

Feedback Control Engineering for Cooperative Community Development

TOOLS FOR FINANCIAL MANAGEMENT ADVICE FOR LOW-INCOME INDIVIDUALS

HUGO GONZALEZ VILLASANTI,
LUIS FELIPE GIRALDO, and
KEVIN M. PASSINO



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Individuals who manage their financial life to “make ends meet” have a complicated decision-making task. Their challenge is especially arduous if they live in an underprivileged community and have very low income (that is, they are part of the 10.7% of the world’s population that survives on under \$1.90 per day [1]). Most individuals in this situation are self-employed and face uncertain income levels (that is, they are day laborers and only hired periodically). They have inefficient or even nonexis-

tent health services in their community and possibly poor available educational training. Furthermore, they may not have the financial services like insurance to help them cope with sudden and unpredictable events or “shocks” (the effects of adverse weather on crops).

As discussed in “Summary,” the goal of this article is to employ tools provided by feedback control theory to offer financial management advice to low-income individuals seeking to ensure present and future subsistence and rise above poverty. To do this, this work takes advantage of person-to-person financial interactions that provide efficient cooperation plans for community development.

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The financial management problem for low-income individuals, and the current tools employed to cope with scarcity constraints, are explained in [2], where three basic challenges are outlined: 1) dealing with uncertain income, 2) withstanding shocks, and 3) finding strategies to save money. The reach of microfinance institutions that could help mitigate some of these problems is still limited as is the use of technology to provide advice to individuals [3]. Hence, cooperative saving/credit plans play an important role in many low-income communities, since they are relatively easy to implement and provide some degree of financial stability to participants. Examples of cooperation plans that have been implemented in many communities around the world include the accumulating savings and credit association (ASCA) [4], [5] and rotating savings and credit association (ROSCA) [5].

Although these cooperation plans are a useful way to promote socioeconomic development of the community, ROSCAs are limited to instruments for savings, not credits, while ASCAs demand actuarial skills and training for members [2]. There is a need to introduce computer tools that provide a better understanding of a community's behavior that implements these strategies and to design new ones where such disadvantages are minimized. The objective of this article is to

provide these tools. Based on concepts in feedback control theory, we present a model that captures the dynamics of cooperative savings and credit plans and propose a new strategy that improves the community's robustness to shocks. We also introduce several performance measures that evaluate the impact of these strategies in a small community and are consistent with the ones used by the United Nations to quantify human development at the country level (that is, the human development index (HDI) [6]). To the best of our knowledge, this is the first attempt to develop mathematical and computational tools to study the social dynamics of a community that implements cooperative financial plans. We envision this work to be the first step toward building a body of research that looks for more effective cooperation strategies that empower poor communities around the world, where the theory of control systems plays a key role.

We use feedback controllers to provide advice to low-income individuals to make financial decisions. Our approach is related to past work. For example, applications of proportional-integral-derivative (PID) control on savings can be found in [7], while a financial advisor system based on optimal control is found in [8]. In the field of portfolio management, model predictive control [9], fuzzy control [10], and PID control [11] are found in the literature. However, none of these approaches deal with the inherent constraints imposed by the scarcity of resources that people living in low-income communities must endure [2]. Taking into account these constraints, the model we propose represents the dynamics of the community in terms of the interaction of the three dimensions that define human development according to the HDI: income/wealth, health, and education [6]. The influence of health in the development of a community is modeled using the concept of "health capital" introduced by Grossman in his seminal work [12]. Here, health is seen as a capital stock that depreciates with time and is increased by investment. The same approach is adopted for education, which has been addressed in the human capital literature [13]. We integrate these three dimensions and their interaction in a single model of the community.

The article first introduces a decision-making model that describes the behavior of an individual in terms of wealth, health, and education. This model is based on personal financial management and optimal control theory and assumes that individuals have the ability to make decisions that depend on their interests and priorities, ability to generate wealth, health condition, current education, unexpected events, and time horizon for decision making. Individuals are then interconnected to create a heterogeneous community, where they interact according to a cooperative strategy. We implement the ASCA in the simulated community and propose a new strategy based on donations. We introduce measurements such as the management failure rate (MFR) and community development index (CDI) to quantify the performance of the community in terms of the three dimensions that describe human development. Through Monte

Summary

Individuals who live in underprivileged communities have complicated decision-making tasks regarding their financial lives. Our goal is to employ feedback control theory tools to offer strategic financial advice to low-income individuals to ensure subsistence and improve well-being. A model that accounts for scarcity constraints and interaction among wealth, health, and education is explained, and a model predictive controller (MPC) is designed at the individual level, using behavioral characteristics, to offer reliable advice on asset allocation. Simulations show that the MPC successfully manages the individuals' assets, provided their initial assets and abilities are high enough. Results on the individual management are scaled to study efficient cooperation plans for a community of individuals by modeling an accumulating savings and credit association (ASCA) and a decentralized donation strategy. Simulations show that both programs improved our well-being performance measures, for intermediate values of the individuals' financial contribution to the cooperation plans, with performance degrading for low and high contributions. Furthermore, the ASCA plan shows better performance than the donations strategy in communities with low inequality in assets and abilities, while the later plan showing better results in unequal communities. These results suggest that these plans should be tailored for each group to maximize positive impacts.

Carlo simulations, we show how the strategy based on donations allows communities to be significantly more robust to shocks than the ASCA.

FEEDBACK CONTROL FOR INDIVIDUAL FINANCIAL DECISION MAKING

We introduce a nonlinear stochastic difference equation that describes the relationship among wealth, health, and education of a low-income individual along with a feedback predictive controller that models the individual's decision-making process based on his/her priorities and prediction time horizon.

Model of Individual Management Dynamics

We adopt the model's parameters with respect to a single individual, assuming that a household model can be scaled from this single individual model. Considering that the majority of low-income individuals in the developing world are self-employed, mostly in agriculture [14], we model the well-being dynamics of an individual who must use part of his/her wealth to invest in a risky business in a low-income community. Note that providing management advice for wage employees can be accomplished with small modifications to the following model. Time k is measured in months, and the individual's working capital at time k is $x_1(k) \in \mathbb{R}$, measured in U.S. dollars using the purchasing power parity (PPP) rate. The individual accumulates savings $x_2(k) \in \mathbb{R}$, measured in PPP dollars, to insure himself from uncertainties in the model. Health and education capital are denoted by the variables $x_3(k) \in \mathbb{R}$ and $x_4(k) \in \mathbb{R}$, respectively. Studies suggest that the body mass index (BMI), measured in kg/m^2 , constitutes an appropriate index to measure health [15], particularly in developing countries [16], [17]. Hence, we adopt the BMI as a measure for the health capital $x_3(k)$. Education capital $x_4(k)$, typically measured in years of school, is adapted here to be measured in months of school (MS). The financial management of the individual's life is modeled according to the classical problem of optimal consumer-investor intertemporal portfolio choice in discrete time [18]. However, a particular difference of our model is that, instead of choosing the ratios of capital $x_1(k)$ to invest between risky assets (health and education) and risk-free assets (savings), we choose to model them as transfers between the different assets, $u_m(k)$, for $m = 2, 3, 4$, to better reflect the nonlinearities (for example, BMI and MS cannot be directly converted into dollars via disinvesting) and the intrinsic dynamics of BMI and MS. The model is of the form

$$x_1(k+1) = \left(x_1(k)(1-s(k)) - \sum_{m=1}^4 u_m(k) \right) (1+r(k)), \quad (1)$$

$$x_2(k+1) = (x_2(k) + u_2(k))(1+d_2), \quad (2)$$

$$x_3(k+1) = x_3(k)(1-d_3) + c_3 u_3(k), \quad (3)$$

$$x_4(k+1) = x_4(k)(1-d_4) + c_4 u_4(k), \quad (4)$$

where, at the beginning of time period k , the individual observes the current assets and must decide how to transfer the working capital $x_1(k)$ between consumption of non-capital goods $u_1(k) \in \mathbb{R}$, the amount to save $u_2(k) \in \mathbb{R}$, the amount to spend in food $u_3(k) \in \mathbb{R}$ to manage the BMI, and the amount to spend on education $u_4(k) \in \mathbb{R}$. Later, the individual engages in a risky business by investing the remaining working capital and expects to obtain, at the end of that period, an amount depending on the return on investment (ROI) at time k , $r(k)$. By employing a linear approximation to the semilogarithmic (Mincerian) wage function and data found in [17], $r(k)$ is modeled as

$$r(k) = a_1 + a_3 x_3(k) + a_4 x_4(k) + z(k),$$

where a_1, a_2 , and a_3 are nonnegative scalars representing the basic return and the returns on health and education, respectively. Together, these scalars represent the individual's capability to generate income. For example, jobs requiring extensive manual labor will have higher values of a_3 , while jobs demanding "intellectual labor" will have higher values of a_4 . A similar model was used in [19] with linear returns on financial and human wealth. In any risky investment, the individual business is subject to uncertainty, commonly referred as "volatility," modeled here with a normally distributed, zero-mean random variable $z(k) \sim \mathcal{N}(0, \sigma)$, and it acknowledges that investments in low-income markets usually fluctuate significantly. Another source of uncertainty is the one produced by sudden losses of cash and capital, common in uninsured communities affected by natural disasters, thefts, machinery malfunctions, and social or health events [2]. These discrete-time events are called "shocks" [20], and it is assumed that they occur at the beginning of the time period, thus, the individual observes the shock before deciding how to spend the wealth. Shocks are modeled with the independent random variable $s(k) = v_s p(k)$, where $v_s \geq 0$ is the shock ratio and $p(k)$ is a Bernoulli random variable with mean p_s . The savings $x_2(k)$ are subject to a net return $d_2 \in \mathbb{R}$, which might be negative if the depreciation is higher than the interest paid, if any.

The dynamics of health $x_3(k)$ and education $x_4(k)$ are modeled according to typical capital dynamics and defined in (3) and (4), respectively. As the investment in these assets is distributed across the time period, we assume that the depreciation rates $d_3 \geq 0$ and $d_4 \geq 0$ affect only the respective capitals. Assuming that the height does not change significantly during the simulation period (three years), the BMI dynamics (3) represents a first-order approximation of the human body weight dynamics, which can be found in [21]. The scalars $c_3 \geq 0$ and $c_4 \geq 0$ are conversion terms between monetary units and kg/m^2 and MS, respectively. With the basic assumption that the individual must ingest food to manage his/her BMI, the term c_3 can be obtained using the price of food per kilogram [22], while c_4 can be obtained using the average cost of education per month

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found in [23]. The health depreciation rate d_3 is related to the amount of food needed to manage the BMI and can be found by converting the monthly energy requirement per unit of mass (kcal/kg/month) found in [24] to the required amount of food (kg), per unit of body mass (kg), per month. Education depreciation d_4 was obtained in [25] and [26], and it can be explained by the need for ongoing education to adapt to technological and social advances.

Along with the definition of the desired levels for each of the states in the next section, the model represents a wealth allocation problem very similar to what any individual or household in a developed (wealthy) community faces. However, individuals in low-income communities must deal with uncertainty, lack of insurance mechanisms, and limitations imposed by the scarcity of resources, leading to subsistence constraints and credit availability constraints [2]. Some of these challenges can be taken into account by the following constraints:

$$u_4(k) \leq \frac{1}{c_4}, u_m^i(k) \geq 0, \forall m = \{1, 3, 4\}, \quad (5)$$

$$x_m(k) \geq 0, \forall m = \{1, 2, 4\}, h \leq x_3(k) \leq H, x_4(k) \leq E, \quad (6)$$

where the lower bound on working capital $x_1(k)$ is imposed due to the individual's inability to access credits. Consumption $u_1(k)$ must be nonnegative, as well as $u_3(k)$ and $u_4(k)$, to consider that the individual cannot disinvest from his/her health and education capitals. The BMI $x_3(k)$ should be above severe malnutrition values $h \geq 0$, which are around 16 kg/m^2 , and it should be below the optimal value $H \geq h$ to avoid obesity and maintain the linearity of $r(k)$ with respect to $x_3(k)$. Education should be nonnegative and lower than a value $E \geq 0$, which depends on the available levels of education in the individual's community. The upper bound in $u_4(k)$ accounts for the fact that the individuals cannot acquire more than a month of school training per month.

Feedback Control as a Management Advisor

Ensuring present and future subsistence in a risky environment by allocating wealth across the different dimensions in the model is not a trivial problem, but it is one that people in low-income communities must solve to overcome poverty. Given the model in (1)–(4), a feedback controller could provide advice, in the form of a control input $u(k) = [u_1(k), u_2(k), u_3(k), u_4(k)]^\top$, to increase wealth, health,

and education and, at the same time, insure against uncertainty. In [27], three feedback controllers were tested on a similar financial model to the one in (1) and its financial constraints. It was shown that a model predictive controller (MPC) and a PID controller performed similar to a dynamic-programming-based optimal controller in finding feasible control inputs and achieving the desired values of wealth. Furthermore, the MPC allowed for "smoother" consumption trajectories compared to the PID, which is desirable for actual implementation settings. Thus, we choose to implement the MPC to describe the individual's behavior to address his/her financial management problem. The MPC features the explicit use of a model to predict the individual's financial, health, and educational status and computes a control sequence that minimizes the defined cost function up to a fixed time horizon. The process is later repeated at each time step, using the output information of the previous time step. We introduce the formulation using the terminology found in [28], with polyhedral constraints. At time k , we solve the T -horizon optimal control problem

$$\begin{aligned} \min_{u_0} J_T(x(k), U_0) &= J_f(x_T) + \sum_{t=0}^{T-1} g(x_t, u_t) \\ \text{subject to: } x_{t+1} &= f(x_t, u_t), t = 0, \dots, T-1 \\ A_x x_t &\leq b_x, A_u u_t \leq b_u, t = 0, \dots, T-1 \\ A_f x_T &\leq b_f, x_0 = x(k), \end{aligned} \quad (7)$$

where $U_0 = [u_0, \dots, u_{T-1}]$. The predictive controller applies the first element of the solution as the control input at time k , that is, $u(k) = u_0^*$. Solving an open-loop problem allows us to avoid dimensionality complications and, at the same time, provides robustness against external disturbances with the recomputation of the control input at each step in a feedback loop (as has been seen in various industrial applications [29]).

Various alternatives exist for the choice of the cost function $g(x, u)$. In economics, the standard isoelastic utility functions have been employed in individual consumption and investment decisions under shocks in [20], while in [30], another utility function that employs risk and time preferences was used to model a consumer's decision with human capital returns (the Epstein–Zin–Weil utility function). The main parameters in these functions are the risk and time preferences of the individual. In modern portfolio theory, a quadratic function involving the mean and

variance of the returns as a measure of risk has been employed [31]. However, as the objectives of this article are to provide management tools in midterm, well-being goals focusing on performance, reliability, and future scalability of the controller, we take a *normative* approach and employ the quadratic cost function

$$g(x(k), u(k)) = (x(k) - x^*)^\top Q(x(k) - x^*) + (u(k) - u^*)^\top R(u(k) - u^*), \quad (8)$$

with $x(k) = [x_1(k), x_2(k), x_3(k), x_4(k)]^\top$ and $u(k) = [u_1(k), u_2(k), u_3(k), u_4(k)]^\top$ as the state and control inputs, respectively. The vectors $x^* = [x_1^*, x_2^*, x_3^*, x_4^*]^\top$ and $u^* = [u_1^*, u_2^*, u_3^*, u_4^*]^\top$ store the desired state and expenditure (input) values, respectively. For all individuals, the values are $x_3^* = H$, the optimal BMI, and $x_4^* = x_4(0) + P$, where P is the total time period considered in this study (36 months). The desired savings level should be set with knowledge of the expected value of the shock $E[s]$, such that $x_2^* = \gamma E[s]$, with $\gamma \geq 0$. The desired working capital x_1^* and final consumption u_1^* should be set to an attainable level, avoiding long periods of low consumption to achieve the desired states. Furthermore, x_1^* and u_1^* should be chosen such that x^* and u^* constitute an equilibrium of the system in (1)–(4), with $u_2^* = d_2 x_2^* / (1 + d_2)$, $u_3^* = x_3^* d_3 / c_3$, and $u_4^* = x_4^* d_4 / c_4$. The rest of the desired values can be obtained by setting the desired states as an equilibrium point.

The weights on the matrices Q and R are chosen based on the desired midterm goals of the article, which are to ensure subsistence and increase consumption, health, and the education capital during the studied period. We define two metrics to help design the controller: the *individual development index* (IDI) and the MFR. The IDI measures the performance of the advisor in terms of consumption u_1 , health x_3 , and education x_4 , achieved by the advised individual. We chose consumption as a measure of financial well-being, as it is employed in the utility functions of the previously cited economic studies and the average consumption across time will remain above zero, making it suitable for the geometric mean calculation below. To formulate the IDI, we employ the approach found in [32] and compute a normalized mean with respect to wealth (consumption) \bar{w}_i , health \bar{h}_i , and education \bar{e}_i . We used the desired values for each dimension to normalize these variables

$$\bar{w} = \frac{\sum_{k=t_0}^{t_1} u_1(k)}{(1 + t_1 - t_0)u_1^*}, \quad \bar{h} = \frac{\sum_{k=t_0}^{t_1} x_3(k)}{(1 + t_1 - t_0)x_3^*},$$

$$\bar{e} = \frac{\sum_{k=t_0}^{t_1} x_4(k)}{(1 + t_1 - t_0)x_4^*},$$

where t_0 and t_1 are the initial and final time, respectively. The IDI is the geometric mean of the three dimensions

$$\text{IDI} = (\bar{w}\bar{h}\bar{e})^{\frac{1}{3}}.$$

A management failure occurs when the controller cannot find a feasible control input $u(k)$, such that the constraints in (5) are not violated. This occurs when the individual is not endowed enough to maintain his/her BMI above the lower bound h .

To simplify the design of the weights of the matrices Q and R and the amount to save x_2^* as an emergency fund, we define two design parameters inspired by the individual preferences from economics: the risk-seeking parameter $p_1 \in (0, 1)$ and the well-being parameter $p_2 > 0$. Using these parameters, the matrices Q and R are defined as

$$Q = \begin{bmatrix} \frac{1}{x_1^*} & 0 & 0 & 0 \\ 0 & \frac{1-p_1}{x_2^*} & 0 & 0 \\ 0 & 0 & \frac{p_2}{x_3^*} & 0 \\ 0 & 0 & 0 & \frac{p_2}{x_4^*} \end{bmatrix}, \quad R = \begin{bmatrix} \frac{p_2}{u_1^*} & 0 & 0 & 0 \\ 0 & \frac{p_1}{u_1^*} & 0 & 0 \\ 0 & 0 & \frac{p_2}{u_3^*} & 0 \\ 0 & 0 & 0 & \frac{1}{u_1^*} \end{bmatrix}.$$

The desired states and inputs are employed to scale the model dimensions, where u_1^* replaces u_2^* and u_4^* , as the latter are small or close to zero. Note that higher values of p_2 assign higher penalties to control inputs that are away from the desired consumption u_1^* and food expenses u_3^* , as well as BMI x_3^* and education x_4^* . In contrast, lower values of the well-being parameter may result in delayed spending to accumulate working capital and savings. The parameter p_1 affects the weight assigned to accumulating emergency funds $x_2(k)$ to insure the individual from the shocks $s(k)$. Higher values of p_1 assign a higher penalty to expenses in savings $u_2(k)$ and reduce the weight assigned to track the desired savings level x_2^* . This latter term can be set as

$$x_2^* = (1 - p_1)v_s x_1^*,$$

such that a risk averse plan with p_1 close to zero will save the amount that would be lost in the event of a shock (that is, $p(k) = 1$). Finally, the MPC in (7) is computed using a linearized model of (1)–(4) introduced in the following section, employing the expected values of the random variables $s(k)$ and $z(k)$.

Controller Formulation

Due to the difficulties of performing a theoretical analysis of the stability properties of the system in (7) under the MPC feedback control, we use Monte Carlo simulation to estimate the average behavior and variability of different measures that quantify the individual's performance, where it is assumed that the individual is prone to unexpected variations of state variables. However, this approach is computationally expensive, due to the repeated solution of the optimization problems in the MPC. To alleviate this computational burden, we formulate the controller using a

linear approximation of the system, employing this formulation during the simulation. One of the main advantages of using a linear model in predictive control is that the predicted states can be efficiently computed using matrix multiplications, and the optimization problems are reduced to quadratic programming problems.

We first compute the linear approximation of the model in (7). Let \hat{x} and \hat{u} be the points where the nonlinear equation is linearized at time step k . The update rule of the dynamical system is

$$\Delta x(k+1) \approx A\Delta x(k) + B\Delta u(k) + f(\hat{x}, \hat{u}) - \hat{x}, \quad (9)$$

where $\Delta x(k) = x(k) - \hat{x}$ and $\Delta u(k) = u(k) - \hat{u}$ are the state and control vectors of the individual relative to the linearization point and

$$A = \left. \frac{\partial f(x, u)}{\partial x} \right|_{x=\hat{x}, u=\hat{u}}, \quad B = \left. \frac{\partial f(x, u)}{\partial u} \right|_{x=\hat{x}, u=\hat{u}}.$$

To formulate the cost function associated with the MPC, we derive the predicted states over the time horizon using matrix multiplications. Let $\Delta U_0 = [\Delta u_0^\top, \Delta u_1^\top, \dots, \Delta u_{T-1}^\top]^\top$ be the concatenation of the given control vectors over a horizon of T time steps starting at k . The predicted states over the horizon are given by

$$\Delta X = S^x \Delta x(k) + S^u \Delta U_0 + L(f(\hat{x}, \hat{u}) - \hat{x}), \quad (10)$$

where $\Delta X = [\Delta x(k)^\top, \Delta x_1^\top, \dots, \Delta x_T^\top]^\top$ is a concatenation of the predicted state vectors, and S^x, S^u , and L are matrices that depend on A and B defined as

$$S^x = \begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ A^T \end{bmatrix}, \quad S^u = \begin{bmatrix} 0 & & & \\ AB & & & \\ \vdots & B & \dots & \\ \vdots & \vdots & \ddots & \\ A^{T-1}BA^{T-2}B & \dots & & \end{bmatrix}, \quad L = \begin{bmatrix} 0 \\ I \\ I+A \\ \vdots \\ \sum_{i=0}^{T-1} A^i \end{bmatrix}.$$

Using this matrix formulation of the predicted states over a horizon, we are able to formulate the MPC problem as the quadratic programming problem

$$\begin{aligned} \min_{\Delta U_0} J_T(\Delta x(k), \Delta U_0) &= \frac{1}{2} \Delta U_0^\top H \Delta U_0 + F^\top \Delta U_0 \\ \text{subject to } G_0 \Delta U_0 &\leq w_0 + E_0 \Delta x(k), \end{aligned}$$

with

$$\begin{aligned} H &= 2(S^{u\top} \bar{Q} S^u + \bar{R}) \\ F &= 2[S^{u\top} \bar{Q} (S^x \Delta x(k) + L(f(\hat{x}, \hat{u}) - \hat{x}) + 1_T \otimes (\hat{x} - x^*)) \\ &\quad + 2\bar{R}^\top 1_T \otimes (\hat{u} - u^*)], \end{aligned}$$

where 1_T is a column vector with T ones, and \otimes denotes the Kronecker product, while $\bar{Q} = \text{diag}\{Q, \dots, Q, Q_f\}$ and $\bar{R} = \text{diag}\{R, \dots, R\}$, with Q_f corresponding to a quadratic terminal cost J_f . The matrices in the constraint inequality are

$$G_0 = \begin{bmatrix} \bar{A}_u \\ \bar{A}_x S^u \end{bmatrix}, \quad w_0 = \begin{bmatrix} \bar{b}_u \\ \bar{b}_x - \bar{A}_x L(f(\hat{x}, \hat{u}) - \hat{x}) \end{bmatrix}, \quad E_0 = \begin{bmatrix} 0 \\ -\bar{A}_x S^x \end{bmatrix}$$

where $\bar{A}_u = \text{diag}\{A_u, \dots, A_u\}$, $\bar{b}_u = \text{col}\{b_u, \dots, b_u\}$, $\bar{A}_x = \text{diag}\{A_x, \dots, A_x, A_f\}$, and $\bar{b}_x = \{b_x, \dots, b_x, b_f\}$. We note that the expected values of the random variables were employed to compute the matrices A and B .

Controller Design and Performance

We set the simulation horizon to $P = 36$ months, with a sample time of one month, and a controller horizon of one year or $T = 12$ months. Longer horizons do not improve the performance. To design the controller parameters p_1 and p_2 , we use Monte Carlo simulations to measure the average MFR and IDI for different initial conditions. We employ model parameters that represent a low-income individual with low initial financial and human capital and low skills, reflected in low ROIs a_1, a_3, a_4 . We choose $a_1 = 0$ and $a_3 = 0.009\alpha$, with α selected uniformly between $\alpha \in [1, 1.5]$ and $a_4 \in a_3/10$. These parameters lie within the lower returns found in [17], once conversion to the monthly rate of returns is completed. The savings depreciation rate d_2 is assumed to be zero. According to [24], the daily energy requirement for men with an active or vigorous life is between $\bar{d} \in [47, 64]$ kcal/kg of body mass. The average energy density and price of food found in [22] are 1240 kcal/kg and 1.7606 US\$/kg, respectively. Assuming that the height remains constant during the time frame of the application of the controllers, the health depreciation term, representing the monthly amount of food (in kg) per kg of body mass recommended, is $d_3 = (30/1240)\bar{d}$. The conversion term is $c_3 = 1.7606/h_i^2$, where h_i is the i th individual's height, which we assume is equal to 1.7 m. Rapid advances in technology prompted studies to estimate the depreciation rate of education among workers [25], [33]. However, none of these results have been performed on rural areas of developing countries. We are inclined to assume that this value is very small as, typically, primary and secondary education depreciate slower than higher education at 8% per year [33], hence $d_4 = 0.005$. With respect to the conversion cost, using the data found in [23] for 15 African countries, one year of education costs an average of US\$102 per child. Hence, the monthly conversion cost is $c_3^i = 12/102$. The lower bound on BMI is $h = 16$, below which the individual is considered to be in severe malnutrition, while the upper bound on BMI is $H = 25$ to avoid obesity. The upper bound on education is $E = 109$, equivalent to nine years of schooling available in the individual's community. The random variable $z(k)$ has a standard deviation of 0.08, while the values of the shock are $v_s = 0.5$ and $p_s = 0.06$, consistent with the frequency of events requiring loans found in [2]. The initial conditions are $x_2(0) = 0$, $x_3(0) \in [0, 20]$, $x_4(0) \in [0, 36]$ and

$$x_1(0) = \beta \left(\frac{r_0}{r_0 - 1} \right) \frac{d_3 x_3(0)}{c_3}, \quad (11)$$

where $\beta \in [1, 1.5]$ is selected uniformly and $r_0 = a_1 + a_3x_3(0) + a_4x_4(0)$. Hence, all individuals are spending ($\sum_{j=1}^4 u_j(k)$), between US\$1 and US\$2 and per day, and are sufficiently endowed to at least maintain the initial state without considering income volatility and shocks. The desired consumption is

$$u_1^* = 31.5 + 8u_1^0, \quad (12)$$

where this expression is employed to consider that better endowed individuals with higher ROI should plan to achieve higher consumption. For this, the *excess initial consumption*, $u_1^0 = (((r_0 - 1)/r_0)x_1(0) - u_3(0))$ contains information about the returns and initial conditions. The parameters in (12) were obtained to equalize the time at which the desired consumption is achieved, which on average is 20 months. The scalar 31.5 is the basic desired consumption and corresponds to a total expenditure of US\$2 per day, that is, $\sum_{j=1}^4 u_j(k) = 60$, with which, according to [34], the individuals will exit extreme poverty and become low-income individuals. The corresponding x_1^* is

$$x_1^* = \frac{r^*}{r^* - 1} \sum_{j=1}^4 u_j^*,$$

where $r^* = a_1 + a_3x_3^* + a_4x_4^*$.

The results of a Monte Carlo simulation with 800 runs are shown in Figure 1 for mean of mean MFR as a percentage of the number of cases when a management failure event occurred and for mean of mean IDI in Figure 2 for values of the risk parameter $p_1 \in [0.05, 0.95]$ and the well-being parameter $p_2 \in [0, 20]$. We obtained convergence of both the mean and standard deviation of the variables after performing the mentioned number of runs (determined to be a sufficient number for means and standard deviations to converge). Recall that the MFR estimates the probability of an advised individual failing to achieve any goals for the given decision process. It can be seen that lower management failures are achieved by plans having lower risk and welfare parameters, but this is achieved at the expense of a lower IDI, resulting in lower expenses. Higher emergency savings counteract the shocks and income uncertainty. For the following simulations in this article, we choose $p_1 = 0.3$ and $p_2 = 8$, which provide a decent compromise between the two measures. Figure 3 shows the time trajectories of state and input for an individual under the advice of the designed MPC. Note that consumption is delayed until enough capital is accumulated to generate higher profits. These profits are later spent on food, education, savings, and consumption. In the event of a shock hitting the working capital of the individual, savings are transferred to x_1 and expenses are reduced to facilitate a quick recovery. However, not all individuals can escape extreme poverty with the advice of the MPC. Figure 4 shows the results for the mean of mean MFR employing an 800-run Monte Carlo simulation for individuals with the same model parameters as in the

previous simulations, except for a_3 and $x_1(0)$, which are generated using $\alpha \in [0.8, 3.5]$ and $\beta \in [0.8, 3.5]$. We obtained convergence of both the mean and standard deviation of the variables after performing the mentioned number of runs. It can be seen that individuals with low returns and low initial capital are not endowed enough to accumulate wealth and improve their health and education. This result is consistent with the concept of a “poverty trap” [20], in this case as a consequence of the nonlinearities imposed by the input constraints as seen in [27], and expanded in this work by considering health and education variables.

DISTRIBUTED FEEDBACK CONTROL FOR PROMOTING COMMUNITY CHANGE

The analysis in the previous section showed that irregularities and uncertainty in an individual’s income and health

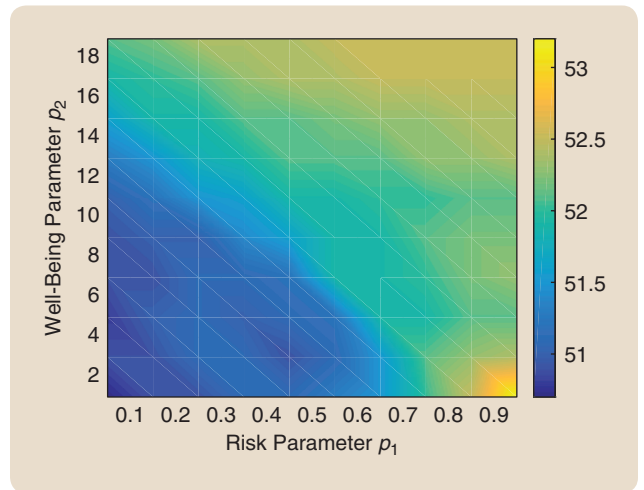


FIGURE 1 The mean of mean management failure rate for an 800-run Monte Carlo simulation for different values of risk parameter p_1 and well-being parameter p_2 .

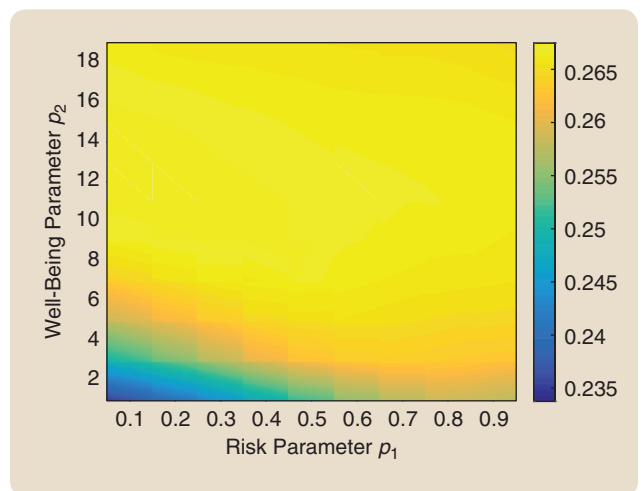


FIGURE 2 The mean of mean individual development index for an 800-run Monte Carlo simulation for different values of risk parameter p_1 and well-being parameter p_2 .

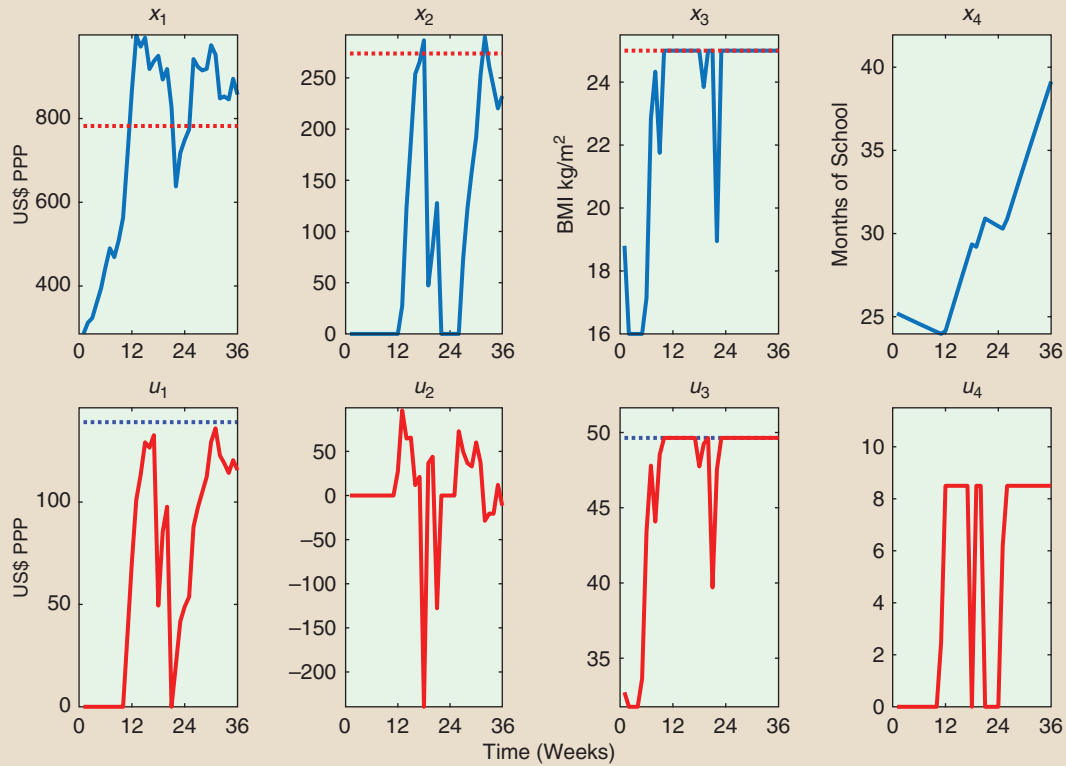


FIGURE 3 The trajectories of working capital $x_1(k)$, savings $x_2(k)$, health measured by the body mass index (BMI) $x_3(k)$, education $x_4(k)$, consumption $u_1(k)$, savings rate $u_2(k)$, health expenditure $u_3(k)$, and education expenditure $u_4(k)$ under the advice of the model predictive controller.

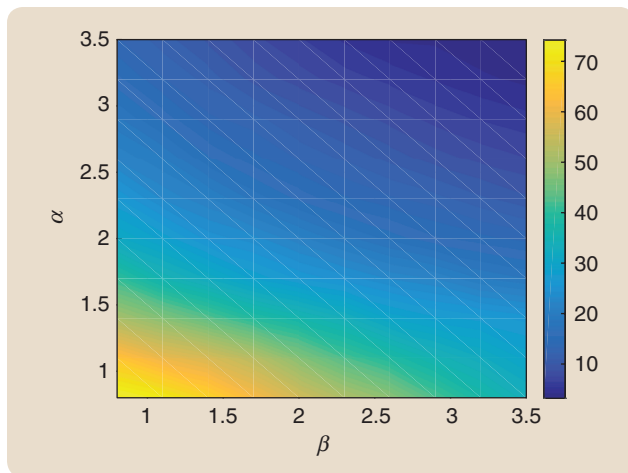


FIGURE 4 The mean management failure rate for a 1000-run Monte Carlo simulation for different values of return on investment multiplier α and initial capital multiplier β .

can be such that the decision-making process is not able to guide him/her to survival. Many people in poor communities do not have access to organizations that provide financial services to help with such situations [2]. In such cases, person-to-person cooperation plays a very important role for the survival of a community. Communities that use cooperation

strategies are usually called “self-help groups,” where groups manage financial transactions without the intervention of financial organizations. These strategies have been implemented in many different parts of the world, and they have demonstrated that they can help with the socioeconomic development of communities [2], [35]–[37].

Using the concept of cooperation in a self-help community, we extend the model of the individual’s financial decision making such that it now incorporates variables associated with the individual’s participation in a cooperation plan. Accordingly, individuals in the community are represented as interconnected controlled systems, where the control strategy takes information from and acts on the individual’s state variables and those in the community who interact with him/her. We refer to the collective action of individuals during the cooperation process as a distributed predictive control in the community.

First, we study the community behavior when the distributed predictive control is based on an ASCA [4], [5], a well-known method of cooperation for saving and lending money in low-income communities. Then, we propose a second strategy that defines a cooperation network based on a donation plan.

The performance of the community that employs the distributed controller is measured by determining the number

ASCAs and ROSCAs are the most popular forms of financial cooperation among individuals in low-income communities.

of individuals who fail to survive during the time interval the state variables are observed and by quantifying the development of the community. The first measurement is the MFR. The second measurement used is CDI, which assesses the achievements of a community in terms of health, wealth, and education, taking into account inequalities in their distribution among the members of the community. The CDI is inspired by the “inequality-adjusted HDI” [38], [39], a criterion that evaluates a country’s development by quantifying the average achievements of the country with respect to the three dimensions of human development (income, health, and education) and then penalizing unequal distributions in and between dimensions.

To define the CDI, we employ the approach found in [32], and, for each individual i in community m , we compute a normalized mean with respect to wealth \bar{w}_i , health \bar{h}_i , and education \bar{e}_i . These variables are normalized using the desired values for each dimension

$$\bar{w}_i = \frac{\sum_{k=t_0}^{t_1} u_1^i(k)}{(1+t_1-t_0)u_1^{i*}}, \quad \bar{h}_i = \frac{\sum_{k=t_0}^{t_1} x_3^i(k)}{(1+t_1-t_0)x_3^{i*}},$$

$$\bar{e}_i = \frac{\sum_{k=t_0}^{t_1} x_4^i(k)}{(1+t_1-t_0)x_4^{i*}}.$$

Note that for computing the normalized mean on wealth, we employ consumption $u_1^i(k)$, as it is bounded to be above zero, making it appropriate for the geometric mean computations described below. Consumption has been employed in economic models and is an adequate signal for wealth status. The mean over all individuals is computed with

$$\bar{\bar{w}} = \frac{1}{N} \sum_{i=1}^N \bar{w}_i, \quad \bar{\bar{h}} = \frac{1}{N} \sum_{i=1}^N \bar{h}_i, \quad \bar{\bar{e}} = \frac{1}{N} \sum_{i=1}^N \bar{e}_i.$$

The Atkinson inequality index [40] for each dimension is computed by

$$Aw = 1 - \frac{\prod_{i=1}^N \bar{w}_i}{\bar{\bar{w}}^N}, \quad Ah = 1 - \frac{\prod_{i=1}^N \bar{h}_i}{\bar{\bar{h}}^N}, \quad Ae = 1 - \frac{\prod_{i=1}^N \bar{e}_i}{\bar{\bar{e}}^N}.$$

This index provides a measure of inequality between individuals. These values are employed in the final computation of the CDI for a community m in

$$\text{CDI}^m = \left((1 - Aw) \left(\frac{\bar{\bar{w}}}{\bar{w}} \right) (1 - Ah) \left(\frac{\bar{\bar{h}}}{\bar{h}} \right) (1 - Ae) \left(\frac{\bar{\bar{e}}}{\bar{e}} \right) \right)^{\frac{1}{3}}, \quad (13)$$

where we normalize with the maximum instantaneous value of consumption, health, and education across all individuals and communities

$$\hat{w} = \max_m \left(\max_i \left(\max_k (u_1^{i \in m}(k)) \right) \right),$$

$$\hat{h} = \max_m \left(\max_i \left(\max_k (x_3^{i \in m}(k)) \right) \right),$$

$$\hat{e} = \max_m \left(\max_i \left(\max_k (x_4^{i \in m}(k)) \right) \right).$$

The CDI in (13) can be used to assess if *all* the community members are knowledgeable and live a decent and healthy life.

Accumulating Savings and Credit Associations (ASCAs)

ASCAs and ROSCAs are the most popular forms of financial cooperation among individuals in low-income communities, as they are used on almost every continent, as detailed in “Financial Cooperation Strategies in Low-Income Communities.” Due its greater reach and relatively low operational complexity, we model the ASCA and study its impact on the CDI and MFR of several simulated communities.

The ASCA Strategy

According to [2], an ASCA is usually composed of 20–50 members and operates as follows. Community members agree on the value of a “share” of the ASCA and the minimum and maximum amount of shares that each member can purchase per week or month. The money gathered from the sales of shares is stored, and at a later time, when enough resources have been pooled, that money is offered to the members in the form of loans. The community agrees on the amount and time of the loan and the interest to be charged. At the end of the process (usually after one year), the stored funds of the ASCA (a product of the sale of shares and the loan interest) are divided among members according to the amount of shares that each individual owns. The process can then start again if members agree.

To study the effectiveness of the ASCAs in terms of CDI and MFR, we model the financial decision process of a member of the ASCA with an augmented model of the individual decision making. Here, we introduce the superscript i to denote the i th individual in the community. We expand the state space by including two additional financial dimensions, the total shares owned in the ASCA $x_5^i(k) \in \mathbb{R}$ and the debt contracted with the ASCA $x_6^i(k) \in \mathbb{R}$. We define the input $u_5^i(k) \in \mathbb{R}$ to be the number of purchased shares and $u_6^i(k) \in \mathbb{R}$ is the debt payment (withdrawal) in the ASCA. The augmented part of the dynamic model employed by the MPC is

$$\begin{aligned}x_5^i(k+1) &= x_5^i(k) + c_5 u_5^i(k), \\x_6^i(k+1) &= (x_6^i(k) - u_6^i(k))(1 + i_6),\end{aligned}$$

$$\bar{u}_5^i(k) = \begin{cases} -x_a(k)x_5^i(k) / \sum_{j=1}^N x_5^j(k) & \text{if } k = nK \\ u_5(k) & \text{if } k \neq nK \end{cases},$$

with constraints for the augmented part of the input and state space as $u_5^i(k) = C_a$ and $x_5^i(k) \geq 0$, $x_6^i(k) \geq 0$. Here, $c_5 \geq 0$ represents the cost of the shares in monetary units and $i_6 \geq 0$ is the monthly interest rate charged for credits by the ASCA. We constrain $u_5^i(k)$ to the constant $C_a \geq 0$ representing a fixed amount of shares to be purchased monthly by the members. The rest of the model remains unchanged. While the model defined previously is to be employed in the MPC formulation, the input $u_6^i(k)$ is also constrained by the credit amount that the ASCA is allowed to offer. To account for this, we consider the dynamics on the ASCA's funds $x_a(k) \in \mathbb{R}$ with N members as

$$x_a(k+1) = x_a(k) + \sum_{i=1}^N \bar{u}_5^i(k) - \sum_{i=1}^N \bar{u}_6^i(k),$$

where the terms $\bar{u}_5^i(k)$ are of the form

and represent at time nK with $K > 0$ and $n = 1, 2, \dots$ the return of the shares of the ASCA in the form of cash to the members at the specified end of cycle nK times, while time $k \neq nK$ represents the monthly contributions to the ASCA in the form of shares. The second sum corresponds to the total amount granted in loans to the members at time k , with $\bar{u}_6^i(k) = u_6^i(k)$, for all i in the community if the total amount requested is less or equal than the amount available in the ASCA's fund at time k , that is,

$$x_a(k) + \sum_{i=1}^N \bar{u}_5^i(k) \geq \sum_{i=1}^N \bar{u}_6^i(k).$$

We assume that the ASCA's funds cannot become negative or incur debt to finance members loans. If the last inequality does not hold, the available funds are allocated between the individuals with lower health index $x_3^i(k)$ in amounts proportional to $u_6(k)$, their desired loan at time k . During

Financial Cooperation Strategies in Low-Income Communities

Individuals in low-income communities are subject to great risks, generated by unhealthy environments and a lack of opportunity. Unable to access formal markets of credit and insurance to maintain a decent living, low-income individuals resort to cooperation to cope with risk [S1]. This population generally resides in rural areas, not yet reached by the microfinance movement, which is serving at least 750 million individuals in the developing world [S2]. In [2], three main strategies are identified among populations from developing countries. Savings clubs are among the simplest implementations of financial cooperation, and often they feature individuals contributing a fixed amount per month. The members retrieve the lump sum of their savings in a later date, which usually coincides with festivities or events requiring big expenses. The pooled money is usually stored in a member's house, which does not eliminate the risk of losing the money due to a robbery, fire, flood, or other events. In rotating savings and credit associations (ROSCAs), members agree on the monthly contribution amount. The total amount pooled each month is awarded to one of the members, usually via lottery or auctions, and the process continues until all members have been awarded the monthly pool. The simplicity, flexibility, and security provided are the key benefits of ROSCAs as the money pooled does not need to be stored. The peer effect also encourages compliance with the rules, boosting savings. However, the reach of ROSCAs is limited because they usually have been implemented between individuals that share strong social links to reduce the risk of fraud by a member that was awarded an early pool and did not contribute afterward.

On the other hand, in accumulating savings and credit associations (ASCAs), part of the pooled money contributed is offered in loans to the members and nonmembers, charging an interest rate. At the end of the cycle, the pooled money, along with the generated interest, is returned to the members. Although the return on investment is positive for well-functioning ASCAs, it requires extensive bookkeeping. More recently, crowdfunding and peer-to-peer mobile lending have reached out to low-income individuals and provided socially conscious entrepreneurs with an opportunity to earn profits by lending capital to low-income communities, enabling investments in agriculture and small enterprises [S3]. The use of devices for these online financial services by low-income individuals provides valuable access to data, which can be instrumental in improving the reach and impact of financial services to the poor [S4].

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the simulations we found that using the BMI to allocate the loans produces the lower MFR for the ASCA, as individuals reduce their food expenses in times of financial need.

The MPC problem is locally solved for each individual. To consider the cyclic nature of the ASCA, at the end each ASCA period, constraints are modeled such that the MPC is aware of the return of the contributions and the need to repay debt. The ASCA funds are then allocated using the procedure detailed in the previous paragraph.

Simulations for the ASCA Strategy

To simulate the community dynamics with the ASCA, we select $N = 30$ members. To add heterogeneity to the groups, we choose the return parameter α and initial capital parameter β according to a probability distribution often employed in modeling human groups' wealth heterogeneity, the Pareto distribution [1]. The scale parameter is $x_m = 0.08$ and shape parameter $\alpha = 12.5$, making it a fairly homogeneous community. Other parameters remain the same as the previous study. The ASCA begins granting loans after the first months and the cycle ends at the eighth month, distributing all the funds among the surviving members. We assume that the ASCA charges a monthly interest rate of $i_6 = 0.012$.

We achieved convergence in the mean and standard deviation on an 800-run Monte Carlo simulation (determined to be a sufficient number for means and standard deviations to converge). The results for various amounts of monthly ASCA contribution C_a are shown in Figure 5 for the MFR and in Figure 6 for the CDI. The plots show the mean (circles), median (horizontal line), and first and third quartiles (box edges) for both performance variables. The results show that values of ASCA monthly contributions of $C_a = \text{US}\$32$ produce lower MFR mean, median, and first and third quartiles. Similar positive results can be seen in the CDI, when the highest value is attained with $C_a = \text{US}\$32$. However, by increasing C_a beyond 48 requires a significant portion of the income during the first weeks, thus the MFRs increase and the CDI decreases. Overall, the results show that, if properly tailored to each community, the cooperation in the form of ASCAs is effective in mitigating risks, reducing management failures, and promoting community development. In addition to the relative ease of implementation in low-income communities, these facts have made the ASCA an instrumental financial device to combat poverty. However, the cooperation strategy to be introduced in the next section overshadows these results with a more direct interaction between members of the community.

Donations in Cooperation Networks

In the previous section we studied the beneficial effects of having shared funds in the community for saving and lending among the community members. Now, we consider a strategy in which cooperation relies on altruistic acts. This strategy is based on the assumption that individuals are willing to give away money to help other members

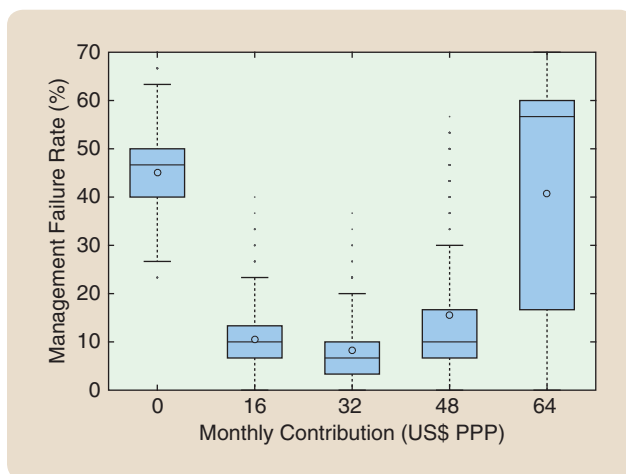


FIGURE 5 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the management failure rate resulting from different values for the members' monthly contributions to the accumulating savings and credit association.

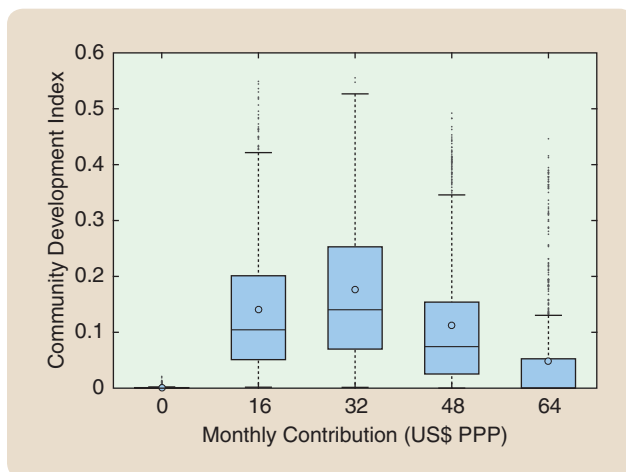


FIGURE 6 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the community development index resulting from different values for the members' monthly contributions to the accumulating savings and credit association.

of the community when it is needed, so long as each person does this. We propose a model in which donations are dynamically assigned by a distributed predictive control that follows a network of interactions among the community members. We show, through simulations, how this strategy can be a very powerful mechanism for the survival and development of the whole community and discuss the situations that can facilitate the emergence of cooperation or make it fail.

The Donation Strategy

The members of the community interact with each other following a given network, where V is the set of vertices representing the individuals, and E is the set of edges or links that represent the interconnections between pairs of

individuals. Edge $(i, j) \in E$ indicates that individual i cooperates with individual j , that is, individual i can transfer money to individual j . Let $N_i = \{j: (i, j) \in E\}$ be the set of neighbors of individual i . "Neighbor" in this context does not refer to spatial closeness but to a member of the community that individual i cooperates with. An individual can interact with other individuals at different levels of cooperation. Let $g_{ij} \geq 0$ be a "generosity parameter" that quantifies the cooperation level from individual i to $j \in N_i$ [32]. For individual i , a larger value of g_{ij} implies a larger priority for cooperating with j . We then refer to the set of individuals in the community, their cooperative relations, and the level of cooperation between them as a "cooperation network," which is represented by the weighted directed graph

$C = (V, E, G)$, where $G = \{g_{ij}: i \in V, j \in N_i\}$ is the set of generosity parameters.

Cooperation in the community is based on a donation strategy. With no expectations of future returns, individuals give part of their current assets (x_1 variable) to their neighbors who are considered to be in need. Again, in this strategy, it is assumed that an individual cooperates with his/her network neighbors based on the information about their health status and how much money they need to maintain the optimal BMI. Let $u_{ij}(k)$ be the amount of money that individual i will donate to individual $j \in N_i$ at time step k . The desired donation from i to j will then be given by

$$u_{ij}^*(k) = g_{ij} \max[u_3^* - u_3^j(k-1), 0].$$

Here, the local MPC takes into consideration not only the individual's goals but also the health status of his/her neighbors in the community, distributing the donations according to their needs and the generosity parameters. The update for state variable x_1^i is given by

$$x_1^i(k+1) = x_1^i(k) - \sum_{l=1}^4 u_l^i(k) - \sum_{j \in N_i} u_{ij}, \quad (14)$$

with variable constraints $\sum_{j \in N_i} u_{ij} \leq D_{i_r}$ and $u_{ij} \geq 0$, for every $j \in N_i$. Here, $D_{i_r} \geq 0$ is the maximum amount of money that individual i can use for donations. The cost matrix associated with the control signal \hat{R}_i is defined as

$$\hat{R}_i = \begin{bmatrix} R_i & 0 \\ 0 & G_i \end{bmatrix}, \quad (15)$$

where R_i is a 4×4 block defined in (8), and G_i is an $|N_i| \times |N_i|$ diagonal matrix with nonzero entries given by the generosity parameters for individual i multiplied by $R(1, 1)$, the penalty term for deviations from desired consumption. The desired values for the control variables associated with the donation strategy change over time and correspond to $u_{ij}^*(k)$. Following (9) with the update in (14), the MPC calculates the expenses that allow the individuals to achieve their goals and participation in the community development through the cooperation network.

Simulations for the Donation Strategy

We study the effect of the donation strategy on the dynamics of a community of 30 individuals. The individuals' parameters are chosen to be the same as the ones employed in the ASCA simulation as well as the distribution of the random variables. We assume that all the individuals have the same generosity parameter, that is, g_{ij} for all $i \in V$ and $j \in N_i$. The mean and standard deviation of the CDI and MFR are estimated using 800 simulation runs (determined to be a sufficient number for means and standard deviations to converge), where the uncertainty comes from the random shocks. These measurements are computed for different values of the generosity parameter. Figures 7 and 8

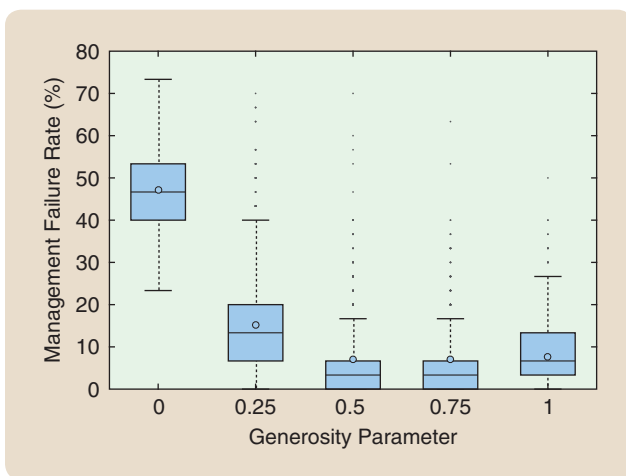


FIGURE 7 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the management failure rate resulting from using a donation strategy for different values of the generosity parameter and cooperation network with a fully connected topology.

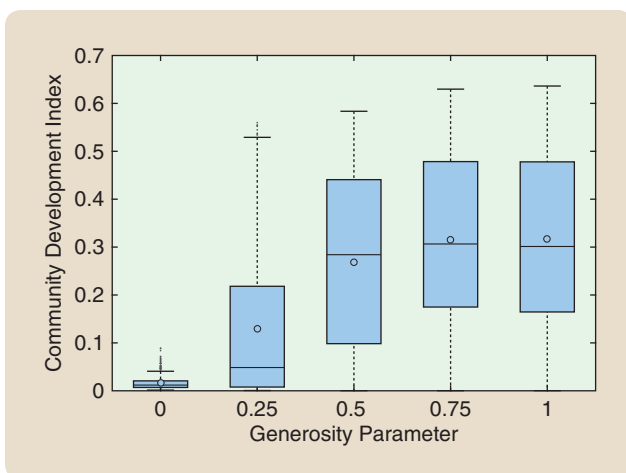


FIGURE 8 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the community development index resulting from using a donation strategy for different values of the generosity parameter and cooperation network with a fully connected topology.

show the MFR and CDI results for various values of generosity parameter. The plots show mean (circles), median (horizontal line), and first and third quartiles (box edges) for both performance variables. The results show that when individuals cooperate by donating to their less-endowed neighbors by providing the adequate amount of resources at adequate timing to individuals in need, the community achieves improved results in the CDI and MFR, compared to the results for ASCA (compared to Figures 5 and 6). However, for generosity values greater than 0.75, the community performance degrades with the increase in MFR. This can be explained by the fact that donations are so big that they leave donors vulnerable to shocks.

The cooperation strategy based on donations relies on the assumption that all the members of the community have some degree of a disinterested prosocial behavior, quantified by the generosity parameter in the model in (15), acting for the benefit of others at the cost of themselves. Studies on cooperation have shown that there are several situations in the community that facilitate the emergence of this behavior. For example, altruistic cooperation arises more easily among community members who are closely related or have developed trust through previous interactions [41], [42]. Also, members tend to behave altruistically to develop a reputation that will allow them to receive benefits from someone else [41, Ch. 2]. On the other hand, a donation plan can also prevent individuals from cooperating in the community. Giving without expecting any return can make individuals behave in a way that they choose not to get involved in cooperation. This is known as a *social dilemma* [43], where the shortsighted actions of the individuals prevent the entire community from obtaining long-term benefits. In this case, “individual rationality leads to collective irrationality” [43, p. 183].

Unequal Communities: Comparing ASCA and Donations

The previous subsections showed simulation results in which we assumed a somewhat equal community, with similar skills and initial assets across individuals and simulation runs. However, more than 75% of the population in developing countries live in communities where the income is not equally distributed, and this income inequality tends to increase over time [44]. This report also provides evidence that supports the negative impacts of income inequality in economic development, poverty reduction, and its correlation with other non-income inequalities and lack of opportunities.

In this section, we compare both cooperation approaches considering more unequal communities by generating the inequality via a variable commonly employed in inequality studies in developing countries, the per capita consumption [45]. To make both approaches comparable, we employ a variable that does not change in time and is equal for both approaches, the individual’s desired consumption found in (12). Thus, to generate inequality in the community we must gen-

erate values for the ROI multiplier α and the initial capital multiplier β . From economic research, it is known that the distribution of income follows a unimodal, right-skewed and long-tail distribution and is generally modeled following a Pareto distribution [46]. Assuming that $\beta = \alpha$ and that the consumption distribution parallels income distribution [47], we generate random values for α following a Pareto distribution. We aim at generating Gini coefficients on desired consumption that match what was reported in [45], that is, Gini values within [0, 0.6] (with “0” representing perfect equality and inequality increasing as the value increases). To avoid differences in total initial assets/skills across Monte Carlo runs, we maintained these totals constant by normalizing the outcome of the random number generator. We employ $C_a = 32$ for the ASCA and generosity $g = 0.75$ for the donations strategy, which gave the best results in the previous section.

Figures 9 and 10 show results of the MFR and CDI, respectively, for a Monte Carlo simulation with 800 runs. This number of runs was enough to achieve convergence in the mean and standard deviation of both variables. The horizontal axis denotes the mean Gini coefficient of the individuals’ desired consumption, the boxes correspond to first and third quartiles, the horizontal lines denote the medians, and the circles denote the means. The mean Gini coefficient is our measure of inequality, and increasing values of this coefficient correspond to an expected MFR and CDI performance degradation for both ASCA and the donation plan. At lower inequality values, the ASCA produces higher CDI mean and median values than the donation, for similar values of MFR. However, as the inequality increases, the donation strategy achieves higher mean and median CDI values with lower values of the MFR than the ASCA. This result suggests that, as the inequality increases in the community, the fixed contribution implementation

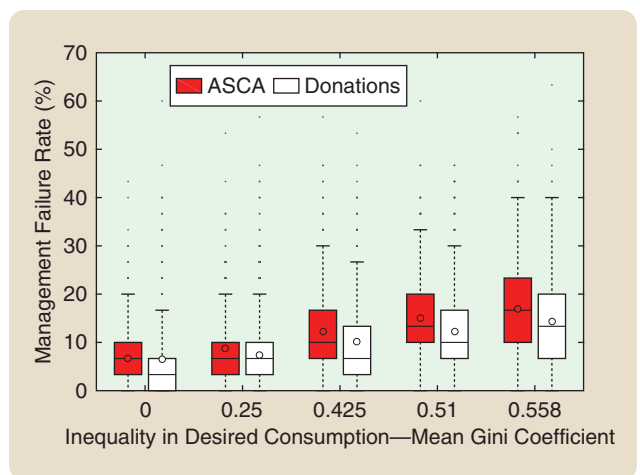


FIGURE 9 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the management failure rate for groups using the accumulating savings and credit association (ASCA, in red) and donation strategy (white) for different values of consumption inequality.

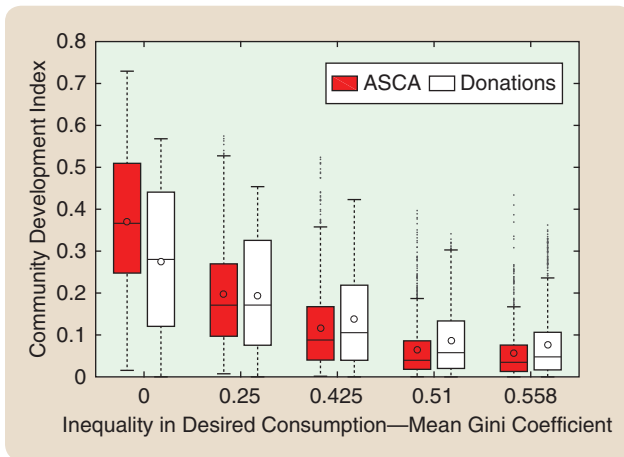


FIGURE 10 The mean (dots), median (middle solid line), and first and third quartile (box edges) of the community development index for groups using accumulating savings and credit association (ASCA, in red) and donation strategy (white) for different values of consumption inequality.

of the ASCA may not be as efficient as the direct donation strategy in allocating the community resources to achieve the desired social outcome of a high CDI and low MFR. This is consistent with the findings in [48], where it is concluded that savings association members tend to share similar characteristics to minimize the risk of default.

DISCUSSION

Using key studies in the literature, a model is formulated that represents an individual's financial life that includes effects from health and education. Using scarcity and subsistence constraints, the model is fit to what has been found for very low-income individuals. The MPC was chosen to provide advice on the financial decision making of the individuals because it allows us to introduce parameters that specify their preferences, priorities, and prediction horizon. For instance, it was shown in simulation that financial plans favoring higher savings funds, possibly delaying higher consumption, are more effective in coping with shocks than those that do not. Next, the performance of the MPC was considered as a function of an individual's returns and initial capital. It was shown that there is a poverty trap where individuals with low returns and low initial capital are not endowed enough to accumulate wealth and increase their health and education, since they have an MFR that is considerably high. These results for individual MPC financial decision making are novel, as the past results did not consider the impact of health and education.

The CDI is defined to represent community development along the dimensions of wealth, health, and education, along with discounting for inequalities among individuals on these dimensions. The ASCA strategy is defined, and the MPC for each individual is modified to include the basic (group) ASCA variables. Simulations showed that the ASCA can be tailored to the community by picking an appropriate value of the weekly contributions, one that simultaneously results in a low

MFR and a high CDI. Next, a donation strategy over a cooperation network is defined. Simulations show that the donation strategy can improve both the MFR and the CDI compared to the ASCA. Finally, considering unequal communities, it was shown that as inequality increased, MFR increased and CDI decreased, so that both strategies performed worse. While the ASCA produced a higher CDI for lower measures of inequality, its performance in both the CDI and MFR degraded at a faster rate than the altruistic donation strategy, suggesting that the flexibility of the later one achieves a better distribution of the resources in an unequal community.

Simulations showed that the cooperation strategy based on donations in general outperforms the ASCA. Even though the community members that implement the donation strategy tend to have a lower CDI compared to those that implement the ASCA at the beginning of the cooperation process, the failure rate ultimately decreases significantly in the presence of altruistic cooperation. This implies higher robustness of the community members to shocks and unexpected events than the one resulting from the ASCA. This is consistent with what has been observed in communities where cooperation is based on altruism [49], [50, Ch. 6].

The improvement achieved by the community under the donation strategy suggests that research should be oriented toward more flexible cooperation plans, aided by technology. Auction-based ROSCAs [2] or ASCAs that could achieve efficient and fair allocations employing mechanism design concepts could be implemented with the help of mobile phones in poor communities (via smartphones or using SMS gateways for basic phones) and empowering local crowdfunding initiatives by adding mobile or PC-based financial advisors to loan recipients are two possible implementations of flexible cooperation plans.

AUTHOR INFORMATION

Hugo Gonzalez Villasanti (gonzalezvillasanti.1@osu.edu) is a Ph.D. student in electrical and computer engineering at The Ohio State University. He obtained the B.S. degree in electromechanical engineering at the Universidad Nacional de Asuncion, Paraguay, and the M.S. degree in electrical and computer engineering at The Ohio State University. His current research interests include the modeling and control-theoretic analysis of social and humanitarian systems, cyberphysical systems, and decentralized systems. He can be contacted at 405 Drees Laboratory, 2015 Neil Avenue, Columbus, OH 43210 USA.

Luis Felipe Giraldo is an assistant professor at Universidad de los Andes, Colombia. His current research interests include modeling and analysis of linear and nonlinear dynamical systems, pattern recognition, and humanitarian engineering. For more information, see: <http://www.prof.uniandes.edu.co/~lf.giraldo404/>.

Kevin M. Passino received the Ph.D. degree in electrical engineering from the University of Notre Dame in 1989. He is currently a professor of electrical and computer

engineering and the director of the Humanitarian Engineering Center at The Ohio State University. He is on the IEEE Humanitarian Engineering Activities Committee. He has served as the IEEE Control Systems Society (CSS) vice president of Technical Activities, was an elected member of the CSS Board of Governors, was the program chair of the 2001 IEEE Conference on Decision and Control, and is currently a Distinguished Lecturer for the IEEE Society on Social Implications of Technology. He is a Fellow of the IEEE. For more information, see: <http://www.ece.osu.edu/passino/>.

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