q, k - 1))(1 - \underline{\pi})^{\underline{a}} \quad (22)$$

where  $\underline{\pi} = \min_{m,j,q} \{\pi_{j,q}^m\}$  and  $\underline{a} = \min_{m \in V, j \in D, q \in Q} \{a_{j,q}^m\} > 0$ . From (22), it can be seen that for all  $j \in D, q \in Q, m \in V, i \in H$ , and  $k \geq 0$   $(x(k + 1))_i \leq (x(k))_i$  so that for all  $i \in H$   $(x(k + 1))_i \leq (x(k))_i \leq (x(0))_i \leq \varepsilon$ . Therefore, the set  $\mathcal{X}_\varepsilon$  is invariant.

**ROR maps are outside  $\mathcal{X}_\varepsilon$  between time indices  $k$  and  $k + C(N_q^{m'})$ .** By assumption,  $\max_i \{(x(k))_i\} > \varepsilon$  and  $\max_i \{(x(k + C(N_q^{m'})))_i\} > \varepsilon$ . Also, assume without loss of generality that  $\max_i \{(x(k))_i\} = \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k))$  is the maximizer derived from (6) at time index  $k$  so that AAV  $m' \in V$  chooses to search next for target  $j' \in D$  in cell  $q' \in Q$  at time index  $k$ . There could arise two possible events, denoted a) and b), at time index  $k + C(N_q^{m'})$ .

*Case a:* The maximizer at this time index is not the map associated with target  $j'$  in cell  $q'$  held by AAV  $m'$ . Assume without loss of generality that the map derived from (6) at time index  $k + C(N_q^{m'})$  is  $\max_i \{(x(k + C(N_q^{m'})))_i\} = \rho_j^m(\hat{\ell}^m(q, k + C(N_q^{m'})))$  so that AAV  $m$  chooses to search next for target  $j$  located in cell  $q$ .

Now we use (14) to obtain

$$\begin{aligned} & V(k + C(N_q^{m'})) - V(k) \\ &= \max_i \{(x(k + C(N_q^{m'})))_i - \varepsilon\} - \max_i \{(x(k))_i - \varepsilon\} \\ &= \rho_j^m(\hat{\ell}^m(q, k + C(N_q^{m'}))) - \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k)). \end{aligned} \quad (23)$$

Here, two other possibilities could arise as well.

TABLE I  
SUMMARY OF ALL THE VARIABLES DEFINED IN THIS STUDY

Variables	Description
$x_1^m, x_2^m, v$	Horizontal position, vertical position, and velocity of $m^{th}$ AAV
$\theta^m, \omega_{max}, u^m$	Heading direction, maximum angular velocity, and steering input of $m^{th}$ AAV
$L, W$	Length and width of search environment
$r, s$	Number of cells in the search environment along the horizontal and vertical direction
$N_Q$	Total number of discrete cells
$d_{f_1}^m, d_{f_2}^m$	Depth and width of $m^{th}$ AAV's sensor footprint
$d_s^m$	Distance from $m^{th}$ AAV to the center of the sensor footprint
$p_j^m(q)$	Probability that target $j$ is in cell $q$ for $m^{th}$ AAV
$\ell^m(q, k)$	Number of looks performed in cell $q$ by $m^{th}$ AAV at time $k$
$B^{m' m}$	Maximum delay between AAV $m'$ and AAV $m$
$\tau_k^{m' m}$	Random delay between AAV $m'$ and AAV $m$ at time $k$
$L^m(q, k)$	AAV $m$ receives at time $k$ the set of the AAVs $m', m' \neq m$ that has looked at cells $q$
$n^m(\ell^m(q, k))$	AAV $m$ receives at time $k$ the number of times that AAVs $m', m' \neq m$ have looked at cells $q$
$a_{j,q}^m$	Sensor capabilities of AAV $m$ for target $j$ in cell $q$
$\pi_{j,q}^m$	Probability that AAV $m$ detects target $j$ in cell $q$ on a single look
$b_j^m(\ell^m(q, k))$	Conditional probability of detecting target $j$ in cell $q$ by AAV $m$ at time $k$
$\beta_j^m(\ell^m(q, k) + 1)$	$m^{th}$ AAV's probability of succeeding to detect target $j$ in cell $q$ on an additional look
$\hat{\rho}_j^m(\ell^m(q, k) + 1)$	Ratio of increase in probability to increase in cost for target $j$ in cell $q$ when AAV $m$ executes an additional look
$x^m(k), x(k)$	State of subsystem, AAV $m$ , and the system at time index $k$
$q^*[m, k, P]$	Planning cooperative surveillance strategy of length $P$ for AAV $m$ at time $k$
$C_m(k, i)$	Cells covered by the sensor footprint of the $m^{th}$ AAV's $i^{th}$ plan at time $k$
$Q^{m' m}[k, P]$	Set held by AAV $m$ at time $k$ that contains the sequence of the cells planned to be visited by AAV $m' \neq m$
$Q^m[k, P]$	Sequence of cells planned to be visited by AAV $m$ at index $k$

Case a.1: This is the case where the map of target  $j$  in cell  $q$  has not been updated by any AAV between time indices  $k$  and  $k + C(N_q^{m'})$ . This means that, on one hand, it is known that

$$\rho_j^m(\hat{\ell}^m(q, k + C(N_q^{m'}))) = \rho_j^m(\hat{\ell}^m(q, k)) \quad (24)$$

and, on the other one,  $\rho_j^m(\hat{\ell}^m(q, k)) < \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k))$  since the map of target  $j$  in cell  $q$  has not been modified between time indices  $k$  and  $k + C(N_q^{m'})$ , it means that this map is not equal to the one that maximizes (6) at time index  $k$  (see the lemma where it is studied the worst case scenario). Therefore, there exists a positive constant  $\sigma \in (0, 1)$  such that the following expression is satisfied:

$$\rho_j^m(\hat{\ell}^m(q, k)) = \sigma \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k)). \quad (25)$$

Now, using (24) and (25) in (23)

$$V(k + C(N_q^{m'})) - V(k) \leq -(1 - \sigma)d(x(k), \mathcal{X}_\varepsilon). \quad (26)$$

Case a.2: For this case AAV  $m$  has updated the map of target  $j$  in cell  $q$   $n^m(\ell^m(q, k + C(N_q^{m'}))) + \sum_{m'' \in L_j^m(q, k + C(N_q^{m'}))} n^{m''}(\ell^{m''}(q, k + C(N_q^{m'})))$  times between time indices  $k$  and  $k + C(N_q^{m'})$ . Notice that  $n^m(\ell^m(q, k + C(N_q^{m'}))) \geq 1$  corresponds to the number of times that AAV  $m$  has visited cell  $q$  looking for any target while  $n^m(\ell^{m''}(q, k + C(N_q^{m'}))) \geq 1$  indicates that AAV  $m$  have received new look values performed by any AAV  $m''$  in

cell  $q$  between time indices  $k$  and  $k + C(N_q^{m'})$ . Hence, the ROR map  $\rho_j^m(\hat{\ell}^m(q, k + C(N_q^{m'})))$  of cell  $q$  is modified as

$$\begin{aligned} \rho_j^m(\cdot) &= \rho_j^m(\hat{\ell}^m(q, k)) (1 - \pi_{j,q}^m)^{a_{j,q}^m n^m(\ell^m(q, k + C(N_q^{m'})))} \\ &\times \prod_{m'' \in L^m(q, k + C(N_q^{m'}))} (1 - \pi_{j,q}^{m''})^{a_{j,q}^{m''} n^{m''}(\ell^{m''}(q, k + C(N_q^{m'})))} \\ &\leq \rho_j^m(\hat{\ell}^m(q, k)) (1 - \pi)^{\underline{a}} \end{aligned} \quad (27)$$

where the last inequality is obtained using (22). Moreover, it is also known that

$$\rho_j^m(\hat{\ell}^m(q, k)) \leq \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k)). \quad (28)$$

Now (27) and (28) can be used in (23) to obtain

$$V(k + C(N_q^{m'})) - V(k) \leq -(1 - (1 - \pi)^{\underline{a}})d(x(k), \mathcal{X}_\varepsilon). \quad (29)$$

Case b: The maximizer at time index  $k + C(N_q^{m'})$  happens to be the map associated with target  $j'$  in cell  $q'$  held by AAV  $m'$ . Thus,  $\varepsilon < \rho_j^m(\hat{\ell}^m(q, k + C(N_q^{m'}))) \leq \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k + C(N_q^{m'}))) = \max_i \{x(k + C(N_q^{m'}))\}_i$  and AAV  $m'$  could have updated the map of target  $j'$  in cell  $q'$   $n^{m'}(\ell^{m'}(q', k + C(N_q^{m'}))) + \sum_{m'' \in L^{m'}(q', k + C(N_q^{m'}))} n^{m''}(\ell^{m''}(q', k + C(N_q^{m'})))$  times between time indices  $k$  and  $k + C(N_q^{m'})$ , so that the inequality for map  $\rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k + C(N_q^{m'})))$ , shown at the bottom of the page, holds.

$$\begin{aligned} \rho_{j'}^{m'}(\cdot) &= \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k)) (1 - \pi_{j',q'}^{m'})^{a_{j',q'}^{m'} n^{m'}(\ell^{m'}(q', k + C(N_q^{m'})))} \\ &\times \prod_{m'' \in L^{m'}(q', k + C(N_q^{m'}))} (1 - \pi_{j',q'}^{m''})^{a_{j',q'}^{m''} n^{m''}(\ell^{m''}(q', k + C(N_q^{m'})))} \\ &\leq \rho_{j'}^{m'}(\hat{\ell}^{m'}(q', k)) (1 - \pi)^{\underline{a}} \end{aligned} \quad (30)$$

Using (14) and (30) we have

$$V\left(k + C\left(N_q^{m'}\right)\right) - V(k) \leq -(1 - (1 - \underline{\pi})^a)d(x(k), \mathcal{X}_\varepsilon). \quad (31)$$

**ROR maps enter into the set  $\mathcal{X}_\varepsilon$ .** Here we know that  $\varepsilon \geq \max_i\{x(k + C(N_q^{m'}))\}_i$  so using (14) we have

$$V\left(k + C(N_q^{m'})\right) - V(k) \leq -\bar{\pi}^a d(x(k), \mathcal{X}_\varepsilon). \quad (32)$$

$\mathcal{X}_\varepsilon$  is exponentially stable in the large w.r.t.  $E_v$ . Here, it is sufficient to show that there exists positive constants  $c_1, c_2, c_3$  and that (33) and (34) are satisfied [31]

$$c_1 d(x(k), \mathcal{X}_\varepsilon) \leq V(k) \leq c_2 d(x(k), \mathcal{X}_\varepsilon) \quad (33)$$

$$V\left(k + C\left(N_q^{m'}\right)\right) - V(k) \leq -c_3 d(x(k), \mathcal{X}_\varepsilon) \quad (34)$$

for all  $x(0) \in \mathcal{X}, k \geq 0$  with  $0 < c_3/c_2 < 1$  for some specified  $C(N_q^{m'}) > 0$ . We use the metric defined in (13) and the Lyapunov function introduced in (14). Clearly, (33) is satisfied for  $c_1 = c_2 = 1$ . From (26), (29), (31), and (32), it can be seen that (34) is also satisfied by

$$V(k + C(N_q^{m'})) - V(k) \leq -\max\{1 - \max\{(1 - \underline{\pi})^a, \sigma\}, \bar{\pi}^a\} d(x(k), \mathcal{X}_\varepsilon) \quad (35)$$

where  $c_1 > 0, c_2 > 0$  and  $0 < c_3 = \max\{\bar{\pi}^a, 1 - \max\{(1 - \underline{\pi})^a, \sigma\}\} < c_2$ , which implies that the invariant set  $\mathcal{X}_\varepsilon$  is exponentially stable in the large w.r.t.  $E_v$ . ■

3) *Proof of Lemma IV.1:* Different from the cooperative problem, the derivation of  $C(\bar{N}_q^{m'})$  only depends on spatial issues of the noncooperative surveillance problem. As in the cooperative problem, we assume the worst case scenario for the spatial factors, which are defined by 2) and 3) in the proof of Lemma III.1.

AAV  $m$  generates  $N_Q$  time indices of  $C(\bar{N}_q^{m'})$  since it visits all the suspected cells. AAV  $m' \neq m$  visits each cell  $q \bar{N}_q^{m'}$  times so it contributes a total of  $\sum_{q=1}^{N_Q} \bar{N}_q^{m'}$  time indices of  $C(\bar{N}_q^{m'})$ . Since these events are generated by each AAV  $m' \neq m$ , and adding the indices from AAV  $m$ , the total is obtained. ■

4) *Proof of Theorem IV.2:* The same logic applied on Theorem III.2 can be used here to obtain that  $c_1 = c_2 = 1, c_3 = \max\{\bar{\pi}^a, 1 - \max\{(1 - \underline{\pi})^a, \sigma\}\}$  and that

$$V(k + C(\bar{N}_q^{m'})) - V(k) \leq -c_3 d(x(k), \mathcal{X}_\varepsilon). \quad (36)$$

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