An Analysis of Blocking Switches Using Error Control Codes

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Abstract — **We study the relation between the degree of blocking and the amount of resource speedup necessary for blocking switches to possess the capabilities of non-blocking switches. We consider both multicast and point to point scenarios. We show that the number of configurations that must be supported by a switching interconnection fabric can be significantly reduced even with small speedup values.**

We construct an analogy between switch configurations and the codewords of certain error control codes for which we use space covering ideas to derive relations between speedup and number of switch configurations. To derive the necessary speedup for non-blocking, we use two sphere packing bounds: the Hamming bound and the Gilbert-Varshamov bound To construct non-blocking switches with a given speedup we use maximum distance separable codes.

I. Introduction

At the core of every switch lies a switching interconnection fabric. The function of the fabric is to set up connections between the input and output units of the switch.

A configuration is a set, λ which is composed of I-O pairs, (v_i, v_o). We assume that the number, N, of inputs and outputs of a fabric is identical, and the cardinality of any configuration, $|\lambda| = N$. We also assume that exactly one connection terminates at each $v_o \in \{1, \ldots, N\}$, i.e., each one of the N pairs in λ will have a distinct output. On the other hand, an input can be found in more than one of these pairs depending on whether the fabric has multicast capability. If it does not, exactly one connection terminates at each $v_i \in \{1, \ldots, N\}$.

A fabric, $\mathcal{N}$, can be represented with a set, $\Lambda_{\mathcal{N}}$, of *feasible* configurations. A configuration is *feasible* if all the connections in the configuration can be made simultaneously. Let the set of all possible configurations be Γ. Hence, for any $\mathcal{N}$, $\Lambda_{\mathcal{N}} \subset \Gamma$. With multicast capability, $\Gamma = \{1, \ldots, N\}^N$. A fabric is called non-blocking if it *supports* all possible configurations, i.e., $\Lambda_{\mathcal{N}} \equiv \Gamma$. Thus, for $\mathcal{N}$ to be non-blocking, $|\Lambda_{\mathcal{N}}| = |\Gamma| = N^N$ feasible configurations is necessary. Note that without the multicast capability, Γ is identical to the set of all permutations that can be made with the elements of $\{1, \ldots, N\}$. In that case, a network is blocking if the number of feasible configurations is less than $N!$.

A fabric, $\mathcal{N}$, is said to have a *speedup* of S, $S \in \mathbb{Z}^+$ if it can support S distinct feasible configurations simultaneously. More precisely, with a speedup of S any $\lambda_1, \ldots, \lambda_S$ such that $\lambda_i \in \Lambda_{\mathcal{N}}$, $1 \leqslant i \leqslant S$ can be in effect simultaneously.

II. Results

In this paper, we study the relation between the speedup and the non-blocking behavior of fabrics. We view the set of feasible configurations, $\Lambda_{\mathcal{N}}$ of the fabric $\mathcal{N}$ as a (possibly non-linear code.

A Necessary Conditions

Theorem 1 *For an $N \times N$ fabric $\mathcal{N}$ to be non-blocking with a speedup of S, $S \in \mathbb{Z}^+ \cap (1, N-1)$, $L(S) = \frac{1}{2} N^{N/(S+1)} / \binom{N}{\lfloor N/(S-1) \rfloor}$ configurations is necessary.*

We show that if $|\Lambda_{\mathcal{N}}| < L(S)$ then there exists a $\gamma \in \Gamma$ such that for any set of S feasible configurations $\{\lambda_1, \ldots, \lambda_S\}$, $\gamma \not\subset \bigcup_{l=1}^{S} \lambda_i$ by proving that in the Hamming space, the total volume of $L(S)$ spheres each with radius $\lfloor N/S \rfloor$ is barely identical to N^N.

Theorem 2 *For an $N \times N$ no multicast fabric, $\mathcal{N}^*$ to be non-blocking with a speedup of S, $S \in \mathbb{Z}^+ \cap [1, N-1)$, $|\Lambda_{\mathcal{N}^*}| = \frac{1}{2} \left\lfloor \frac{N}{S+1} \right\rfloor !$ configurations is necessary.*

We also derive the speedup necessary to make a fabric non blocking for a given number of configurations for both multicast and no multicast fabrics.

B Sufficient Conditions

Theorem 3 *If $|\Lambda_{\mathcal{N}}| = N^{\lceil N/(S-1) \rceil}$, for some S, $S \in \mathbb{Z}^+$, then a speedup of S is sufficient to make the interconnection, $\mathcal{N}$ non-blocking and multicast capable.*

We prove this by choosing $\Lambda_{\mathcal{N}}$ to be the linear MDS code $(N, k, N-k+1)$ and using the information set property of the maximum distance separable codes. Note that for any given S,

$$\lim_{N \to \infty} \log \frac{N^{\lceil N/(S-1) \rceil}}{\frac{1}{2} \frac{N^{N/(S+1)}}{\binom{N}{\lfloor N/(S-1) \rfloor}}} = 1$$

which shows that the necessary and sufficient number of configurations for non-blocking have similar asymptotic exponents.

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