Scheduling with Rate Adaptation under Imperfect Channel-Estimator Joint Statistics

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Abstract

In time varying wireless networks, the state of communication channels are subject to random variations, and hence need to be estimated for efficient rate adaptation and scheduling. The estimation mechanism possesses inaccuracies that need to be tackled in a probabilistic framework. In this work we study scheduling with rate adaptation in single-hop queueing networks under two levels of channel uncertainty: when the channel estimates are inaccurate but the statistics of the mismatch is known and when the mismatch statistics is unknown. In the former case, we characterize the network stability region and show that a back-pressure type scheduling policy is throughput optimal. For the latter case, we propose a joint channel learning and scheduling policy and show that it has a stability region that can be arbitrarily close to the network stability region with an associated trade-off in the average packet delay.

I. Introduction

Scheduling in wireless queueing networks is an important design paradigm that aims to maximize network utility subject to link interference and queue stability constraints. Since the seminal paper by Tassiulas and Ephremedis [1], the back-pressure algorithm is well studied [2]-[7] and is found to be throughput optimal in various network settings. The simplicity of the back-pressure algorithm - schedule the user with the largest queue weighted rate - is based on the assumption that full knowledge of channel state information is available at the scheduler. In realistic scenarios, however, due to random variations in the channel, full CSI is not readily available at the scheduler. The dynamics of the scheduling problem, therefore, is vastly different in the following two ways (1) a non-trivial amount of network resource, that could otherwise be used for data transmission, is spent in learning the channel; (2) the acquired information on the channel is potentially inaccurate, essentially underlining the need for intelligent rate adaptation and user scheduling. Realistic queueing networks are thus characterized by a convolved interplay between channel estimation, rate adaptation and multiuser scheduling mechanisms. This complicated dynamics is studied under various network settings in recent works [8]-[13].

In [8], the authors study scheduling in single-hop wireless networks with Markov-modeled binary ON-OFF channels. Here scheduling decisions are made based on cost-free estimates of the channel obtained once every few slots. The authors show that a back-pressure type scheduling policy, that takes into account the probabilistic inaccuracy in the channel estimates and the memory in the Markovian channel, is throughput optimal. Ying et al. [9] study decentralized scheduling under partial CSI in multi-hop wireless networks with Markov-modeled channels. Here, each user knows its channel perfectly and has access to delayed CSI of other users' channels. The authors characterize the stability region of the network and show that a back-pressure type, threshold policy implemented in a decentralized fashion at each user is throughput optimal. In [11], the authors study scheduling under uncertainty in single-hop networks with i.i.d. channels. They consider a two-stage decision setup: In the first stage the scheduler decides whether to estimate the channel with a corresponding energy cost. In the second stage, scheduling with rate adaptation is performed based on the outcome of the first stage. Under this setup, the authors propose a back-pressure type scheduling policy that minimizes the energy consumption subject to queue stability.

While understanding scheduling under imperfect CSI is a first step in the right direction, these works assume that the knowledge of the joint channel-estimator statistics, that is crucial for the success of opportunistic scheduling, is readily available at the scheduler. This is another simplifying assumption that need not always hold in reality. Taking note of this, we study scheduling in single-hop networks under imperfect CSI and when no prior information on the channel-estimator statistics is available at the scheduler. We propose a joint channel learning, scheduling policy that allocates a fraction of time slots (the exploration slots) to continuously learn the channel-estimator statistics, which are in turn used for scheduling and rate adaptation during data transmission slots. Note that our setup has similarities to that of [13]. Here the author considers a two stage decision setup. When applied to the scheduling problem this work can be interpreted as follows: In the first stage, one of K estimators is chosen to estimate the channel, without an explicit knowledge of the joint channel-estimator statistics for any of the K estimators. In the second stage another decision is to be made that minimizes a penalty vector that is a *known* function of the estimate obtained in the first stage. Our problem is different from this setup in the following sense: The channel-estimator statistics is necessary to optimize the second stage decision in our problem - i.e., scheduling with rate adaptation. This is not the case in [13] where a *known* function of the estimate is optimized and the channel-estimator statistics is required only in the first stage that decides one of K estimators. Our contribution is in two parts:

- When full channel-estimator statistics is available at the scheduler, we characterize the network stability region and show that a simple back-pressure type scheduling policy is throughput optimal.
- Using the preceding system level results as benchmark, we study scheduling when no prior knowledge on the channel-estimator statistics is available at the scheduler. We propose a scheduling policy with an in-built channel learning mechanism. We show that, with a corresponding trade-off in convergence time of the channel learning mechanism and hence the packet delay, the stability region of the proposed policy can be pushed arbitrarily close to the network stability region under full knowledge of channel-estimator statistics.

The paper is organized as follows: Section II formalizes the system model. In Section III we characterize the stability region of the network and propose a throughput optimal scheduling policy. In Section IV, we study joint channel learning, scheduling and rate adaptation when the scheduler does not have access to channel-estimator statistics. Concluding remarks are provided in Section V.

II. System Model

We consider a single-hop centralized wireless network with one base-station and N mobile users. Data packets to be transmitted from the base station to the users are stored in N separate queues at the base station. Time is slotted with the slots of all the users synchronized. In each slot, the channel between the base station and each user is ***i.i.d*** across time and independent across users. We do not assign any specific distribution to the channels throughout this work. The channel state of an user in a slot denotes the number of packets that can be successfully transmitted without outage to that user, in that slot. Transmission at a rate below the channel state always succeeds, while transmission at a rate above the channel state always fails. We assume the channel state lies in a finite discrete state space $\mathcal{S}$. Let $C_i[t]$ be the random variable denoting the channel state of user i in slot t. The channel state of the network in slot t is denoted by the vector $\boldsymbol{C}[t] = \left[C_1[t], C_2[t], \cdots, C_N[t]\right] \in \mathcal{S}^N$. In each slot, the scheduler has (cost-free) access to estimates of the channel states, i.e., $\widehat{\boldsymbol{C}}[t] = \left[\widehat{C}_1[t], \widehat{C}_2[t], \cdots, \widehat{C}_N[t]\right] \in \mathcal{S}^N$. The estimator is fixed for each user and the estimates are independent across users. Define the joint channel-estimator statistics for user i as the $|\mathcal{S}|^2$ probabilities $P(C_i = c_i, \widehat{C}_i = \hat{c}_i)$, $\forall c_i \in \mathcal{S}, \hat{c}_i \in \mathcal{S}$.

We adopt the one-hop interference model, where, in each slot, only one user is scheduled for data transmission. The scheduler (base station), based on the channel estimate and the queue length information, decides which user to schedule and performs rate adaptation in order to maximize the overall network stability region. Let $I[t]$ and $R[t]$ denote the index of the user scheduled to transmit and the corresponding rate of transmission, respectively, at slot t. Due to potential mismatch between channel estimates and the actual channels, it is possible that the allocated rate is larger than the actual channel rate, thus leading to outage. In this case the packet is retained at the head of the queue and a retransmission will be attempted later. Let $A_i[t]$ denote the number of exogenous packet arrivals at queue i at the beginning of slot t with $E[A_i[t]] = \lambda_i$. Let $Q_i[t]$ denote the state (length) of queue i at the beginning of slot t. The queue state evolution can now be written as a discrete stochastic process:

$$Q_i[t+1] = \max\{0, Q_i[t] - \mathbf{1}(I[t]=i) \cdot R[t] \cdot \mathbf{1}(R[t] \leq C_i[t])\} + A_i[t]. \tag{1}$$

We adopt the following definition of queue stability [[ref]]: queue i is stable if there exists a limiting stationary distribution F_i such that $P(\lim_{t\to\infty} Q_i[t] < q) = F_i(q)$.

III. Part I: Full Knowledge of Channel-Estimator Joint Statistics

In this section, we consider the scenario when scheduler has full access to the channel-estimator joint statistics, i.e., $P(C_i = c_i, \widehat{C}_i = \hat{c}_i)$, $\forall c_i \in \mathcal{S}, \hat{c}_i \in \mathcal{S}$ for every user $i \in \{1, \ldots, N\}$. We characterize the network stability region next.

A. Network Stability Region

Consider the class of stationary scheduling policies G that base its decision on the current queue length information $[Q_1[t] \ldots Q_N[t]]$, the channel estimates $[\widehat{C}_i, \ldots, \widehat{C}_N]$ and full knowledge of channel-estimator joint statistics. Define the network stability region as the closure of arrival rates that can be stably supported by the policies in G. Let $P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}} = [\hat{c}_1, \ldots, \hat{c}_N])$ denote the probability of the channel estimate vector. Thus

$$P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}} = [\hat{c}_1, \ldots, \hat{c}_N]) = \prod_{i=1}^{N} P(\widehat{C}_i = \hat{c}_i) \tag{2}$$

where the probabilities $P(\widehat{C}_i = \hat{c}_i)$ are evaluated from the channel-estimate joint statistics. Defining $\mathcal{CH}[A]$ as the convex hull of set A and $\vec{\mathbf{1}}_i$ as the i^{th} coordinate vector, we record our result on the network stability region below.

Proposition 1. *The stability region of the network is given by*

$$\boldsymbol{\Lambda} = \sum_{\hat{\boldsymbol{c}} \in \mathcal{S}^N} P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}}) \cdot \mathcal{CH}\Big[\ \mathbf{0}, P(C_i \geq r_i^*(\hat{c}_i) \big| \widehat{C}_i = \hat{c}_i\ ) \cdot r_i^*(\hat{c}_i) \cdot \vec{\mathbf{1}}_i;\ i = 1, \cdots, N\Big],$$

where $r_i^*(\hat{c}) = \arg\max_{r \in S} \left\{ P(C_i \geq r \Big| \widehat{C}_i = \hat{c}\ ) \cdot r \right\}$ *and the conditional probability* $P(C_i \geq r_i^*(\hat{c}_i) | \widehat{C}_i = \hat{c}_i)$ *are evaluated from the channel-estimator joint statistics.*

Proof: The proof contains two parts. We first show that any rate vector $\boldsymbol{\lambda}$ strictly within $\boldsymbol{\Lambda}$ is stably supportable by some randomized stationary policy. In the second part, we establish that any arrival rate $\boldsymbol{\lambda}$ outside $\boldsymbol{\Lambda}$ is not supportable by any policy. We show this by first identifying a hyperplane that separates $\boldsymbol{\lambda}$ and $\boldsymbol{\Lambda}$ using strict separation theorem [15]. We then define an appropriate Lyapunov function and show that, for any scheduling policy, there exists a positive drift thus rendering the queues unstable. Details of the proof is available in [22]. ■

B. Optimal Scheduling and Rate Allocation

In this section, we propose a back-pressure type scheduling policy with rate adaptation and show that it is throughput optimal, i.e., it can support any arrival rate that can be supported by any other policy in G. The policy is introduced next.

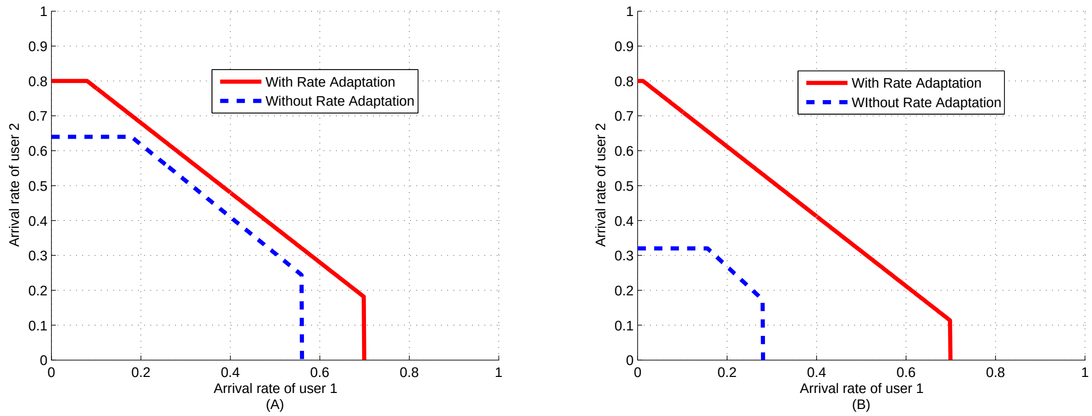

Fig. 1. Illustration of the system level gains associated with joint scheduling and rate control. P_i denote the probability that the channel of user i is in ON state. $P_i(\widehat{C} = \hat{c} | C = c)$ denotes the probability that the channel estimate of user i is $\hat{c}$ given the actual channel of user i is c

Scheduling Policy Ψ

At time slot t, the base station makes the scheduling and rate adaptation decisions based on the channel-estimator joint statistics and the channel estimate vector $\widehat{\boldsymbol{C}} = \hat{\boldsymbol{c}}$ (the time index is dropped for notational simplicity).

(1) Rate Adaptation: For each user i, assign rate R_i such that,

$$R_i = \arg\max_{r \in S} \left\{ P(C_i \geq r \big| \widehat{C}_i = \hat{c}_i) \cdot r \right\}$$

(2) Scheduling Decision: Schedule the user i^* that maximizes the queue-weighted rate R_i, as follows:

$$i^* = \arg\max_{i} \left\{ Q_i \cdot P(C_i \geq R_i \big| \widehat{C}_i = \hat{c}_i) \cdot R_i \right\}$$

Note that when the channel state estimation is perfect, the conditional probability $P(C_i \geq r \big| \widehat{C}_i = \hat{c}_i)$ is a step function, with Ψ essentially becoming the classic back-pressure policy. The next proposition establishes the throughput optimality of policy Ψ.

Proposition 2. *The scheduling policy* Ψ *supports all arrival rates that lie in the interior of the stability region* $\boldsymbol{\Lambda}$.

Proof: The proof proceeds as follows: Consider a Lyapunov function $L(\boldsymbol{Q}[t]) = \sum_{i=1}^{N} Q_i^2[t]$. For any arrival rate $\boldsymbol{\lambda}$ that lies strictly within the stability region $\boldsymbol{\Lambda}$, we know it is stably supportable by some policy G_0. Under G_0, we show that the Lyapunov drift corresponding to L is negative. We then show that policy Ψ minimizes the Lyapunov drift and hence will have a negative drift, thus establishing the throughput optimality of Ψ. ∎

IV. Part II: No Prior Knowledge of Channel-Estimator Joint Statistics

Using the results from the previous section as benchmarks, we study scheduling with rate adaptation when no prior knowledge of channel-estimator statistics, i.e., $P(C_i = c_i, \widehat{C}_i = \hat{c}_i)$, $\forall c_i \in \mathcal{S}, \hat{c}_i \in \mathcal{S}$ for $i \in \{1, \ldots, N\}$, is available at the scheduler. We first illustrate the system level loss incurred when no effort is made to learn these statistics, and hence no rate adaptation is performed, with a simple example.

A. Illustration of the Gains from Rate Adaptation

With no information on the channel-estimate joint statistics, the scheduler naively trusts the channel estimates to be actual channel states and transmits at the rate allowed in this state. Under this scheduling structure, for the single-hop network we consider, the stability region is given [22] by

$$\tilde{\Lambda} = \sum_{\hat{\boldsymbol{c}} \in \mathcal{S}^N} P_{\widehat{\boldsymbol{C}}}(\hat{\boldsymbol{c}}) \cdot \mathcal{CH}\Big[\ \mathbf{0}, P(C_i \geq \hat{c}_i \big| \widehat{C}_i = \hat{c}_i\ ) \cdot \hat{c}_i \cdot \vec{\mathbf{1}}_i;\ i = 1, \cdots, N\Big].$$

This region is plotted along-side the network stability region when full channel-estimator statistics is available at the scheduler and hence rate adaptation is performed. The following network is considered: a two-user single-hop network with independent, binary ON-OFF channel ($\mathcal{S} = \{1, 0\}$) between the base station and each user. The ON state allows successful transmission, while the OFF state corresponds to outage. For different mismatch between the estimate and the channel, Fig 1 plots the stability region of the system when rate adaptation is performed and when it is not. Notice the significant loss in stability region when rate adaptation is not performed. This loss increases with increase in the degree of channel-estimator mismatch. The preceding example underlines the importance of rate adaptation and hence the need to learn the channel-estimator statistics. We now proceed to introduce our joint channel learning, scheduling policy.

B. Joint Channel Learning, Scheduling Policy

We design the policy with the following main components: (1) The fraction of time slots the policy spends in learning the channel-estimator statistics is fixed at $\gamma \in (0, 1)$ (2) The worst-case rate of convergence of the channel-estimator statistics learning process is maximized. We formally introduce the policy next, followed by a discussion on the policy design.

Joint channel learning, scheduling policy (parameterized by γ)

(1) In each slot, the scheduler first decides whether to explore the channel of one of the users or transmit data to one of the users. Specifically, it randomly decides to explore the channel of user i with probability $\frac{x^i_{\hat{c}_i}}{N}$ where $\sum_{i=1}^N \frac{x^i_{\hat{c}_i}}{N} < 1$. The quantity $x^i_{\hat{c}_i} \in (0, 1]$ is a function of γ and the channel estimate $\hat{c}_i$ of user i. It is optimized to maximize the worst-case rate of convergence of the learning mechanism subject to the γ constraint. We postpone a discussion on this optimization to a later stage. Note that we have dropped the time index from the estimates for ease of notation.

(2) If a user is chosen for exploration, this time slot becomes an observing slot. Call the chosen user as e. The scheduler now sends data at a rate r_e that is uniformly chosen from the set $\mathcal{S}$. Let the quantity $\xi(t)$ indicate whether the transmission was successful or not, i.e.,

$$\xi(t) = \mathbf{1}(c_e \geq r_e\big).$$

where, recall, c_e denotes the current channel state of user e. Let $\Theta_{i,\hat{c},r}$ denote the set of exploration time slots when the channel estimate of user i was $\hat{c}$ and user i was explored with rate r. Thus the current slot is added to the set $\Theta_{e,\hat{c}_e,r_e}$. Now, an estimate of the quantity $P(C_e \geq r \Big| \widehat{C}_e = \hat{c}_e)$ is obtained using the following update:

$$\widehat{P}_t(C_e \geq r \Big| \widehat{C}_e = \hat{c}_e) = \frac{\sum_{k \in \Theta_{e,\hat{c}_e,r_e}} \xi(k)}{|\Theta_{e,\hat{c}_e,r_e}|}$$

where $|\mathcal{V}|$ denotes the cardinality of set $\mathcal{V}$.

(3) With probability $1 - \frac{1}{N}\sum_{i=1}^N x^i_{\hat{c}_i}$, no user is chosen for exploration and the slot is used for data transmission. The scheduler follows policy Ψ introduced in the previous section with $P(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i\ )$ replaced by the estimate $\widehat{P}_t(C_i \geq r \Big| \widehat{C}_i = \hat{c}_i\ )$.

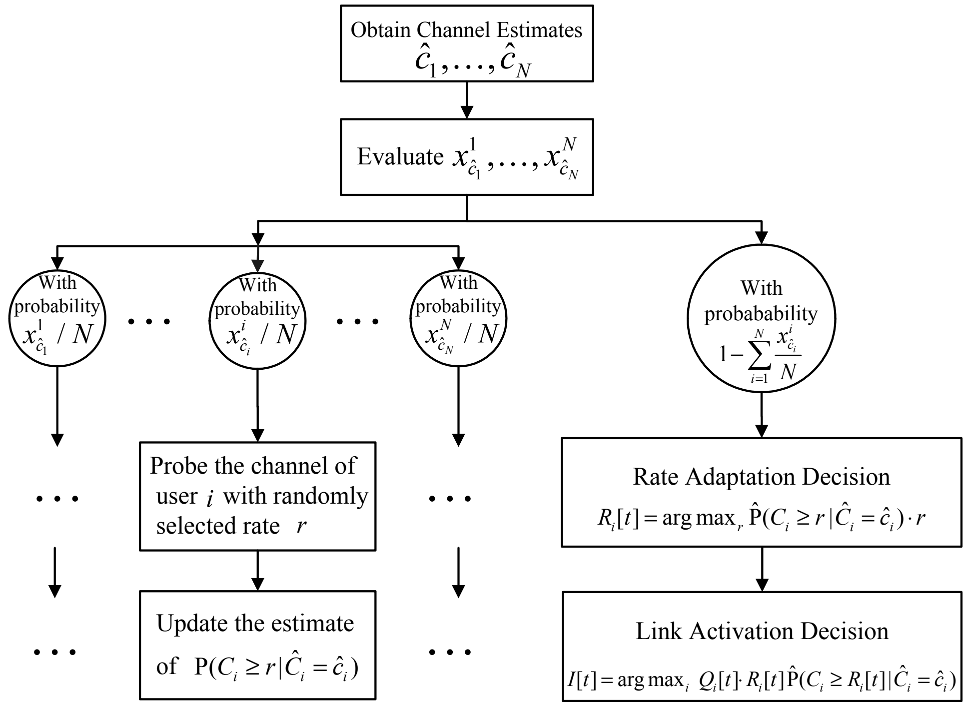

Fig. 2. Illustration of the joint channel learning - scheduling policy

An illustration of the proposed policy is provided in Fig 2. We now discuss the design of the quantities $x^i_{\hat{c}}$, $i \in \{1,\ldots,N\}$. Let $\eta_{i,\hat{c}} = P(\widehat{C}_i = \hat{c})\frac{x_{\hat{c}_k}}{N}$ be a measure of how often the channel of user i is explored when the estimate is $\hat{c}$. For fairness considerations, we impose the following constraint in addition to the γ-constraint discussed earlier:

$$\sum_{\hat{c}\in S} \eta_{i,\hat{c}} = \gamma/N.$$

The preceding constraint ensures that each user's channel is explored for equal fraction, γ/N , of total time slots. The rate of convergence of the channel statistics estimate, parameterized by the user and the channel estimate, is given by the following Lemma.

Lemma 3.

$$\limsup_{t\to\infty} \frac{\widehat{P}_t(C_i \geq r\big|\widehat{C}_i = \hat{c}) - P(C_i \geq r\big|\widehat{C}_i = \hat{c})}{\sqrt{\frac{\log\log(\frac{\eta_{i,\hat{c}}t}{|\mathcal{S}|})}{(\frac{\eta_{i,\hat{c}}t}{|\mathcal{S}|})}}} = \sqrt{2\sigma} \quad a.s.,$$

where $\sigma = P(C_i \geq r\big|\widehat{C}_i = \hat{c})(1 - P(C_i \geq r\big|\widehat{C}_i = \hat{c}))$.

Proof: The proof uses law of iterated logarithm [17] and elementary renewal theorem [18]. Details can be found in [22]. ■

Note from the preceding lemma that, for each $\{i, \hat{c}\}$, the higher the quantity $\eta_{i,\hat{c}}$ is, the faster is the convergence of $\widehat{P}_t(C_i \geq r\big|\widehat{C}_i = \hat{c})$. Also note that, for each user i, the channel estimate $\hat{c}$ with the slowest convergence affects the overall convergence performance for that user i. Taking note of this, we proceed to design $x^i_{\hat{c}}$ that maximizes the lowest convergence rate – the bottleneck.

The optimization problem (U) for each user i is given by

$$\begin{aligned}
\max_{x_{\hat{c}}} \quad \min_{i,\hat{c}} \quad & \eta_{i,\hat{c}} = \frac{1}{N} P(\widehat{C}_i = \hat{c}) x^i_{\hat{c}} \\
s.t. \quad & \sum_{\hat{c} \in \mathcal{S}} \eta_{i,\hat{c}} = \frac{\gamma}{N} \\
& \sum_{\hat{c} \in \mathcal{S}} P(\widehat{C}_i = \hat{c}) = 1 \qquad (3) \\
& 0 < x^i_{\hat{c}} \leq 1, \quad for\ all \quad \hat{c} \in \mathcal{S}
\end{aligned}$$

For ease of analysis, without loss of generality, we assume $P(\widehat{C}_i = s_1) \leq P(\widehat{C}_i = s_2) \leq \cdots \leq P(\widehat{C}_i = s_{|\mathcal{S}|})$. Let $[x^*_{s_1}, x^*_{s_2}, \cdots, x^*_{s_{|\mathcal{S}|}}]$ be the optimal solution to the above problem. We now record the structural properties of the optimal solution to (U).

Proposition 4. *The optimal solution $x^{i*}_{s_k}$, $\forall k \in \{1, \ldots, |\mathcal{S}|\}$ can be found with the following algorithm:*

(1) *Initialization: Let $k = 1$; $\Gamma = \emptyset$, $\omega = 0$;*
(2) *If $P(\widehat{C} = s_k) \geq \frac{\gamma - \sum_{s_j \in \Gamma} P(\widehat{C}_i = s_j)}{|\mathcal{S}| - w}$, then $\forall l \geq k$*

$$x^{i*}_{s_l} = \frac{\gamma - \sum_{s_j \in \Gamma} P(\widehat{C}_i = s_j)}{(|\mathcal{S}| - \omega) \cdot P(\widehat{C} = s_l)}.$$

Algorithm terminates.
(3) *Otherwise $x^{i*}_{s_k} = 1$, $\Gamma = \Gamma \cup s_k$, $\omega = \omega + 1$, $k = k + 1$. If $\Gamma = S$, algorithm terminates, otherwise repeat Step (2).*

Proof: The proof proceeds by establishing two crucial properties of the optimal solution. First, define Ω_i as the set of all channel estimates s_k such that the optimal $x^{i*}_{s_k} = 1$. Thus $\Omega_i = \cup_k \{s_k : \ x^{i*}_{s_k} = 1\}$. If no such estimate exists, $\Omega_i = \emptyset$. The optimal solution has the following properties:
(1) If $\Omega_i = \emptyset$ then $Pr(\widehat{C}_i = s_k) x^{i*}_{s_k} = \gamma / |\mathcal{S}|, \forall i$.
(2) If $\Omega_i \neq \emptyset$, then $x^{i*}_{s_1} = 1$.
Recall that the channel states are ordered such that $P(\widehat{C}_i = s_1) \leq P(\widehat{C}_i = s_2) \leq \cdots \leq P(\widehat{C}_i = s_{|\mathcal{S}|})$. The first property essentially says that if $\nexists$ a channel estimate s, for which $x^{i*}_s = 1$, then the optimal solution is such that the learning rate ($\frac{Pr(\widehat{C} = s_k) x^{i*}_{s_k}}{N}$) is uniform ($\frac{\gamma / |\mathcal{S}|}{N}$) for all s_k. Because, otherwise there is always room to improve the bottleneck convergence rate by redesigning the quantities $x^{i*}_{s_k}$. The second property says that whenever there exist an estimate s_j for which $x^{i*}_{s_j} = 1$, the estimate s_1 acts as a bottleneck and the optimal value of $x^{i*}_{s_1}$ must be 1. The proposed algorithm now checks if a solution yielding uniform convergence rate is feasible. If so, the solution is trivially given by $x^{i*}_{s_k} = \frac{1}{P(\widehat{C}_i = s_k)} \frac{\gamma}{\mathcal{S}}$. Otherwise, using the preceding properties the algorithm assigns $x^{i*}_{s_1} = 1$ and goes on to solve the reduced optimization problem over $x^{i*}_{s_2} \ldots, x^{i*}_{s_{|\mathcal{S}|}}$, iteratively. ■

The proposed algorithm is illustrated in Fig 3 when $\gamma = 0.2$, $N = 2$ and $\mathcal{S} = \{1, \ldots, 6\}$. Focusing on user 1, Fig 3(a) plots the probability of the estimated channels and the optimal values of x^*_s. Note that the lower the $P(\widehat{C} = s)$, the higher is the assigned x^*_s since the algorithm maximizes the bottleneck convergence rate $\frac{P(\widehat{C} = s) x^*_s}{N}$. This is further illustrated in Fig 3(b) where the optimized convergence rate is shown to be 'near uniform', underlining the min-max nature of the optimization.

We now perform a stability region analysis of the proposed policy. Define the stability region of a policy as the exhaustive set of arrival rates such that the network queues are rendered stable under the policy. The stability region of the proposed policy, parameterized by $\gamma \in (0, 1)$ is recorded below.

Proposition 5. *The stability region of the proposed policy is given by*

$$\mathbf{\Lambda}'_\gamma = \{\boldsymbol{\lambda} \ \ s.t. \ \ \frac{\boldsymbol{\lambda}}{1 - \gamma} \in \mathbf{\Lambda}\} \triangleq (1 - \gamma)\mathbf{\Lambda}.$$

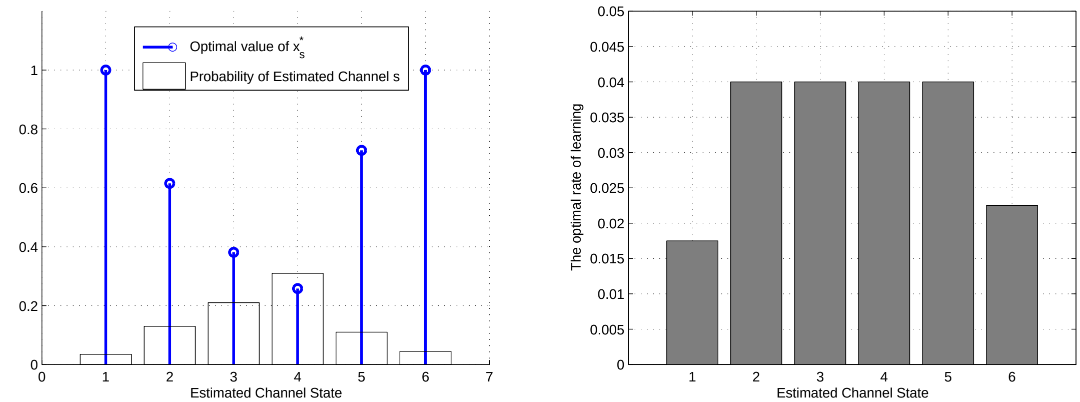

Fig. 3. Illustration of the design of optimal $x_{\hat{c}}^*$ when $N = 2$, $\gamma = 0.2$ and $\mathcal{S} = \{1, \ldots, 6\}$

where $\mathbf{\Lambda}$ *is the stability region of the network when accurate channel statistics information is available at the scheduler.*

Proof: The proof proceeds by showing that, under the proposed joint channel learning - scheduling policy, the instantaneous maximal sum of the queue weighted achievable rates, with sufficient time, can be arbitrarily close to the case when perfect channel statistics is available. ■

C. Numerical Results and Discussion

As $\gamma \rightarrow 0$, the proposed policy has a stability region that can be arbitrarily close to the system stability region Λ. The trade-off involved here is the speed of convergence and hence queueing delays. Since an analytical study of this trade-off appears complicated, we proceed to perform a numerical study. The simulation setup is described next.

We use *i.i.d* Rayleigh fading channels with minimum mean square error (MMSE) channel estimator as seen in [19] and [20]. The channel model is given by $Y = \sqrt{\rho} h X + \nu$, where X, Y correspond to transmitted and received signals, ρ is the average SNR at the receiver, and w is the additive noise. Both h and ν are zero mean complex Gaussian random variables, i.e., with probability density $\mathcal{CN}(0, 1)$. let $\hat{h}$ denote the estimate of the channel and $\tilde{h}$ denote the estimation error. Under the channel statistics assumed, $\tilde{h}$ is zero mean Gaussian with variance β, where the value of β depends on the resources allocated for estimation [21]. Given the value of h, the channel rate is $R = \log(1 + \rho|h|^2)$. We quantize the transmission rate to make the channel state space to be discrete and finite. We assume a two user network and fix $\beta = 0.1$ and $\rho = 50$ for both users' channels. We study the average behavior of the proposed policy by implementing it over 10000 parallel queuing systems.

We first study the time evolution of the probability of transmission success for different values of γ. Fig. 4(a) shows that, for any γ, the probability of successful transmission increases as the channel-estimator statistics information at the scheduler improves with time. Also, as expected, the larger the value of γ, the faster is the improvement in the probability of successful transmission. Note that higher transmission success probability essentially means lesser number of retransmissions. This is illustrated in Fig. 4(b). In Fig. 5, we study the average packet delay - delay between the time a packet enters the queue and the time it leaves the head of the queue - with respect to time for various values of γ. Once again, higher γ ensures faster learning of the channel-estimator statistics and consequently better rate adaptation and lesser retransmissions. This in turn reduces the average packet delay. Fig. 5 essentially illustrates a larger phenomenon: the trade-off between throughput (stability region) and delay.

V. Conclusion

We studied scheduling and rate adaptation in single-hop queueing networks, under imperfect channel state information. When the joint statistics of the channel-estimator mismatch is known at the scheduler, we characterized the network stability region and proposed a back-pressure type scheduling policy that is throughput optimal.

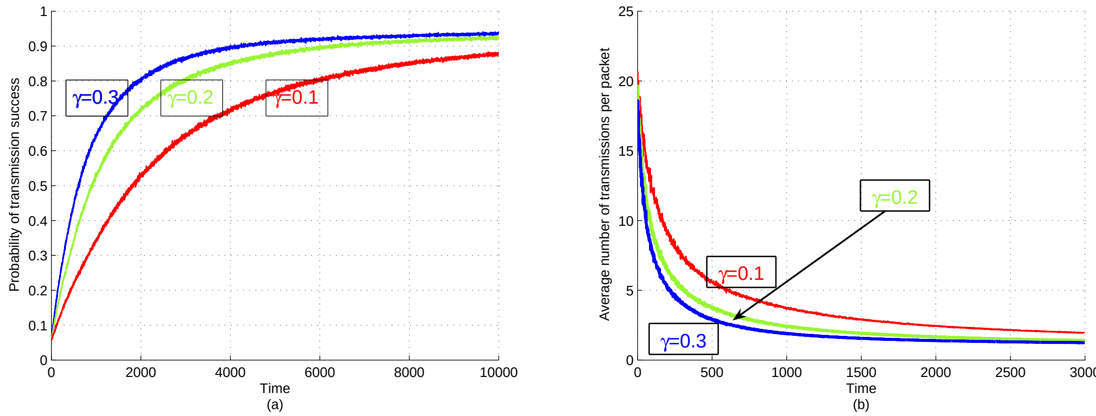

Fig. 4. Illustration of the time evolution of the (a) Probability of successful packet transmission (b) Average number of retransmissions per packet for various values of γ.

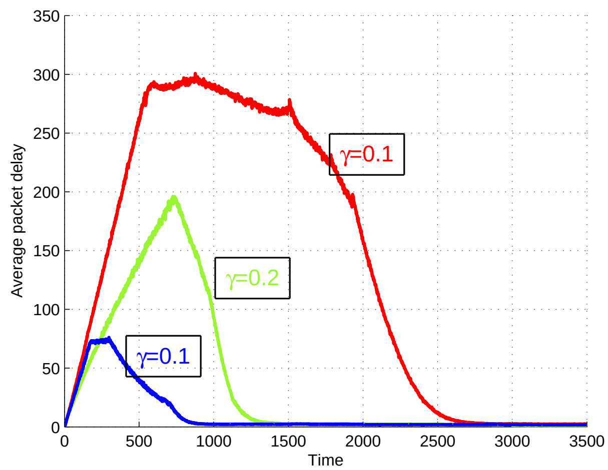

Fig. 5. Illustration of the average packet delay over time for various values of γ.

When the channel-estimator joint statistics is not known at the scheduler, we design a joint channel learning - scheduling policy that maximizes the worst case rate of convergence of the channel learning mechanism. We show that the proposed policy can be tuned to achieve a stability region arbitrarily close to the network stability region with an associated trade-off in the average packet delay.

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