# Distributed Channel Probing for Efficient Transmission Scheduling Over Wireless Fading Channels

Bin Li and Atilla Eryilmaz

*Abstract*—**It is energy-consuming and operationally cumbersome for all users to continuously estimate the channel quality before each data transmission decision in opportunistic scheduling over wireless fading channels. This observation motivates us to understand whether and how opportunistic gains can still be achieved with significant reductions in channel probing requirements and without centralized coordination amongst the competing users. In this work, we first provide an optimal centralized probing and transmission algorithm under the probing constraints. Noting the difficulties in the implementation of the centralized solution, we develop a novel Sequential Greedy Probing (SGP) algorithm by using the maximum-minimums identity, which is naturally well-suited for physical implementation and distributed operation. We show that the SGP algorithm is optimal in the important scenario of symmetric and independent ON-OFF fading channels. Then, we study a variant of the SGP algorithm in general fading channels to obtain its efficiency ratio as an explicit function of the channel statistics and rates, and note its tightness in the symmetric and independent ON-OFF fading scenario. We further expand on the distributed implementation of these greedy solutions by using the Fast-CSMA technique.**

## I. Introduction

Opportunistic scheduling has long been observed (e.g., [9], [8]) to improve communication performance in wireless fading systems by selectively transmitting over channels that are in good condition. This presumes the knowledge of channel state information (CSI) at the outset of each transmission decision. However, in the presence of many contending users that utilize the time-varying channel, acquiring CSI per user is not only energy-consuming, but, more importantly, operationally difficult since it typically requires non-overlapping pilot training phases to obtain reliable channel quality estimates. Moreover, such persistent probing is likely unnecessary given that only few of them may be allowed to transmit due to the interference constraints. Yet, opportunistic gains from multi-user diversity cannot be realized if sufficient CSI is not present. This implies a natural tradeoff between exploring the multi-user diversity and energy consumption for channel acquisition, and raises a fundamental question on the design of opportunistic scheduling towards the determination of which subset of users to probe the channel given limited average probing rates.

The seminal works of Tassiulas and Ephremides (e.g., [16], [17] and [15]) have showed the throughput-optimality of the opportunistic scheduling, which prioritize activation of links with the largest product of backlog awaiting service and corresponding channel rate given the full knowledge of CSI, also called Maximum Weight Scheduling (MWS). Recently, there has been an increasing understanding on efficient scheduling with limited CSI (e.g., [3], [7], [1], [12]). In [3], the authors propose a two-stage throughput-optimal MWS-type algorithm given partial CSI under the assumption that only users with known channel states can contend for the channel. However, they do not answer how to select a subset of users to probe the channel. In [7], the authors also develop a similar MWS-type algorithm that minimizes the energy consumption. However, the resulting decision space being exponentially increasing with the number of users appears to limit its applicability in multi-user environments. In fact, existing works in the design of joint probing and transmission strategies assume centralized controllers that utilize all state information, and hence are not suitable for distributed operation in large-scale networks. However, as we shall point out, the design for distributed probing strategies generates difficult challenges that require novel techniques beyond existing approaches discussed next.

In an exciting thread of work, it has been shown that Carrier Sense Multiple Access (CSMA) based distributed scheduling strategies (e.g., [4], [11], [2], [13]) can maximize long-term average throughput for general non-fading wireless topologies. Yet, the design of distributed schedulers in a fading environment has been proven to be much more difficult. Nevertheless, when CSI is available, a distributed Fast-CSMA (FCSMA) algorithm has also been developed [6] that guarantees throughput-optimal scheduling over wireless fading channels in a fully-connected network topology. Yet, to the best of our knowledge, there does not exist a distributed solution that also accounts for the energy and operational limitations in the CSI acquisition.

With this motivation, in this work, we address the problem of distributed joint probing and transmission scheduling when users have heterogeneous loads, probing rate constraints, and channel statistics. The following items list our main contributions along with references on where they appear in the text:

- In Section III, we first characterize the capacity region given the allowable probing rate for general fading channels. Then, we develop a centralized throughput-optimal joint probing and transmission algorithm. This algorithm, while impractical as is, forms the basis for the subsequent design of algorithms that are suitable for distributed operation.
- In Section IV, based on the maximum-minimums identity [14], we first develop a novel Sequential Greedy Probing (SGP) algorithm where users probe the channel sequentially.

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Then, we show that the SGP algorithm can get the optimal probing schedule, leading to throughput-optimal performance over symmetric and independent ON-OFF fading channels.

• In Section V, we introduce and analyze a Modified SGP (MSGP) algorithm that is adapted to general fading channels, and explicitly characterize its efficiency ratio as an explicit function of the channel statistics and rates. The efficiency ratio is tight for symmetric and independent ON-OFF fading channels and independent of the number of users.

• In Section VI, we utilize the FCSMA strategy [6] to develop distributed implementations of proposed greedy algorithms, and analyze the performance of the resulting algorithm.

## II. System Model

We consider a wireless system where a set of $N$ users with independently fading conditions contend for data transmission over a single channel. We assume that the channel for each user has $M+1$ possible rates $c_0, c_1, c_2, ..., c_M$, where $c_0 < c_1 < c_2 < ... < c_M$ and $c_0 = 0$. Let $C_i[t]$ denote the maximum amount of service available in slot $t$ if user $i$ is scheduled. We assume that $\mathbf{C}[t] = (C_i[t])_{i=1}^N$ are independently distributed random variables over users and identically distributed over time, with $p_{ij} \triangleq \Pr\{C_i[t] = c_j\}, \forall i = 1, ..., N; j = 0, 1, ..., M$. We reasonably assume that the channel for each user is unavailable with a strictly positive probability[1], that is, $p_{i0} > 0, \forall i$. In the rest of paper, we also use $\mathbf{C}$ to denote the fading channel.

In order to get CSI, each user needs to probe the channel by transmitting small control packets. We denote the probing schedule as $\mathbf{X} = (X_i)_{i=1}^N$, where $X_i = 1$ if user $i$ probes the channel and $X_i = 0$ otherwise. We also treat $\mathbf{X}$ as a set of probing users. Let $\mathcal{X}$ be the collection of probing schedules. Due to the interference constraints, at most one user can transmit in each slot. We call a schedule where at most one user is active in each slot as a *feasible schedule* and denote it as $\mathbf{S} = (S_i)_{i=1}^N$, where $S_i = 1$ if user $i$ grabs the channel at slot $t$ and $S_i = 0$ otherwise. We use $\mathcal{S}$ to denote the collection of feasible schedules.

If the user does not probe the channel at the beginning of each time slot, it may underestimate the channel rate or may even fail to transmit due to a bad channel condition. Thus, it is reasonable to assume (as in [3]) that each user will not start a transmission if it does not observe the channel state at the beginning of each time slot. We denote the *allowable probing rate* for each user $i$ as $m_i \in (0, 1], \forall i$, which puts an upper bound on the average number of probing operations that each user is allowed to make. This bound, as noted in the introduction, may be due to energy or operational constraints associated with the channel estimation operation.

We assume that each user $i$ serves its own traffic load and maintains them in a data queue with $Q_i[t]$ denoting its queue length at the beginning of slot $t$. Let $A_i[t]$ denote the number of packets arriving at user $i$ in slot $t$ that are independently distributed over links and identically distributed over time with mean $\lambda_i$, and $\mathbb{E}[A_i^2[t]] < A_{\max}$ for some $A_{\max} < \infty$. Then, the evolution of data queue $i$ is described as follows:

$$Q_i[t+1] = (Q_i[t] + A_i[t] - X_i[t]S_i[t]C_i[t])^+, \forall i, \quad (1)$$

where $(y)^+ \triangleq \max\{y, 0\}$. Our goal is to find the optimal joint probing and transmission schedule $\{\mathbf{X}[t], \mathbf{S}[t]\}_{t\geq 1}$ under the scheduling constraint that at most one user can be scheduled at each time slot and probing constraint that the average probing rate of each user should not be greater than its allowable probing rate. A key difficulty in the solution of this problem is that the information available at the transmission scheduling decision $\mathbf{S}[t]$ critically depends on the previously made probing decision $\mathbf{X}[t]$, which in turn must be performed distributively with only local information. We will first address the problem of optimal centralized control, and then return to the distributiveness challenge.

To resolve the centralized controller design problem, instead of reverting to optimal control methods, we use the clever technique in [10] to introduce and guarantee stability of a virtual queue for each user that conveniently measures the degree of violation of the average probing constraint. Specifically, we let $U_i[t]$ denote the virtual queue length for user $i$ at the beginning of slot $t$. The number of packets entering the virtual queue $i$ at slot $t$ is just $X_i[t]$. We use $I_i[t]$ to denote the service for virtual queue $i$ at slot $t$ that are independently distributed over links and identically distributed over time with mean $m_i$, and $\mathbb{E}[I_i^2[t]] \leq I_{\max}$ for some $I_{\max} < \infty$. Then, the evolution of the virtual queue $i$ is as follows:

$$U_i[t+1] = (U_i[t] + X_i[t] - I_i[t])^+, \forall i. \quad (2)$$

We consider the class $\mathcal{P}$ of stationary probing and transmission policies with well-defined long-term average throughput that first decide the probing schedule $\mathbf{X}[t]$ at slot $t$ based on the available information ($\mathbf{Q}[t]$, $\mathbf{U}[t]$) and then determine the transmission schedule $\mathbf{S}[t]$ based on the observed vector ($\mathbf{Q}[t] \otimes \mathbf{X}[t]$, $\mathbf{C}[t] \otimes \mathbf{X}[t]$), where $\otimes$ stands for component-wise multiplication. Thus, a joint probing and transmission policy in $\mathcal{P}$ is a two-stage mapping where it first maps from the space of ($\mathbf{Q}[t]$, $\mathbf{U}[t]$) to the space of probing schedules $\mathcal{X}$ during the probing stage and then maps from the space of ($\mathbf{Q}[t] \otimes \mathbf{X}[t]$, $\mathbf{C}[t] \otimes \mathbf{X}[t]$) to the space of feasible schedules $\mathcal{S}$ during the transmission stage. Under any policy in $\mathcal{P}$, both data queue length process $\{\mathbf{Q}[t]\}_{t=1}^{\infty}$ and virtual queue length process $\{\mathbf{U}[t]\}_{t=1}^{\infty}$ form a Markov Chain.

We say that data queue $i$ is *strongly stable* if it satisfies $\text{limsup}_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[Q_i[t]] < \infty$ and virtual queue $i$ is *mean rate stable* if $\lim_{T\to\infty} \frac{\mathbb{E}[U_i[T]]}{T} = 0$. The system is *stable* if all its data queues are strongly stable and all its virtual queues are mean rate stable. We define the *capacity region* as a maximum set of arrival rate vectors $\lambda = (\lambda_i)_{i=1}^N$ for which the system is stable under any policy. We call an algorithm *optimal* if it can make the system stable for any arrival rate vector that lies

[1] In practice, the probing packets and data packets are transmitted in low-rate (e.g., 1Mbps in IEEE 802.11b) and high-rate (e.g., 2/5.5/11Mbps in IEEE 802.11b) respectively, which implies that the transmission of probing packets requires lower signal-to-noise-ratio than that of data packets. Thus, it is reasonable to assume that when the channel is very poor, the user can still probe the channel but cannot transmit the data packets.

strictly inside the capacity region. An algorithm can achieve the *efficiency ratio* $\rho$ if it can stabilize the system for any $\lambda$ strictly within a fraction $\rho$ of the capacity region. Next, we provide an optimal centralized joint probing and transmission algorithm.

## III. Optimal Centralized Probing and Transmission

In this section, we first study the capacity region given the allowable probing rate in a general fading channel. Then, we propose a centralized probing and transmission algorithm that supports any throughput in it.

### A. Characterization of the capacity Region

The next lemma gives the capacity region $\Lambda(\mathbf{m}, \mathbf{C})$ under the allowable probing rate vector $\mathbf{m} = (m_i)_{i=1}^N$ in a general fading channel $\mathbf{C}$.

*Lemma 1:* The capacity region $\Lambda(\mathbf{m}, \mathbf{C})$ is a set of arrival rate vectors $\lambda$ such that there exist non-negative numbers $\alpha(\mathbf{x})$ and $\beta(\mathbf{x}, \mathbf{c}; \mathbf{s})$ satisfying

$$\lambda_i < \sum_{\mathbf{x}} \alpha(\mathbf{x}) \sum_{\mathbf{c}} P(\mathbf{C}[t] = \mathbf{c}) \sum_{\mathbf{s} \in \mathbf{S}} \beta(\mathbf{x}, \mathbf{c}; \mathbf{s}) x_i c_i s_i, \forall i; \quad (3)$$

$$\sum_{\mathbf{s} \in \mathcal{S}} \beta(\mathbf{x}, \mathbf{c}; \mathbf{s}) = 1, \forall \mathbf{x}, \mathbf{c}; \quad (4)$$

$$\sum_{\mathbf{x}} \alpha(\mathbf{x}) = 1; \quad (5)$$

$$\sum_{\mathbf{x}} \alpha(\mathbf{x}) x_i \leq m_i, \forall i. \quad (6)$$

*Proof:* Please see our technical report [5] for the proof. ■

### B. An Optimal Joint Probing and Transmission Algorithm

We are now ready to develop an optimal joint probing and transmission algorithm.

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**Joint Probing and Transmission (JPT) Algorithm:**

In each slot $t$, given $(\mathbf{Q}[t], \mathbf{U}[t])$, perform:
(1) *Probing Decision*: Set the probing vector $\mathbf{X}^*[t]$ as

$$\mathbf{X}^*[t] = \text{RAN}\left\{\arg\max_{\mathbf{X}} \left(\mathbb{E}\left[\max_i Q_i[t] X_i C_i[t]\right] - \sum_{i=1}^N U_i[t] X_i\right)\right\}, \quad (7)$$

where RAN$\{\cdot\}$ denotes that ties are broken uniformly at random.
(2) *Transmission Scheduling Decision*: After the channel states of the selected users are probed, schedule the transmission of user $i^*[t]$ that satisfies:

$$i^*[t] = \text{RAN}\left\{\arg\max_i Q_i[t] X_i^*[t] C_i[t]\right\}. \quad (8)$$

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Next, we will show that the JPT algorithm is optimal in the sense that it can stabilize the system for any arrival rate vector within the capacity region.

*Proposition 1:* The JPT algorithm is optimal, i.e., for any arrival rate $\lambda$ that is strictly inside the capacity region $\Lambda(\mathbf{m}, \mathbf{C})$, the JPT algorithm stabilizes the system subject to the allowable probing rate constraints.

*Proof:* Please see our technical report [5] for the proof. ■

Even though the JPT algorithm is optimal, it cannot directly be applied in practice due to the need of centralized coordination. We can use the FCSMA algorithm in [6] to solve the transmission scheduling component (8) of the JPT algorithm distributively if we know the optimal probing schedule. However, how to solve the probing component of the JPT algorithm in a distributed way still remains an open question. Next, we develop a sequential greedy probing algorithm that is well-suited for distributed computation of (7) and analyze its performance. From now on, we always use the well-known MWS algorithm or its distributed variants (e.g., the FCSMA algorithm) in the transmission stage.

## IV. Sequential Greedy Probing and Analysis

In this section, we propose a sequential greedy algorithm for the probing component of the JPT algorithm, which can be implemented distributively as we will explain in Section VI. Then, we show that it can get an optimal probing schedule in symmetric and independent ON-OFF fading channels.

We need to establish some new notations to introduce our proposed algorithm. Let $\mathbf{N}$ be a collection of $N$ users. For any non-empty set $\mathbf{E} \subseteq \mathbf{N}$, we define the function $f(\mathbf{E}, e)$ as follows:

$$f(\mathbf{E}, e) := \mathbb{E}[\max_{i \in \mathbf{E}} \min\{Q_i C_i, Q_e C_e\}], \quad (9)$$

where $e \notin \mathbf{E}$. Our technical report [5] explores more properties of $f(\mathbf{E}, e)$. Here, it is worth noting that, by using the maximum-minimums identity [14], $f(\mathbf{E}, e)$ can be computed recursively.

Also, let $\phi_i := \mathbb{E}[Q_i C_i] - U_i, \forall i \in \mathbf{N}$, and consider a set $\mathbf{F} \subseteq \mathbf{N}$ of probing users and $r \in \mathbf{N} \backslash \mathbf{F}$. By using the maximum-minimums identity, we have the key relationship:

$$\begin{aligned} &\mathbb{E}\left[\max_{i \in \mathbf{F} \bigcup \{r\}} Q_i C_i\right] - \sum_{i \in \mathbf{F} \bigcup \{r\}} U_i \\ &= \left(\mathbb{E}[\max_{i \in \mathbf{F}} Q_i C_i] - \sum_{i \in \mathbf{F}} U_i\right) + \phi_r - f(\mathbf{F}, r). \end{aligned} \quad (10)$$

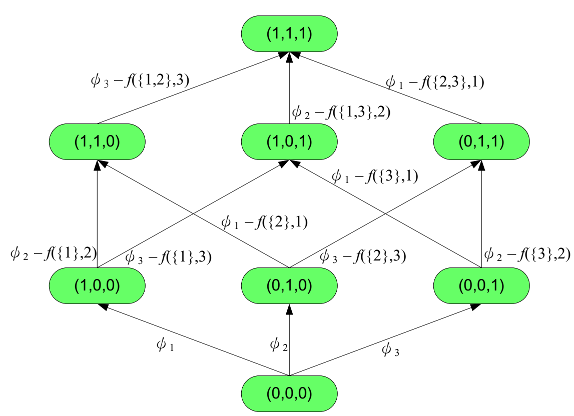


Fig. 1: The directed graph $\mathcal{G} = (\mathcal{X}, \mathcal{E})$ when $N = 3$

For the derivation of this identity, please see our technical report [5] for details. Based on the iterative equation (10), we can define a directed graph $\mathcal{G}$, where each probing schedule $\mathbf{X}$ denotes a node with an associated *value* of $\mathbb{E}[\max_{i\in\mathbf{X}} Q_iC_i] - \sum_{i\in\mathbf{X}} U_i$. Thus, $\mathcal{X}$ also represents the collection of all nodes. Note that $|\mathcal{X}| = 2^N$ since each node is a binary vector of $N$ dimensions. For two nodes $\mathbf{X_1}$ and $\mathbf{X_2}$, there is a *directed link* from node $\mathbf{X_1}$ to node $\mathbf{X_2}$ if and only if $\mathbf{X_1}$ is a subset of $\mathbf{X_2}$ with the cardinality $|\mathbf{X_2}|-1$. Let $q = \mathbf{X_2} \setminus \mathbf{X_1}$. We define the *weight* of a link from node $\mathbf{X_1}$ to node $\mathbf{X_2}$ as $\phi_q - f(\mathbf{X_1}, q)$. Let $\mathcal{E}$ be the collection of edges, and let node $\mathbf{X_0}$ denote the all-zero probing schedule where no user probes the channel, and thus the value of node $\mathbf{X_0}$ is 0. Finally, let $\mathbf{I} = \{i \in \mathbf{N} : \phi_i > 0\}$. Figure 1 shows the directed graph for $N = 3$.

We first divide each time slot into a control slot and a data slot. The purpose of the control slot is to determine the probing schedule used for transmission in the data slot. To achieve this goal, we further subdivide the control slot into $N$ mini-slots.

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**Sequential Greedy Probing (SGP) Algorithm:**

(1) In the first mini-slot, let $i_1 = \arg\max_{i\in\mathbf{I}} \phi_i$. User $i_1$ probes the channel while also announcing its queue-length. If no users probe the channel, then all users keep silent in the rest of current slot and restarts in the next time slot.

(2) In the $k^{th}$ $(1 < k \leq N)$ mini-slot, let

$$i_k = \text{RAN}\left\{ \underset{i\in\mathbf{I}\setminus\{i_1,...,i_{k-1}\}}{\arg\max} \left(\phi_i - f\left(\{i_1, ..., i_{k-1}\}, i\right)\right) \right\}. \quad (11)$$

If $\phi_{i_k} > f\left(\{i_1, ..., i_{k-1}\}, i_k\right)$, then user $i_k$ probes the channel. Otherwise, all users stop probing and all probing users with non-zero channel states are candidates for transmission scheduling as dictated in (8).

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Next, we show the optimality of the SGP algorithm in symmetric and independent ON-OFF fading channels.

*Proposition 2:* The SGP algorithm can achieve the optimal value of the optimization problem (7) in symmetric and independent ON-OFF fading channels.

*Proof:* Please see our technical report [5] for the proof. ∎

In general wireless fading channels, the SGP algorithm cannot always find the optimal value of (7) as in the above symmetric setup, and thus its performance is unclear. Instead, we consider a Modified SGP (MSGP) algorithm.

## V. The Modified SGP Policy and Analysis

In this section, we allow the most general channel statistics, and introduce a slightly modified version of the SGP algorithm studied in the previous section. Then, we explicitly characterize the efficiency ratio that this modified algorithm is guaranteed to achieve as a function of the channel statistics and rates.

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**Modified SGP (MSGP) Algorithm:**

The MSGP algorithm operates exactly the same as the SGP algorithm, except that steps are computed assuming the identical and independent distributed ON-OFF fading channels with $\Pr\{C_i^{\min}[t] = c_1\} = p_{\min}$ and $\Pr\{C_i^{\min}[t] = 0\} = 1-p_{\min}, \forall i$, where $p_{\min} := 1 - \max_j p_{j0}$ and $p_{\max} := 1 - \min_j p_{j0}$.

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*Remarks:* The MSGP algorithm differs from the SGP algorithm only in the assumed channel statistics and rates.

*Proposition 3:* The MSGP algorithm combined with the MWS algorithm in the transmission stage (see equation (8)) can at least achieve a fraction $\rho := \frac{p_{\min}}{p_{\max}} \frac{c_1}{c_M}$ of the capacity region in general fading channels.

*Remarks:* In symmetric and independent ON-OFF channels, the MSGP algorithm can achieve the full capacity region, which matches the result in Proposition 2.

*Proof:* Please see our technical report [5] for the proof. ∎

## VI. Distributed Implementation with Fast CSMA

Here, we expand on the distributed implementation of the greedy sequential probing algorithms developed in the previous two sections by using the FCSMA technique developed in [6]. Since the MSGP algorithm has the same performance as the SGP Algorithm in the special case of symmetric ON-OFF channels, we focus on the distributed implementation of the MSGP Algorithm in the control slot.

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**Distributed MSGP (DMSGP) Algorithm:**

In the first mini-slot, each user $i$ with $\phi_i > 0$ independently generates an exponentially distributed random variable with rate $\exp(G\phi_i)$ $(G > 0)$, and starts transmitting a small probing packet after this random duration unless it senses another transmission before. The user that grabs the channel transmits its probing packet until the end of the mini-slot. After probing, all other users know the queue length of the current probing user. If no users transmit the probing packet during this mini-slot, then all users keep silent in the rest of current slot and restarts in the next time slot.

In the $k^{th}$ $(1 < k \leq N)$ mini-slot, the remaining non-probing user $i$ with $\phi_i - f\left(\{i_1, ..., i_{k-1}\}, i\right) > 0$ generates an exponential distributed random variable with rate $\exp(G(\phi_i - f\left(\{i_1, ..., i_{k-1}\}, i\right)))$ and uses the same produce as in the first mini-slot to probe the channel. If no users probe the channel in the current mini-slot or the control slot is over, then all the probing users with the available channel state start to contend for data transmission.

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The above procedure leads to a probing schedule $\mathbf{X}^{DMSGP}$ by the end of the control slot, where each selected probing user $i$ knows its channel state $C_i$. Then, to determine the one that transmits the data packet each probing user $i$ distributively runs the FCSMA algorithm as described in [6] with parameter $\exp(Q_iC_i)$. The following main result establishes the performance of such a distributed probing and transmission algorithm.

*Proposition 4:* For any $\zeta > 0$ and arrival rate vector $\lambda$ satisfying $\lambda + \zeta \in \rho\Lambda(\mathbf{m}, \mathbf{C})$, with the efficiency ratio $\rho$ given in Proposition 3, there exists a design parameter $G > 0$ such that the DMSGP algorithm, combined with the FCSMA algorithm in the transmission stage, can support $\lambda$ subject to the given probing rate constraints $\mathbf{m}$ in general fading channels.

*Proof:* Please see our technical report [5] for the proof. ∎

## VII. Numerical Results

In this section, we mainly study the impact of the number of iterative steps allowed in the operation of the SGP algorithm since this is not theoretically explored, but is crucial to the ease of practical implementation. We consider symmetric and independent ON-OFF fading channels with ON probability $p = 0.8$ and $N = 20$ users. The number of arrivals in each slot follows Bernoulli distribution with mean $\lambda$, and all users have the same allowable probing rate of $m = 0.4$. Under this setup, the capacity region is $\Lambda = \{\lambda : \lambda < 0.05\}$ (See Proposition 1 in [5]). We use $K$ to denote the maximum allowable iterative steps. From Figures 2 and 3, we observe that the SGP algorithm with unlimited iterative steps can achieve full capacity. In addition, as $K$ increases, the performance of the SGP algorithm improves. Especially, we can see that 4 iterative steps are enough to reach almost optimal performance. This implies that while the original algorithm may be defined over more steps, in practice, we can limit the iterative steps to a small number virtually without hurting the throughput.

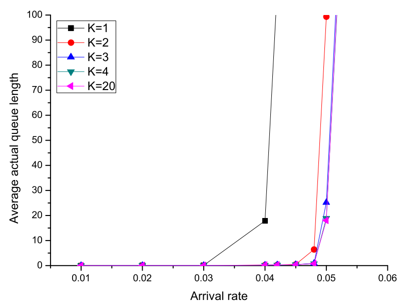


Fig. 2: Average queue length: N=20

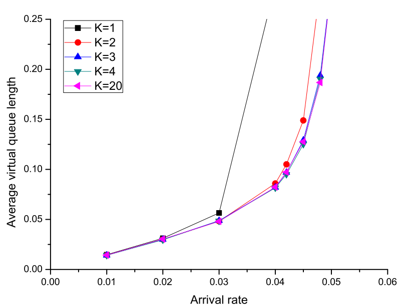


Fig. 3: Average virtual queue length: N=20

## VIII. Conclusion

In this paper, we considered the distributed channel probing for opportunistic scheduling under heterogeneous allowable probing rate constraints. We first characterized the capacity region under the heterogeneous probing rate constraints and provided the optimal centralized JPT algorithm. Realizing the operational difficulty of centralized solution, we put effort in developing a novel SGP algorithm based on the maximum-minimums identity, which is easy for distributed implementation. Also, we showed that the SGP algorithm is optimal in the crucial scenario of symmetric and independent ON-OFF fading channels. In the case of general fading channels, we analyzed a more tractable variant of the SGP algorithm to obtain its efficiency ratio as an explicit function of the channel statistics and rates and showed that this ratio is tight in the symmetric ON-OFF fading scenario. Finally, we discussed the distributed implementation of these greedy probing algorithms by using the FCSMA technique.

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