# Counter-intuitive Characteristics of Rational Decision-Making using Biased Inputs in Information Networks

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***Abstract*—We consider an information network comprised of nodes that are: rational-information-consumers (RICs) and/or biased-information-providers (BIPs). Making the reasonable abstraction that any external event is reported as an answer to a logical statement, we model each node's information-sharing behavior as a binary channel. For various reasons, malicious or otherwise, BIPs might share incorrect reports of the event regardless of their private beliefs. In doing so, a BIP might favor one of the two outcomes, exhibiting intentional or unintentional bias (e.g. human cognitive biases). Inspired by the limitations of humans and low-memory devices in information networks, we previously investigated a graph-blind rational-information-consumer interested in identifying the ground truth. We concluded that to minimize its error probability, graph-blind RIC follows a counter-intuitive but tractable rule. In this work, we build on this foundational knowledge: "graph-blind RICs prefer the combination of information-providers that are all fully-biased against the a-priori likely input, over all other combinations." Upon studying RICs with partial knowledge of the network graph, we find that they act similar to graph-blind RICs when their BIPs "listen to" sufficiently many information-providers of their own. Furthermore, if a common node is informing/influencing all $n$ BIPs of a partially-aware RIC, that RIC anticipates its discovery of the "influential node" to diminish the average error probability by a factor that increases exponentially with $n$. However, from the partially-aware RIC's perspective, choosing $n$ fully-, similarly-biased BIPs outweighs the discovery of influential nodes among its BIPs' sources. These insights might inform the design of consumer-centric information networks.**

***Index Terms*—Rational decision-making, information networks, statistical decision theory, cognitive bias, data fusion.**

## I. Introduction

We consider an information network connecting users that may be devices with limited resources or human, and are biased-information-providers (BIPs) and/or rational-information-consumers (RICs). We assume that reports of a single event that occurred outside this network are propagated as a true (1) or false (0) answer to a logical statement. While our network model and analysis are applicable to any information network, we were primarily motivated by the unique qualities and constraints presented by modern-day social networks.

That is, a single bit of $0/1$ information from a source is being transferred via BIPs to the RICs. However, the BIPs may report incorrectly for a variety of reasons; we model a BIP's reporting behavior as a binary channel to depict the errors in its reporting of the input bit. In particular, each BIP can possess a *bias* favoring either the 0 bit or the 1 bit in its reporting. Since RICs (e.g. low-memory devices, humans) might not know the underlying graph of the information network, we can reasonably assume that they are *graph-blind* (if it only knows a list of its own neighbors), or that they are *partially-aware* (if it also knows the number of neighbors each of its neighbors has, or of any "influential" nodes among the neighbors of its neighbors.) Therefore, when trying to minimize its consumption of false information, *an RIC might be graph-blind or only partially-aware* and it will have to assume that all of its BIPs are acting independently unless there is evidence stating otherwise. This assumption of independence is a reasonable approximation of the typical behavior of RICs that are either human or low-memory devices.

In our prior work [1], we studied the impact of information-providers' biases on the choices of a graph-blind RIC that is attempting to accurately detect the original information. We recall these results in Section IV. The goal of this work is to perform a careful study of the impact of the information providers' biases on the choices of a "partially-aware" RIC that is attempting to accurately detect the original information. Given that BIPs are inevitably unreliable *and* biased, we are especially interested in unearthing "how" the BIPs' biases impact the RIC, and by "how much". In online social networks (OSNs), $0/1$ might represent contradicting depictions/viewpoints of a current event. For a particular social network user, its trusted media outlets and friends are the BIPs. A BIP's favorable opinion of arguments/evidence supporting one viewpoint as opposed to the other, might color its reports of the original/source information.

Even though the design is inspired and constrained by the particular nature of interactions on OSNs, this setting finds application in other information networks. For example, sensor networks where the devices have unequal false alarm and misdetection probabilities. Herein, the $0/1$ information at the source can represent the absence/occurrence of a sensor-triggering event. Akin to the OSN users described earlier, the sensors might have limited capacity to account for the network graph, they might be unreliable, and they might also have asymmetric sensitivities to the $0/1$ triggering event (i.e.,

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This work was supported in part by NSF grants: CNS-NeTS-1514260, CNS-NeTS-1717045, CMMI-SMOR-1562065, CNS-ICN-WEN-1719371, CNS-SpecEES-1824337, CNS-NeTS-2007231; the ONR Grant N00014-19-1-2621; and the DTRA grants: HDTRA1-15-1-0003 and HDTRA1-18-1-0050.

causing more false alarms than misdetections and vice versa).

Our problem statement bears comparison to these topics in literature: Containment of Misinformation, Information Theory, Social Sensing, and Information Fusion.

Influence maximization and containment of misinformation on social networks, by placing influential and protector nodes strategically, is studied in literature (e.g., [2], [3]). Our approach differs from these works in that we are not interested in the NP-hard problem of finding the most influential nodes to target, in a network. Instead, we are interested in the decisions of any rational-information-consumer choosing from a (possibly, large) selection of biased-information-providers.

Reliable transfer of information over unreliable binary channels is widely studied in information theory (e.g. [4]–[7]). Wherein, unlike the problem statement of this paper, the BIPs might choose to encode the information or transmit information about a block of $n_e$ events at the same time, to increase the rate of reliable information transfer (a.k.a. channel capacity). However, in OSNs and some sensor networks, the BIPs lack accountability and it is more practical to transmit information about each event separately and without delay. Here, BIPs are error-prone, biased, *abundantly available and act promptly*. This requires methods for reliable information transfer when *only one event* is processed at a time.

Social sensing literature studied the problem of the true value of a solitary binary quantity based on data arriving from multiple data sources of unknown credibility (e.g., [8]–[12]). These works, however, are focused on identifying duplicates and dependencies in incoming datasets and estimating the credibility of sources to maximize the probability of discovering the ground truth. As such, they do not consider the impact of information-providers' biases.

Information fusion studies the detection of ground truth in information (mostly, sensor) networks (e.g. [13]–[21]). These algorithms assume channel and noise characteristics, and may process signals sequentially, in the order of a sensor's reliability, accounting for channel fading, distance to its measuring target, etc. Our investigation contributes to these efforts, with a unique perspective on how, from abundantly-many BIPs, *graph-blind and partially-aware RICs choose BIPs that are optimally-biased, given their individual error rate.*

While intuition might dictate that an unbiased BIP is a better choice than a biased one, we find that a *rational*-information-consumer will choose the opposite. The choices and actions of the RIC (with limitations similar to those of humans or low-memory devices) can be summarized as follows:

- When choosing independent BIPs to report a $0/1$ event: we find that an RIC is best served by the BIPs that are fully biased against the a-priori likely event. Using this optimal choice of BIPs, the RIC will deduce that the a-priori likely event is true unless all BIPs report that the a-priori unlikely event is true (cf. Corollary 1). Further, if it is not possible to obtain BIPs that are fully-biased against the a-priori likely event, then the set of $n$ BIPs that will best serve the RIC are still guaranteed to be maximally-biased (cf. Theorem 1).
- When a system of unbiased BIPs (that do not favor either outcome) is replaced by a system of fully-biased BIPs (favoring the same outcome), the gain (in terms of probability of error) rises as an exponential function of a positive exponent (cf. Theorem 2) that decreases with the BIPs' error rate and increases linearly with $n$.
- When a RIC is partially-aware of the network graph by being aware of the number of information sources utilized by each of its $n$ BIPs), then we can make reasonable assumptions to deduce that the number-aware RIC will prefer BIPs that utilize sufficiently many sources to estimate the ground truth. The minimum number of sources that each BIP needs to utilize is a number that decreases when the bias of the BIP and its sources increases.
- Lastly, suppose a number-aware RIC, such as the one discussed previously, *discovers a single node influencing all $n$ of its BIPs*. The partially-aware RIC would anticipate this discovery to reduce its error probability for detecting the ground truth to fall exponentially with increasing $n$. However, choosing fully-biased BIPs that utilize sufficiently many sources appears to take precedence over the discovery of a single "influential" node.

We find that an RIC acts according to a counter-intuitive, but very tractable rule. As such, we consider RICs with limited knowledge of the network, model their surprising behaviors such as their perceived benefits of fully-biased information-providers and their perceived precedence of choosing fully-biased BIPs over accounting for influential nodes. These insights inform the models of rational-information-consumers that exhibit varying levels of network awareness and, thus, could enable novel RIC-centric frameworks with mechanisms that can leverage bias to reduce misinformation.

The rest of the paper is organized as follows. In Section II, we introduce the rational agent's model of the multi-BIP system of interest. In Section III, we discuss the characteristics of the optimal decision rule. In Section IV, we state the main results and our analysis of biases and their impact on a graph-blind RIC's choices from our prior work [1]. In Section V, we examine the choices that are perceived as optimal by an RIC that is partially-aware of the information network's underlying graph. We first analyze the behavior of an RIC that is aware of the number of sources that each of its BIPs utilizes. Next, in Section V-C, we analyze the behavior of the number-aware RIC when it discovers a single node that is influencing all $n$ of its BIPs. Finally, in Section V-D, we will numerically evaluate the least number of sources that a number-aware RIC will require from each of its BIPs, and thus, validate the results of Theorem 3. In Section VI, we summarize our analysis of graph-blind and partially-aware rational-information-consumers. The proofs of the results in Sections IV and V are presented in the Appendix.

## II. System Model

In this section, we will discuss the layout of an information network as viewed by a rational agent in the network, a.k.a. the rational-information-consumer (RIC). As we already stated, we are interested in human-like information consumers who can make rational decisions but cannot be made to learn the graph due to their limitations.

We differentiate between RICs that are graph-blind (have no knowledge of the network except the directly observable parameters of its neighbors) and RICs that are partially-aware of the network's graph structure. Partially-aware RICs that we discuss in this paper include: RICs that are number-aware (are knowledgeable of the number of biased-information-providers (BIPs) that each of its own BIPs has used), and RICs that are influence-aware (are knowledgeable of any common biased-information-providers that all of its own BIPs have used).

### *A. Rational Agent's Perspective of Information Sharing*

**Network of Information Providers and Consumers:** We consider any information network where the network users/nodes are biased-information-providers (BIPs) or rational-information-consumers (RICs) or both. We start by visualizing a general information network (for reference, see Fig. A.1a in the Appendix) from a graph-blind RIC's perspective, as shown in Fig. 1a.

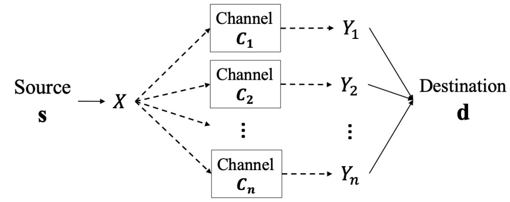
A graph-blind RIC will know the channel characteristics (Fig. 1b) of each of its BIPs, but not much else. It is natural to assume that any RIC would have limited knowledge of the global network and, therefore, will *perceive* its neighboring BIPs as *independent agents who are all accessing the ground truth*. In OSNs, we can assume that an information-provider might make false reports, perhaps biased in favor of a specific outcome. We also reason that an RIC can easily quantify the biases of its information-providers (by using the information-provider's history of reporting for past events where the ground truth is now known, for example).

Building on this, later in Section V, we will consider the information network from the perspective of a partially-aware RIC (Figures 6a). In addition to knowing the BIPs' channel characteristics, a partially-aware RIC will know some parameters pertaining to the BIPs that cannot be directly observed. Such as, the number of nodes that a BIP uses to obtain its raw information from; the presence of a common "influential" node that shares information with many BIPs of the same RIC.

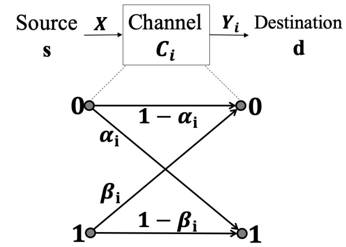
Here, in this section, we present the model of the agents in an information network and their interactions. We discuss the parameters relevant to the biases of the BIPs and the error-optimal decision policy that is relevant to the various RICs.

*Our goal is to identify an RIC's best strategy for maximizing its probability of identifying the ground truth given that it has no/partial network information. Based on the kind of information that is available to the RIC, we assume the RIC adopts a graph-blind perspective (Fig. 1a) or a number-aware perspective (Fig. 6a) or a influence-aware perspective (Fig. 7).*

**Multi-Channel Communication Model:** The system in Figure A.1b can be modeled as shown in Figure 1a, where the information from a source $\mathbf{s}$ arrives at an RIC at destination $\mathbf{d}$ through $n$ BIPs, whose reporting behavior can be modeled as parallel, independent binary channels. In the rest of the paper, we will use the terms "BIP" and "channel" interchangeably.




(a) System of $n$ independent BIPs connecting a source $\mathbf{s}$ and destination $\mathbf{d}$.




(b) Input-output behavior of the $i$-th BIP modeled by channel $C_i$.

Fig. 1: Information transfer through $n$ independent binary channels representing $n$ independent BIPs. Simplified version of Fig A.1a, from the perspective of graph-blind RIC.

Let $\{X(t)\}_t$ be the information stream available to a source $\mathbf{s}$, where $X(t) \in \{0,1\}$ is a binary random variable representing the information available to $\mathbf{s}$ at time $t$. We assume that the bits in the information stream are independent of each other and can be examined individually.

For ease of notation, we fix the time instant $t$ and fix $X := X(t)$. We assume that the prior probability distribution of $X$ is given by $\boldsymbol{\rho} = (\rho_0, \rho_1)$ such that $\rho_0 = P(X=0)$ and $\rho_1 = P(X=1)$. Without loss of generality, we assume $\rho_0 \geq \rho_1$.

Given $X$, we denote the information received by an RIC (at destination $\mathbf{d}$) through its BIPs' channels, using the binary random vector $\mathbf{Y}^{(n)} = \{Y_i\}_{i=1}^n$. The random variable $Y_i$ corresponds to the channel $C_i$, $\forall i \in \{1, \cdots, n\}$. We represent the behavior of the BIP represented by the binary channel $C_i$ in Fig. 1b, where $\alpha_i = P(Y_i = 1 | X = 0) \in [0,1]$ and $\beta_i = P(Y_i = 0 | X = 1) \in [0,1]$. Let the properties of the channel $C_i$ be given by $\mathcal{C}_i := (\alpha_i, \beta_i)$ (cf. Fig. 1b). We denote the properties of all $n$ channels using $\mathcal{C}^n := \{n, \boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)}\}$, where $\boldsymbol{\alpha}^{(n)} = \{\alpha_i\}_{i=1}^n$ and $\boldsymbol{\beta}^{(n)} = \{\beta_i\}_{i=1}^n$. Also, we define $\mathcal{S}_n := \{n, \rho_0, \boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)}\}$ to denote the system parameters.

**Decision Policy:** We are interested in the value of $X \in \{0,1\}$ that is more likely to generate $\mathbf{y}^{(n)} = \{y_i\}_{i=1}^n \in \{0,1\}^n$ as a realization of the random vector $\mathbf{Y}^{(n)}$. In other words, we are interested in a decision policy $\pi$: $\{0,1\}^n \mapsto \{0,1\}$ that achieves the smallest probability of error. We denote the expected probability of error of decision policy $\pi$ in the system $\mathcal{S}_n$ by $P_e^\pi(\mathcal{S}_n)$ and define it as follows.

$$P_e^\pi(\mathcal{S}_n) = \sum_{x \in \{0,1\}} \rho_x P\left\{\pi\left(\mathbf{Y}^{(n)}\right) \neq X | X = x\right\}. \qquad (1)$$

Let $\Pi^*(\mathcal{S}_n)$ be the set of error-optimal decision policies in a system with parameters $\mathcal{S}_n$. Therefore, for an error-optimal decision policy $\pi^* \in \Pi^*(\mathcal{S}_n)$,

$$\pi^*\left(\mathbf{y}^{(n)}\right) = \underset{x\in\{0,1\}}{\arg\max}\, P\left\{X = x; \mathbf{Y}^{(n)} = \mathbf{y}^{(n)} \middle| \mathcal{S}_n\right\},$$
$$P_e^{\pi^*}(\mathcal{S}_n) = \sum_{\mathbf{y}^{(n)}} \min_{x\in\{0,1\}} \rho_x P\left\{\mathbf{Y}^{(n)} = \mathbf{y}^{(n)} \middle| X = x\right\}. \quad (2)$$

**Channel Bias:** The parameters $(\alpha_i, \beta_i)$ capture the *bias* of the BIP modeled by $C_i$ (see Fig. 1b). So, $\alpha_i > \beta_i$ implies that $C_i$ changes a 0 input to a 1 output more often than it changes a 1 input to a 0 output. And if $\alpha_i < \beta_i$, the opposite holds true. For the multi-channel system $\mathcal{S}_n$, we are interested in the effect of the channel biases $(\boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)})$ on the smallest probability of error that the RIC can achieve. The following definition clarifies the concepts of unbiased, biased, and fully-biased (S, Z) BIPs/channels, which will be useful in the analysis.

**Definition 1** (Unbiased/Biased Channels)**.** *Channel $C_i$ is said to be* ***unbiased*** *if $\alpha_i = \beta_i$, and* ***biased*** *if $\alpha_i \neq \beta_i$. Fully-biased channels are special cases of biased channels: an S-channel with $\beta_i = 0$ (Fig. 2a); a Z-channel with $\alpha_i = 0$ (Fig. 2b).*

**Channel Error Rate:** Given the priors $\boldsymbol{\rho} = (\rho_0, \rho_1)$, the average rate at which an erroneous output is received at $\mathbf{d}$ from channel $C_i$ is given by

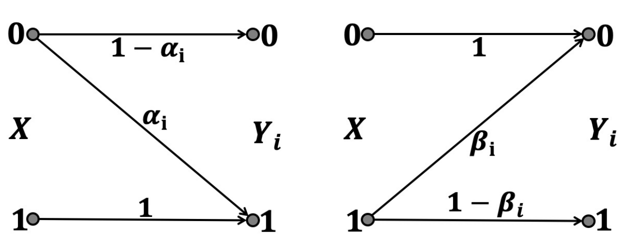
$$r_i^{(\boldsymbol{\rho})} \quad := \quad \rho_0\alpha_i + \rho_1\beta_i. \quad (3)$$

Also, in vector form, we use $\mathbf{r}^{(n)} = \{r_i^{(\boldsymbol{\rho})}\}_{i=1}^n$.

In the next section, we will characterize the optimal decision rule that is relevant to the RICs. Following that, in Section IV, we will state the graph-blind RIC's choices and behaviors that were studied in our earlier work.

## III. Optimal Decision rule

In this section, we will characterize the optimal decision rule and discuss its complexity. Then, we will reiterate our conclusions from our previous work that not only does a graph-blind RIC perceive the set of $n$ BIPs that are fully-biased against the a-priori likely outcome as the optimal choice, but it also perceives the set of $n$ unbiased BIPs to be an exponentially worse choice than the set of $n$ fully-biased BIPs.




(a) S-Channel (b) Z-Channel

Fig. 2: S- and Z-Channels

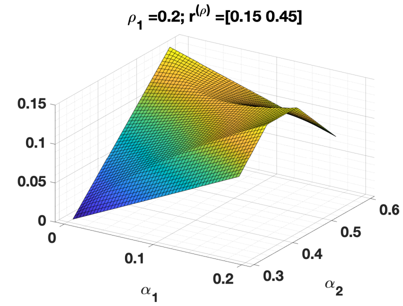



Fig. 3: Minimum error probability for 2 independent channels with fixed error rate, but varying bias reveals a complex form.

### A. Characterization and Discussion of the Optimal Rule

We start by describing the nature of the decision rule that forms the optimal decision policy for $\mathcal{S}_n$. We define

$$\tilde{P}_{\boldsymbol{\alpha}} := \tilde{P}_{\boldsymbol{\alpha}}\left(\mathbf{y}^{(n)}\right) := P(\mathbf{y}^{(n)}|X=0) = \prod_{i=1}^{n} \alpha_i^{y_i}(1-\alpha_i)^{1-y_i},$$
$$\tilde{P}_{\boldsymbol{\beta}} := \tilde{P}_{\boldsymbol{\beta}}\left(\mathbf{y}^{(n)}\right) := P(\mathbf{y}^{(n)}|X=1) = \prod_{i=1}^{n} (1-\beta_i)^{y_i}\beta_i^{1-y_i}.$$

The classical hypothesis testing framework [22] yields the optimal decision rule $\pi^*(\mathbf{y}^{(n)})$ for a given $\mathbf{y}^{(n)}$ as follows:

$$\rho_0\tilde{P}_{\boldsymbol{\alpha}}\left(\mathbf{y}^{(n)}\right) \underset{\pi^*(\mathbf{y}^{(n)})=0}{\overset{\pi^*(\mathbf{y}^{(n)})=1}{\lesseqgtr}} \rho_1\tilde{P}_{\boldsymbol{\beta}}\left(\mathbf{y}^{(n)}\right), \ \forall \mathbf{y}^{(n)}. \quad (4)$$

This decision rule can be further simplified into a Log-Likelihood-Ratio (LLR) with an additive structure. However, it is easy to see that this test is a highly nonlinear function of the bias parameters: $(\boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)})$. This nonlinearity significantly complicates the error analysis of the decision rule w.r.t. bias, which is the main objective of this work.

$$\frac{\partial \log\left(\tilde{P}_{\boldsymbol{\alpha}}/\tilde{P}_{\boldsymbol{\beta}}\right)}{\partial\alpha_i} = \frac{(\rho_1 - r_i^{(\boldsymbol{\rho})})y_i}{\rho_1\alpha_i(1-\beta_i)} + \frac{(\rho_0 - r_i^{(\boldsymbol{\rho})})(1-y_i)}{\rho_1\beta_i(1-\alpha_i)}.$$

In particular, note that: The value of $y_i$ alone (not $r_i^{(\boldsymbol{\rho})}$) decides whether $\tilde{P}_{\boldsymbol{\alpha}}$ and $\tilde{P}_{\boldsymbol{\beta}}$ are both increasing or decreasing in $\alpha_i$. So, even when $\frac{\tilde{P}_{\boldsymbol{\alpha}}}{\tilde{P}_{\boldsymbol{\beta}}}$ is a monotonic function of $\boldsymbol{\alpha}^{(n)}$ for all $\mathbf{y}^{(n)}$, the impact of $\boldsymbol{\alpha}^{(n)}$ on $P_e^{\pi^*}(\mathcal{S}_n) = \sum_{\mathbf{y}^{(n)}} \min\left\{\tilde{P}_{\boldsymbol{\alpha}}, \tilde{P}_{\boldsymbol{\beta}}\right\}$ might not be monotonic. Fig. 3 further illustrates the nontrivial nature of the optimal error as a function of channel biases for $n$ as small as 2, calling for an analysis of the choices of bias.

## IV. Characteristics and Performance of Graph-blind RICs

The graph-blind RIC has a limited perspective of the graph of the information (for reference, see Figure A.1b in the Appendix). A graph-blind RIC has no knowledge of the network except for its own BIPs.

In our previous investigation [1], we derived the choices that the graph-blind RIC will consider optimal. The results are presented in Theorems 1, 2 and Corollary 1. The proofs can be found in the Appendix.

The realization of the difficulty in the direct analysis of the optimal decision rule motivated us to seek a uniform lower bound on the probability of error. By proving that for a given $n$, the minimum error is achieved by a system $\mathcal{S}_n$ of extremely-biased channels, the following theorem later establishes that *systems of fully-biased channels which favor the a-priori unlikely outcome will yield the least error while detecting the source information.*

**Theorem 1** (Performance of the optimal decision policy is coordinate-wise concave in bias)**.**
*Without loss of generality, assume that $\rho_0 \geq \rho_1$. Consider a system of $n$ independent, parallel binary channels described by $\mathcal{S}_n = \left(n, \rho_0, \boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)}\right)$, where $\rho_0 \boldsymbol{\alpha}^{(n)} + \rho_1 \boldsymbol{\beta}^{(n)} = \mathbf{r}^{(n)}$. Assume that $\mathbf{r}^{(n)}$ is fixed to be a constant, and*[1] *$r_k^{(\boldsymbol{\rho})} \in \left[0, \frac{1}{2}\right] \forall k$.*

*For every such $\mathcal{S}_n$, there is an optimal decision policy $\pi^*$ chosen from $\Pi^*\left(\mathcal{S}_n\right)$ and an average probability of error $P_e^{\pi^*}(\mathcal{S}_n)$. This error function $P_e^{\pi^*}(\mathcal{S}_n)$ is a* concave function *in each individual $\alpha_k$, for $k \in \{1, \cdots, n\}$.*

Theorem 1 leads us to the conclusion that: while holding the $\mathbf{r}^{(n)}$ fixed, the error function $P_e^{\pi^*}(\mathcal{S}_n)$ will achieve its least value for a system $\mathcal{S}_n$ such that $\alpha_k \in \{\alpha_{k,\min},\ \alpha_{k,\max}\}\ \forall k$. Here, $\alpha_{k,\min} \geq \frac{\left(r_k^{(\boldsymbol{\rho})} - \rho_1\right)^+}{\rho_0}$ and $\alpha_{k,\max} \leq \frac{r_k^{(\boldsymbol{\rho})}}{\rho_0}$. We will refine the $2^n$ possible combinations of extreme bias to find the one that achieves the lower bound on the probability of error.

**Corollary 1** (Similarly and fully-biased BIPs are optimal)**.** *Assume that $\rho_0 \geq \rho_1$, $\mathbf{r}^{(n)} \in \left[0, \frac{1}{2}\right]^n$ is fixed, and $\rho_0 \prod_{i=1}^{n} \frac{r_i^{(\boldsymbol{\rho})}}{\rho_0} \leq \rho_1$. Then the least probability of error is achieved by an optimal policy on $n$ parallel, binary channels when $\mathcal{S}_n$ is a system of $n$ S-channels (Fig. 2a). That is, $\boldsymbol{\beta}^{(n)} = \mathbf{0}^{(n)}$.*

$$P_e^{\pi^*}(\mathcal{S}_n) \geq \rho_0 \prod_{i=1}^{n} \frac{r_i^{(\boldsymbol{\rho})}}{\rho_0}. \tag{5}$$

*Since $\rho_0 \geq \rho_1$, the optimal decision policy for a system of S-channels is $\pi^*\left(\mathbf{Y}^{(n)}\right) = Y_1 \cdot Y_2 \cdot \cdots \cdot Y_n$.*

Corollary 1 strongly reveals that *systems with $n$ independent, fully-biased BIPs/channels which favor the a-priori unlikely outcome* (i.e., BIPs with S or Z-channels, depending on the priors) are preferable to all other possible BIPs/channels with the same individual average error rates. An RIC with such BIPs is prone to error only when all $n$ of them are wrong.

This result provides an insight that may be against common sense: when looking for $n$ independent biased-information-providers with prefixed error rates $\mathbf{r}^{(n)}$, it is not optimal to choose the diversely biased or totally unbiased BIPs! In fact, it is best to select all similarly and fully biased BIPs, but pay extra attention if one of them reports *against* their bias.

[1]If $\exists k$ such that $r_k^{(\boldsymbol{\rho})} > \frac{1}{2}$, then we can map $(1 - Y_k, 1 - r_k^{(\boldsymbol{\rho})}, 1 - \alpha_k, 1 - \beta_k) \mapsto (Y_k, r_k^{(\boldsymbol{\rho})}, \alpha_k, \beta_k)$.

We note that this finding greatly extends a loosely related result in [6], which proves that only among channels of very low and equal capacity, a maximally asymmetric channel with a noiseless symbol has the least probability of error.

Motivated by the optimality of full bias in minimizing probability of error, we also studied the gains obtained by working with a set of similarly and fully-biased BIPs over a set of unbiased BIPs. For fair comparison, we assume that the average error rates $r_i^{(\boldsymbol{\rho})}$ of the BIPs are all equal to $r^{(\boldsymbol{\rho})}$ in both scenarios. Therefore, the BIPs are equivalent in their average rate of sending erroneous bits, but differ in their biases.

### *A. Characterizing the gains of fully-, similarly-biased BIPs*

Corollary 1 demonstrates "how" the BIPs's affect the RIC's decisions. In this section, we will investigate the gains that the RIC anticipates to obtain from its choices.

The following theorem characterizes these gains with upper and lower bounds, which become asymptotically tight as the number of BIPs, $n$, increases.

**Theorem 2** (Gains of Fully-Biased BIPs vs Unbiased BIPs)**.** *Fix $r^{(\boldsymbol{\rho})} \in \left(0, \frac{1}{2}\right]$ to be the common error rate for all the BIPs' channels. Then, let $\mathcal{S}_n^u = \left\{n, \rho_0, r^{(\boldsymbol{\rho})}\mathbf{1}^{(n)}, r^{(\boldsymbol{\rho})}\mathbf{1}^{(n)}\right\}$ and $\mathcal{S}_n^f = \left\{n, \rho_0, \frac{r^{(\boldsymbol{\rho})}}{\rho_0}\mathbf{1}^{(n)}, \mathbf{0}^{(n)}\right\}$, respectively, describe the unbiased and fully-biased systems, each containing a set of $n$ independent BIPs. Correspondingly, let the error-optimal decision policies for $\mathcal{S}_n^u$ and $\mathcal{S}_n^f$ be denoted as $\pi_u^*$ and $\pi_f^*$, respectively*[2]*. Then, for any $n \in \mathbb{N}$, we have*

$$\ln\left(\frac{P_e^{\pi_u^*}\left(\mathcal{S}_n^u\right)}{P_e^{\pi_f^*}\left(\mathcal{S}_n^f\right)}\right) \leq m \ln\left(4\rho_0^2\left(\frac{1}{r^{(\boldsymbol{\rho})}} - 1\right)\right) + \ln\frac{2(m+1)}{\rho_0}$$

$$\ln\left(\frac{P_e^{\pi_u^*}\left(\mathcal{S}_n^u\right)}{P_e^{\pi_f^*}\left(\mathcal{S}_n^f\right)}\right) \geq m \ln\left(4\rho_0^2\left(\frac{1}{r^{(\boldsymbol{\rho})}} - 1\right)\right) - \ln\frac{4m}{\rho_1},$$

*where $m = \left\lfloor \frac{n}{2} \right\rfloor$. Moreover, asymptotically, these bounds converge to get*

$$\lim_{n\to\infty} \frac{1}{n} \ln\left(\frac{P_e^{\pi_u^*}\left(\mathcal{S}_n^u\right)}{P_e^{\pi_f^*}\left(\mathcal{S}_n^f\right)}\right) = \frac{1}{2}\ln\left(4\rho_0^2\left(\frac{1}{r^{(\boldsymbol{\rho})}} - 1\right)\right). \tag{6}$$

This theorem (proof in Appendix) reveals that the proportional gains of using all fully biased BIPs rather than all unbiased BIPs will increase exponentially with $n$, and the factor that is *explicitly* characterized in (6) in terms of the average error rates $r^{(\boldsymbol{\rho})}$ of the channels. This shows that the gains are particularly high when the average error rates $r^{(\boldsymbol{\rho})}$ are closer to zero. It is expected that the gains will be relatively small when $r \to \frac{1}{2}$, since the probability of error approaches $\frac{1}{2}$ in both scenarios.

Numerical results pertaining to this theorem can be found in the following section.

[2]Note that $\pi_f^*$ is the policy described in Corollary 1.

This result is quite useful in characterizing the conditions under which the use of fully biased BIPs would be especially preferable to unbiased BIPs, and the conditions under which using all unbiased channels may be acceptable. Moreover, the upper and lower bounds on the gains can be reverse engineered to determine the number of BIPs $n$ needed to guarantee a desired limit on the probability of error for a graph-blind RIC.

### *B. Simulations*

In this section, we perform numerical experiments to validate our theoretical results and to develop a broader understanding of the impact of bias on the choices that appear optimal to a graph-blind RIC that seeks $n$ BIPs.

We showed that a graph-blind RIC expects to minimize its error probability, by choosing fully-biased BIPs amongst all BIPs that have the same individual error rates (cf. Corollary 1). Further, we want to know if the RIC expects other choices to yield vastly different error probabilities (as in Theorem 2).

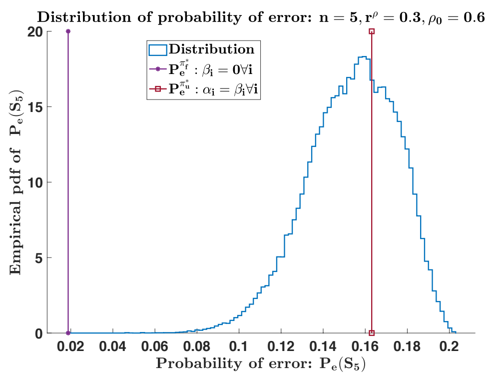



Fig. 4: Distribution of probabilities of error in a system with fixed priors ($\boldsymbol{\rho}$) and fixed channel error rate $r^{(\boldsymbol{\rho})}$.

In Figure 4, we explore this question for $n = 5$ channels, $\boldsymbol{\rho} = (0.6, 0.4)$, and $r^{(\boldsymbol{\rho})} = 0.3$. We choose many different values of $(\boldsymbol{\alpha}^{(n)}, \boldsymbol{\beta}^{(n)})$ that satisfy $r^{(\boldsymbol{\rho})} = 0.3$ in order to generate the plotted histogram of the optimal-probability-of-error values obtained by them. The resulting plot confirms that the fully-biased system with all S-channels ($\beta_i = 0, \forall i$) achieves the minimum value (in this case $\approx 0.02$). Moreover, it reveals that the error probabilities has a wide range between $0.02$ and $0.2$ with most of the distribution centered around higher values of $\approx 0.15$. In particular, the case of unbiased channels ($\alpha_i = \beta_i, \forall i$) yields an error probability of $\approx 0.165$. As such, this figure further highlights the significance of utilizing fully-biased channels as opposed to other choices.

In Theorem 2, we replaced unbiased BIPs ($\alpha_i = \beta_i$) with fully-biased BIPs that favor the same outcome ($\alpha_i = 0, \forall i$ or $\beta_i = 0, \forall i$). There, we established that the RIC's anticipated gains (factor by which probability of error will fall) grow asymptotically at an exponential rate with the exponent being linearly dependent on $n$ (the number of BIPs/channels used). We obtained this result by deriving upper and lower bounds on the gains that become tight as $n$ increases. To check the tightness of these bounds, the rates of change of the exponents with $n$ are plotted in Figure 5 for different $\boldsymbol{\rho}$ and $r^{(\boldsymbol{\rho})}$. From this plot, we verify that the growth rates of the exponents of the gains are, in fact, linear in $n$. Moreover, the upper and lower bounds are tight even for fairly small values of $n$. We also confirm that the growth rates of the exponents in the gains and their bounds all converge to $\frac{1}{2}\ln\left(4\rho_0^2\left(\frac{1}{r^{(\boldsymbol{\rho})}} - 1\right)\right)$. Incidentally, the plot also reveals that the RIC expects to obtain almost no gains, by replacing unbiased BIPs with fully-biased BIPs when $\rho_1 = r^{(\boldsymbol{\rho})} = \frac{1}{2}$, confirming our earlier intuition.

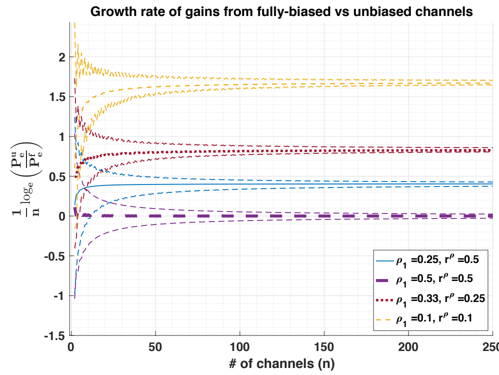



Fig. 5: Asymptotic growth rate of the gain $\frac{P_e^u}{P_e^f}$.

From a graph-blind RIC's perspective, these numerical investigations not only validate the optimality of fully-biased BIPs and their asymptotic gains over unbiased BIPs, but also reveal that: (i) even partially-biased BIPs lose substantially against the fully- and similarly-biased ones, and (ii) the asymptotic performance is approached quite rapidly for increasing $n$.

Recall that, the graph-blind RIC perceives the following behaviors as optimal:

- Among the BIPs with the same channel error rate, the optimal choice is BIP that is extremely-biased (fully-biased, if such a BIP exists).
- When choosing BIPs that are fully-biased, the estimation error is minimized if the BIPs are all biased in the same direction, against the a-priori likely event.

From the conclusions that arise from the above model of a graph-blind RIC, we are given to wonder how partial knowledge of the graph will affect the RIC's choices and how these choices can affect the network. In particular, we are given to wonder whether the optimality of similarly-and-highly-biased BIPs will persist and if the selection of such BIPs takes precedence over the conclusions drawn from the partial knowledge of the graph. To that end, we will study partially-aware RICs in the next section. These are RICs that have partial knowledge of the information network's underlying graph, as opposed to graph-blind RICs which have no knowledge of the information network's underlying graph.

## V. Characteristics and Performance of Number-Aware RICs

Given the behaviors and choices that are perceived as optimal by the graph-blind RIC in the previous section, here we intend to expand our analysis to the behaviors that are perceived as optimal by an RIC that is also aware of the number of information-providers that were employed by its own BIPs.

***Assumptions about the number-aware RIC:*** We note that a number-aware RIC is only aware of the number of information-providers employed by its BIPs, but not their biases. Therefore, in order to conduct our analysis, we start by assuming that the RIC conjectures that: its $i$-th BIP will choose its own information-providers such that they each *independently* have error rates and biases that are identical to those of the $i$-th BIP itself.

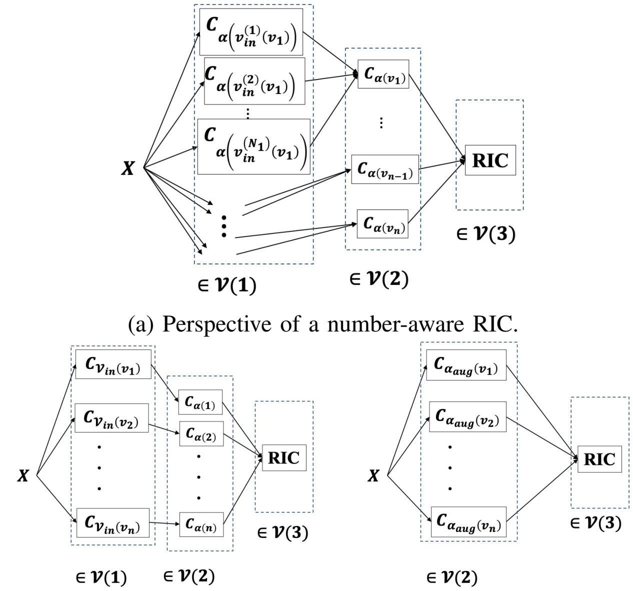
However, we also find that our assumption does not affect the larger implications of our analysis in this section. That is because, based on the optimal decision policy from Sections II and III, we can verify the extension of our conclusions to the case where the $i$-th BIP and its $N_i$ sources have biases and individual error rates which differ from each other. And also, because we find that the broader conclusions of our analysis are that: "the number-aware RIC will still view similarly-and-highly-biased nodes as the optimal choice." And since the channel characteristics are assumed to be independent (even if they are identical), we can see that the broader implications of the analysis in this section will apply even when the channel characteristics are *not* identical. For instance:

- Assume a BIP and its information sources are similarly-and-highly-biased (have a high $\alpha$ to $\beta$ ratio). A number-aware RIC will consider this BIP to be reliable even if its information sources are fewer in number.
- If a BIP and its information sources are unbiased, then a number-aware RIC will consider this BIP to be reliable only if it has a larger number of information sources.
- Lastly, assume a BIP and its information sources are highly-biased, but their biases are in opposite directions. To a number-aware RIC, the combined (augmented) BIP derived from (7)-(9) will appear to have very little bias.

***Model of a number-aware RIC in an information network:*** Consider the network given in Figure A.1a. Let the underlying graph be denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. We will assume that the nodes in $\mathcal{V}$ can be partitioned into subsets $\mathcal{V}(1), \mathcal{V}(2), \cdots, \mathcal{V}(h)$ and so on, where the nodes in $\mathcal{V}(i), i \geq 2$ are the rational information-consumers and derive their BIPs from $\mathcal{V}(i-1)$ alone. From Figure 6a and A.1, it is obvious that the nodes in $\mathcal{V}(1)$ do not have biased-information-providers but get the true information or ground truth directly.

Let the RIC be denoted by the node $v_D \in \mathcal{V}$. Every node $v \in \mathcal{V}$, that acts as a BIP at some point of time, has the individual error rate is given by $r(v) \leq \frac{1}{2}$ and the biases are given by $\alpha(v)$ and $\beta(v) = \dfrac{1}{\rho_1} \cdot (r(v) - \rho_0 \alpha(v))$. To facilitate our analysis we will assume that $\alpha(v) \geq \beta(v)$ and $\alpha(v) + \beta(v) < 1$ for every $v \in \mathcal{V}$.

Now, we note that from the perspective of a number-aware RIC the network appears as shown in Figure 6a. That is, with its limited knowledge of the information network, the number-aware RIC assumes that: it belongs to the subset $\mathcal{V}(3)$ in the network hierarchy, that its BIPs belong to $\mathcal{V}(2)$, and that their BIPs belong to $\mathcal{V}(1)$. Further, we assume that the RIC itself has $n$ BIPs and the $i$-th BIP receives information from $N_i$ BIPs of its own. Let $\mathbf{N}^{(n)} = [N_1, N_2, \cdots, N_n]^T$.



(a) Perspective of a number-aware RIC.

(b) Augmentation of BIPs by the number-aware RIC.

Fig. 6: Augmented BIPs of a number-aware RIC.

Let the number-aware RIC, $v_D$, receive the binary vector $\mathbf{Y}_{in}^{(n)}(v_D)$ from its BIPs. The RIC will be interested in the optimal decision policy,

$$\pi^* \left( \mathbf{Y}_{in}^{(n)}(v_D), \mathbf{N}^{(n)} \right) = \arg\max_X P \left( X; \mathbf{Y}_{in}^{(n)}(v_D) \mid \mathbf{N}^{(n)} \right).$$

### A. *Perspective of a number-aware RIC: Augmented BIPs*

Figure 6b shows how the network, as perceived by the number-aware RIC, can be distilled into the simpler form with $n$ parallel BIPs with the individual error rates $\mathbf{r}_{aug}^{(n)}$ and the biases $\boldsymbol{\alpha}_{aug}^{(n)}$, $\boldsymbol{\beta}_{aug}^{(n)}$.

Let $\mathbf{r}_{aug}^{(n)} = \{r_{aug}^{(\boldsymbol{\rho})}(v_i)\}_{i=1}^n$, $\boldsymbol{\alpha}_{aug}^{(n)} = \{\alpha_{aug}(v_i)\}_{i=1}^n$ and $\boldsymbol{\beta}_{aug}^{(n)} = \{\beta_{aug}(v_i)\}_{i=1}^n$. Note that the channel $\mathcal{C}_{\alpha_{aug}(v_i)}$ is a series combination of the channels $\mathcal{C}_{\alpha(\mathcal{V}_{in}(v_i))}$ and $\mathcal{C}_{\alpha(v_i)}$.

From the perspective of the RIC node $v_D$, all of its BIPs are in $\mathcal{V}(2)$. We notice that every node in $\mathcal{V}(2)$ has BIPs, but those BIPs access the ground truth and do not have (or need) any information-providers of their own. Thus, the number-aware RIC assumes that when each of its BIPs were receiving information and identifying the ground truth, they could only have acted as graph-blind RICs and not number-aware RICs. The channel $\mathcal{C}_{\alpha(\mathcal{V}_{in}(v_i))}$ is a distilled representation of the graph-blind decision policy adopted by $v_i \in \mathcal{V}(2)$, which is the $i$-th BIP of the node $v_D$.

$$\alpha_{\mathcal{V}_{in}(v_i)} = P_{e,\mathcal{V}_{in}(v_i)}^0 := P \left( \pi^*(\mathbf{Y}_{in}^{(n)}(v_i)) = 1 \mid X = 0 \right),$$

$$\beta_{\mathcal{V}_{in}(v_i)} = P_{e,\mathcal{V}_{in}(v_i)}^1 := P \left( \pi^*(\mathbf{Y}_{in}^{(n)}(v_i)) = 0 \mid X = 1 \right).$$

Therefore, the channel $\mathcal{C}_{\alpha(V_{in}(v_i))}$ has an individual error rate denoted by

$$r_{\mathcal{V}_{in}(v_i)} = P_{e,\mathcal{V}_{in}(v_i)} := \rho_0 P^0_{e,\mathcal{V}_{in}(v_i)} + \rho_1 P^1_{e,\mathcal{V}_{in}(v_i)}.$$

The biases and error rate corresponding to $\mathcal{C}_{\alpha_{aug}(v_i)}$ are $\alpha_{aug}(v_i)$, $\beta_{aug}(v_i)$, and $r_{aug}(v_i)$. Since the channel $\mathcal{C}_{\alpha_{aug}(v_i)}$ is a series combination of the channels $\mathcal{C}_{V_{in}(v_i)}$ and $\mathcal{C}_{\alpha(v_i)}$,

$$\alpha_{aug}(v_i) = \alpha(v_i) + (1 - \alpha(v_i) - \beta(v_i))P^0_{e,\mathcal{V}_{in}(v_i)} \quad (7)$$

$$\beta_{aug}(v_i) = \beta(v_i) + (1 - \alpha(v_i) - \beta(v_i))P^1_{e,\mathcal{V}_{in}(v_i)} \quad (8)$$

$$r_{aug}(v_i) = r(v_i) + (1 - \alpha(v_i) - \beta(v_i))P_{e,\mathcal{V}_{in}(v_i)}. \quad (9)$$

*B. Optimal choices of a number-aware RIC*

For the RIC in Figure 6, the following result proves that the minimum probability of error achieved by the number-aware RIC is coordinate-wise concave with respect to the value of $\alpha(v_i)$ assuming that $r_{aug}(v_i)$ is fixed. Thus, using this Corollary, we can extend the result of Theorem 1 that applies only to graph-blind RICs, to number-aware RICs.

**Corollary 2.** *The minimum probability of error $P_e^{\pi^*}(\mathcal{S}_n)$ for the RIC in Figure 6 is coordinate-wise concave in both $\alpha_{aug}(v_i)$ and $\alpha(v_i)$, when $r_{aug}(v_i)$, $P^0_{e,\mathcal{V}_{in}(v_i)}$ and $P^1_{e,\mathcal{V}_{in}(v_i)}$ are fixed and $r_{aug}(v_j) \leq \frac{1}{2}$, $j = 1, 2, \cdots, n$.*

*Proof.* We already know from Theorem 1 that the minimum probability of error $P_e^{\pi^*}(\mathcal{S}_n)$ for the RIC is coordinate-wise concave in $\alpha_{aug}(v_i)$ when $r_{aug}(v_i)$ is fixed.

We know that the minimum probability of error $P_e^{\pi^*}(\mathcal{S}_n)$ is a function of $\boldsymbol{\alpha}^{(n)}_{aug}$ and $\mathbf{r}^{(n)}_{aug}$ alone. So it is enough to show that $\alpha_{aug}(v_i)$ is a linear function of $\alpha(v_i)$ alone, provided $r_{aug}(v_i)$ is constant. Now, we rearrange (7),(8),(9) as follows.

$$\alpha_{aug}(v_i) = \alpha(v_i)(1 - P^0_{e,\mathcal{V}_{in}(v_i)}) - \beta(v_i)P^0_{e,\mathcal{V}_{in}(v_i)} + P^0_{e,\mathcal{V}_{in}(v_i)}$$

$$\beta_{aug}(v_i) = -\alpha(v_i)P^1_{e,\mathcal{V}_{in}(v_i)} + \beta(v_i)(1 - P^1_{e,\mathcal{V}_{in}(v_i)}) + P^1_{e,\mathcal{V}_{in}(v_i)}$$

$$r_{aug}(v_i) = \alpha(\rho_0 - P_{e,\mathcal{V}_{in}(v_i)}) + \beta(v_i)(\rho_1 - P_{e,\mathcal{V}_{in}(v_i)}) + P_{e,\mathcal{V}_{in}(v_i)}.$$

Therefore, we have the following linear mapping between $\alpha_{aug}(v_i)$ and $\alpha(v_i)$.

$$\alpha_{aug}(v_i) = \alpha(v_i) \cdot \rho_1 \cdot \frac{1 - P^0_{e,\mathcal{V}_{in}(v_i)} - P^1_{e,\mathcal{V}_{in}(v_i)}}{\rho_1 - P_{e,\mathcal{V}_{in}(v_i)}} + P^0_{e,\mathcal{V}_{in}(v_i)} \cdot \frac{\rho_1 - r_{aug}(v_i)}{\rho_1 - P_{e,\mathcal{V}_{in}(v_i)}}. \quad (10)$$

From (10), we can conclude that since the minimum probability of error $P_e^{\pi^*}(\mathcal{S}_n)$ for the RIC is coordinate-wise concave in $\alpha_{aug}(v_i)$ for a fixed $r_{aug}(v_i)$, it is also coordinate-wise concave in $\alpha(v_i)$ corresponding to the $i$-th BIP. □

The corollary above extends our insight in Theorem 1 regarding the effect of bias on the optimal choices of the graph-blind RIC to the optimal choices of the number-aware RIC. In both cases, a BIP becomes more preferable to the RIC, if it becomes more biased than it was before while its *augmented* error rate remains the same. While the corollary only requires that $r_{aug}(v_i)$ remain constant and not $r(v_i)$, we find it is more crucial to understand how changes in the biases *as well as* the error rates of the *individual* BIPs and their information-providers will affect the choices that will be perceived as optimal by the number-aware RIC.

*1) Critical Assumption on Number-aware RIC:* $v_D$ is a number-aware RIC and as its name suggests, it only knows the number of information-providers used by its own BIPs. Their individual biases are either unavailable to $v_D$ or not being utilized by $v_D$ or will become irrelevant to $v_D$.

In any such case, the RIC might reasonably assume that all $N_i$ sources of its $i$-th BIP will have biases and individual error rates identical to those of the $i$-th BIP. (Based on the optimal decision policy from Sections II and III, we can verify the extension of our conclusions to the case where the $i$-th BIP and its $N_i$ sources have biases and individual error rates which differ from each other.) We also consider this a reasonable assumption based on the following conjectures.

(i) Since the RIC is only number-aware and it cannot/will not access the identities or individual values of the biases and error rates of the $i$-th BIP's sources.
(ii) However, similar to the graph-blind RIC, the number-aware RIC can either access or empirically estimate the bias and the individual error rate of the $i$-th BIP augmented by its $N_i$ sources.
(iii) For social networks, it is reasonable to assume that the $i$-th BIP acquires information from sources that hold similar biases as it does.
(iv) It also stands to reason that the number-aware RIC might optimize for the number of information sources utilized by each of its BIPs.

Our analysis in the rest of this section shows that conjectures (i)-(iv) are consistent with each other and with (7),(8),(9).

***$i$-th BIP of a number-aware RIC:*** From here on, we will analyze the $i$-th BIP of the number-aware RIC. To facilitate analysis, we will assume that, $\alpha(v) = \alpha$, $\beta(v) = \beta$, and $r(v) = r$ when $v = v_i$ or $v \in$ sources$(v_i)$. With this assumption, it turns out that the individual biases of the BIPs' sources will be of no relevance to the RIC node $v_D$. Which covers under it the conditions where the individual biases of the BIPs' sources are unavailable to $v_D$, or are of no interest to $v_D$.

As stated earlier, here we consider the cases where $\alpha \geq \beta$, $r \leq \frac{1}{2}$, and $\alpha + \beta < 1$. As discussed before, each RIC in the network regardless of its actual location in the hierarchy assumes that it is in $\mathcal{V}(3)$ and that all the BIPs' input-output behavior is characterized by identical binary channels, $\mathcal{C}_{\alpha(v_i)}$.

Therefore, an RIC can only maintain its assumption that all nodes have the same channel characteristics if $\alpha_{aug}(v_i) \approx \alpha$, etc. The assumption can hold, to an approximation, if for each BIP with $N_i$ sources we can find a lower bound on $N_i$ such that $\alpha_{aug}(v_i) - \alpha < \epsilon$, $\beta_{aug}(v_i) - \beta < \epsilon$, and $r_{aug}(v_i) - r < \epsilon$. In essence, the number-aware RIC at node $v_D$, when choosing its $i$-th BIP is enforcing a minimum requirement on the number of sources ($N_i$) of the $i$-th BIP. By doing so, the

RIC is ensuring that its own estimates of $\alpha(v_i)$, $\beta(v_i)$, and $r(v_i)$ do not deviate from their actual values by more than an $\epsilon$ amount. Thus, the number-aware RIC is ensuring that the BIPs utilize sufficiently many sources to arrive at a highly-reliable estimate of the ground truth themselves.

In summary, since the RIC is only number-aware, using our critical assumption from above, we will assume that $\alpha(v) = \alpha$, $\beta(v) = \beta$ for all $v \in \mathcal{V}$ in this analysis. And since any RIC in $\mathcal{V}(h)$, $h \geq 3$ views itself as being in $\mathcal{V}(3)$, we also need this assumption to remain valid regardless of whether the RIC is in $\mathcal{V}(3)$ or not.

Consider $v_i$, the $i$-th BIP of the RIC node $v_D$. From the perspective of the RIC, the node $v_i$ is in $\mathcal{V}(2)$, and thus, $v_i$ is making a graph-blind decision regarding the true value of $X$. Let $\hat{X}_i$ be the estimate of the ground truth made by the node $v_i$ using $N_i$ independent and exclusive BIPs of its own, according to the RIC.

***Decision policy of the $i$-th BIP of the number-aware RIC:*** From the perspective of the RIC, the observations that $v_i$ receives from its own BIPs are unknown and can be denoted by $\mathbf{Y}_{in}^{(n)}(v_i)$. Let $k$ and $N_i - k$ be the number of 0s and 1s in $\mathbf{Y}_{in}^{(n)}(v_i)$, respectively. Since all the nodes are assumed to have identical channel characteristics, the estimation of $X$ made by $v_i$ is as follows.

$$\rho_0 \alpha^{N_i - k}(1-\alpha)^k \underset{\hat{X}_i=1}{\overset{\hat{X}_i=0}{\gtrless}} \rho_1 (1-\beta)^{N-k}\beta^k$$

$$\Longrightarrow k \underset{\hat{X}_i=1}{\overset{\hat{X}_i=0}{\gtrless}} k_i^* = \frac{N_i \cdot L_a - L_p}{L_a + L_b}, \tag{11}$$

where $L_a := \log\left(\frac{1-\beta}{\alpha}\right) \leq L_b := \log\left(\frac{1-\alpha}{\beta}\right)$, and $L_p := \log\left(\frac{\rho_0}{\rho_1}\right) \geq 0$ since $\alpha \geq \beta$, $\alpha + \beta \leq 1$, and $\rho_0 \geq \rho_1$. Since $L_a \leq L_b$, we also see that $k_i^* \leq \frac{1}{2} \cdot N_i$.

According to (7), if $\alpha_{aug}(v_i) - \alpha = (1-\alpha-\beta)P^0_{e,\mathcal{V}_{in}(v_i)} \leq \epsilon > 0$ then we must obtain the value of $N_i$ for which $P^0_{e,\mathcal{V}_{in}(v_i)} \leq \frac{\epsilon}{1-\alpha-\beta} =: \delta > 0$. It is obvious that if $\alpha + \beta = 1$ or if $\epsilon \geq 1 - \alpha - \beta$, then $\alpha_{aug}(v_i) - \alpha \leq \epsilon$ for all $N_i$. The following theorem discusses the acceptable values of $N_i$ if $0 < \epsilon \leq 1 - \alpha - \beta$.

**Theorem 3.** *Consider any $r \leq \frac{1}{2}$ with $\alpha$ such that $\alpha \geq \beta$ and $\alpha + \beta \leq 1$. Then, we have $\alpha_{aug}(v_i) - \alpha < \epsilon$ for some $0 < \epsilon < 1 - \alpha - \beta$ if*

$$N_i \geq \frac{\log\epsilon - \log(1-\alpha-\beta) + \log\left(\dfrac{\sqrt{2\pi f_a(1-f_a)}}{1 + \dfrac{\alpha f_a}{1-\alpha-f_a}}\right)}{f_a \log\left(\dfrac{1-\alpha}{f_a}\right) + (1-fa)\log\left(\dfrac{\alpha}{1-f_a}\right)}, \tag{12}$$

*where $f_a := \dfrac{L_a}{L_a + L_b}$.*

*Similarly, we have $\beta_{aug}(v_i) - \beta < \epsilon$ for some $0 < \epsilon < 1 - \alpha - \beta$ if*

$$N_i \geq \frac{\log\epsilon - \log(1-\alpha-\beta) + \log\left(\dfrac{\sqrt{2\pi f_b(1-f_b)}}{1 + \dfrac{\beta f_b}{1-\beta-f_b}}\right)}{f_b \log\left(\dfrac{1-\beta}{f_b}\right) + (1-fb)\log\left(\dfrac{\beta}{1-f_b}\right)}, \tag{13}$$

*where $f_b := \dfrac{L_b}{L_a + L_b}$ and $N$ is such that $k^* = f_a N - \dfrac{L_p}{L_a + L_b} \geq \beta N$. That is,*

$$N \geq \frac{L_p}{L_a - (L_a + L_b)\beta}.$$

*If $\alpha_{aug}(v_i) - \alpha < \epsilon$, $\beta_{aug}(v_i) - \beta < \epsilon$, then $r_{aug}(v_i) - r < \epsilon$.*

From Theorem 3, we now know the minimum required number of sources $N_i$ for the $i$-th BIP of the number-aware RIC $v_D$ to have $\alpha_{aug}(v_i) - \alpha \in [0, \epsilon)$ and $\beta_{aug}(v_i) - \beta \in [0, \epsilon)$ (and thus, $r_{aug}(v_i) - r \in [0, \epsilon)$). We already know that if an RIC is aware of the number of information-providers that its $i$-th BIP is using, then it can impose restrictions on the number of information-providers that its $i$-th BIP must use in order for its information to be considered trustworthy. We also now know that a number-aware RIC can through reasonable assumptions obtain a minimum number of such information-providers that the $i$-th BIP must use. Further, we notice that such a requirement on the part of the number-aware RIC will also ensure that if the $i$-th BIP is highly-biased or unbiased or otherwise on its own, then the augmented version of the $i$-th BIP that is observable by the RIC is also highly-biased or unbiased or otherwise (i.e., has the same behavior in terms of bias).

In this section, we considered an information network where an RIC assumes that any BIP and its sources will have identical input-output channel characteristics. Using this simplifying assumption, we derived the behavior that is perceived as optimal by the number-aware RIC, a partially-aware RIC that is aware of the number of information-providers its own BIPs referred to.

In the next section, we will consider a number-aware RIC that is also aware that there is an "influential" node that acts as *a common source for all $n$ of its BIPs*. The presence of the "influential" node and the RIC's awareness of it implies that the RIC no longer sees its BIPs as independent.

### C. Effect of one Influential node on a Number-aware RIC

In this section, we will study how the knowledge of the presence of a single "influential" node among the $n$ BIPs of a number-aware RIC could change the RIC's decision policy and the associated probability of error. We note the same analysis holds for scenarios where the number-aware RIC knows of coordinated action among $n$ distinct nodes each of which acts as a source for exactly one of its $n$ BIPs.

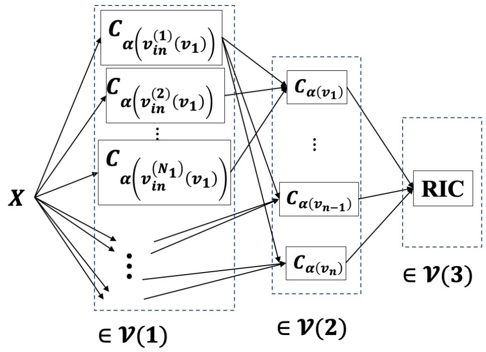



Fig. 7: An influential node serving as a source each of the $n$ BIPs of an RIC, from the perspective of a number-aware RIC.

***Influential Node:*** In this section, when we say that a node $v_I$ is influential with respect to the RIC node $v_D$, we mean that the node $v_I$ is two hierarchical levels above $v_D$ and that it is the information-provider to all $n$ BIPs of $v_D$ as shown in Figure 7.

***Perceived effect of a single influential Node:*** We will assume that all the BIPs of the $v_D$ have $N$ RICs. Let the influential node $v_I$ be the first node indexed in $\mathcal{V}(1)$, as well as the first node indexed in $\mathcal{V}_{in}(v_i)$, $\forall\, i \in \{1, \cdots, n\}$.

We assume that the influential node $v_I$ sends the same outcome, either $Y_I = 0$ or $Y_I = 1$, to all the nodes that it sends reports of the event to, including all the $n$ BIPs of the number-aware RIC ($v_D$). Let us denote by $(\alpha_{aug}(0), \beta_{aug}(0))$ and $(\alpha_{aug}(1), \beta_{aug}(1))$ to be the biases of the $i$-th augmented BIP when the output of the influential node is 0 and 1, respectively. Similarly, we extend the notations and definitions for $r_{aug}(Y_I)$ and $k^*_{aug}(Y_I)$.

Then, we continue to calculate the average probability of error for an RIC that is aware of this dependency.

***Decision policy:*** Let $\pi^*_d(\cdot)$ and $\hat{\pi}^*_d(\cdot)$ be the decision policies that are perceived as optimal by the number-aware RIC when it is aware of the influential node and when it is unaware of the influential node, respectively.

We are interested in the perceived increase in probability of error that the number-aware RIC expects to incur when it either ignores or remains unaware of the dependency. We denote this ratio by $\rho_d(v_I)$.

$$\rho_d(v_I) = \frac{P_e(\hat{\pi}^*_d)}{P_e(\pi^*_d)}. \tag{14}$$

***Biases of the BIPs conditional on*** $Y_I$***:*** Now, let us denote $T_{k,N}(x) := x^{N-k}(1-x)^k$.

Given the $k^*$ in (11), when the influential node $v_I$ broadcasts the outcome $Y_I = 0$ (this occurs w.p. $(1-\alpha)$ when $X = 0$ and w.p. $\beta$ when $X = 1$) to all $n$ BIPs (each of which have $N$ information-providers of their own),

$$\alpha_{aug}(0) = \alpha + \frac{(1-\alpha-\beta)}{N(1-\alpha)} \sum_{k \le k^*} k \binom{N}{k} T_{k,N}(\alpha) \le \alpha_{aug}$$

$$\beta_{aug}(0) = \beta + \frac{(1-\alpha-\beta)}{N\beta} \sum_{k > k^*} k \binom{N}{k} T_{N-k,N}(\beta).$$

Similarly, when the influential node $v_I$ broadcasts the outcome $Y_I = 1$ (this occurs w.p. $\alpha$ when $X = 0$ and w.p. $(1-\beta)$ when $X = 1$) to all $n$ BIPs,

$$\alpha_{aug}(1) = \alpha + \frac{(1-\alpha-\beta)}{N\alpha} \sum_{k \le k^*} (N-k) \binom{N}{k} T_{k,N}(\alpha) \ge \alpha_{aug}$$

$$\beta_{aug}(1) = \beta + \frac{(1-\alpha-\beta)}{N(1-\beta)} \sum_{k > k^*} (N-k) \binom{N}{k} T_{N_k,N}(\beta).$$

Also, we notice that, the characteristics of the augmented channels with and without being conditioned on the influential node's $Y_I$ are related as follows.

$$\alpha_{aug} = \mathbb{E}_{Y_I}\left[\alpha_{aug}(Y_I)\right] = (1-\alpha)\alpha_{aug}(0) + \alpha\alpha_{aug}(1)$$
$$\beta_{aug} = \mathbb{E}_{Y_I}\left[\beta_{aug}(Y_I)\right] = \beta\beta_{aug}(0) + (1-\beta)\beta_{aug}(1).$$

***Influence-aware decision policy:*** To arrive at the optimal decision policy of the number-aware RIC it is sufficient to find $k^*_{aug}$, $k^*_{aug}(0)$, and $k^*_{aug}(1)$ to compare against the number of 0's received by the number-aware RIC from $n$ BIPs.

For each of the cases, using the values of the biases $(\alpha_{aug}, \beta_{aug})$, $(\alpha_{aug}(0), \beta_{aug}(0))$, and $(\alpha_{aug}(1), \beta_{aug}(1))$ we arrive at the values of $k^*_{aug}$, $k^*_{aug}(0)$, and $k^*_{aug}(1)$ from (11).

Now, for a fixed $n$, let us denote $T_k(x) := x^{n-k}(1-x)^k$ and $T_{n-k}(x) := x^k(1-x)^{n-k}$. We can calculate the RIC's perceived probability of error when it is unaware of the influential node $v_I$ and its perceived probability of error when it is aware of $v_I$ as:

$$P_e(\hat{\pi}^*_d) = \sum_{k \le k^*_{aug}} \rho_0 \binom{n}{k} \left[(1-\alpha)T_k(\alpha_{aug}(0)) + \alpha T_k(\alpha_{aug}(1))\right]$$
$$+ \sum_{k > k^*_{aug}} \rho_1 \binom{n}{k} \left[\beta T_{n-k}(\beta_{aug}(0)) + (1-\beta)T_{n-k}(\beta_{aug}(1))\right]. \tag{15}$$

$$\begin{aligned} P_e(\pi^*_d) = {} & \rho_0(1-\alpha) \sum_{k \le k^*_{aug}(0)} \binom{n}{k} T_k(\alpha_{aug}(0)) \\ & + \rho_0\alpha \sum_{k \le k^*_{aug}(1)} \binom{n}{k} T_k(\alpha_{aug}(1)) \\ & + \rho_1\beta \sum_{k > k^*_{aug}(0)} \binom{n}{k} T_{n-k}(\beta_{aug}(0)) \\ & + \rho_1(1-\beta) \sum_{k > k^*_{aug}(1)} \binom{n}{k} T_{n-k}(\beta_{aug}(1)). \end{aligned} \tag{16}$$

Now, in keeping with Section V-B, let us assume that $\alpha$, $\beta$, $N$ are such that $\alpha_{aug} \ge \beta_{aug}$, $\alpha_{aug} + \beta_{aug} < 1$ along with $\alpha_{aug}(0) \ge \beta_{aug}(0)$, $\alpha_{aug}(0) + \beta_{aug}(0) < 1$ and $\alpha_{aug}(1) \ge \beta_{aug}(1)$, $\alpha_{aug}(1) + \beta_{aug}(1) < 1$.

**Theorem 4.** *Fix a target number-aware RIC that has $n$ BIPs and denote it by $v_D$. Assume that each of the $n$ BIPs utilizes $N$*

*sources. Suppose a single influential node $v_I$ and the decision policies $\pi_d^*$ and $\hat{\pi}_d^*$ as described above. Consider the ratio $\rho_d(v_I)$ comparing the two decision policies, as defined in* (14)*.*

*For the case where $k_{aug}^* \leq k_{aug}^*(0), k_{aug}^*(1)$, the magnitude of the ratio $\rho_d(v_I)$ is bounded below by* 1*.*

*On the other hand, $\rho_d(v_I) - 1$ is bounded above by:*

$$\sum_{Y_I} \left[ \frac{P_e\left(\pi_d^*; k \in \left(k_{aug}^*, k_{aug}^*(Y_I)\right] \mid X=0, Y_I\right)}{\frac{P(X=0;Y_I)}{P(X=1;Y_I)}} * e^{n\left(\frac{1-\alpha_{aug}(Y_I)-\beta_{aug}(Y_I)}{\alpha_{aug}(Y_I)}\right)} \right]. \quad (17)$$

*where, $N$ is such that $\alpha_{aug}(0), \alpha_{aug}(1) \leq \alpha + \epsilon$, $\beta_{aug}(0), \beta_{aug}(1) \leq \beta + \epsilon$ (Theorem 3) and $k^* \geq N\beta$.*

*Similar bounds can be derived for other conditions on the values $k_{aug}^*$, $k_{aug}^*(0)$, and $k_{aug}^*(1)$ in relation to each other.*

***Notes on the effect of one influential node on an RIC**:* Recall that, $\alpha > r^{(\boldsymbol{\rho})} > \beta$. In the above theorem, the RIC would perceive its own awareness of the influential node to be *at most exponentially* beneficial with a positive exponent which:

(i) is *vanishingly* small when $\alpha + \beta \to 1$ (i.e., a nearly ineffective BIP with $P(Y=0|X) \approx P(Y=1|X)$);

(ii) is *arbitrarily large only when* $r^{(\boldsymbol{\rho})} \leq \alpha_{aug}(Y_I) \to 0$ (i.e., a nearly perfect BIP).

Further, we note that:

(iii) if $\beta \to 0$, then the error corresponding to $P_e\left(\pi_d^*; k \in \left(k_{aug}^*, k_{aug}^*(Y_I)\right] \mid X=0, Y_I\right)$ also approaches zero. Thus, when the BIPs are completely biased against the a-priori likely outcome, the upper bound on the ratio $\rho_d(v_I)$ approaches one. That is, the error probability of the decision policy $\pi_d*$ approaches that of the sub-optimal decision policy $\hat{\pi}_d^*$.

(iv) if $N$ exceeds the minimum number required by Theorem 3 for $\alpha_{aug}(0), \alpha_{aug}(1), \alpha_{aug} \approx \alpha$ and $\beta_{aug}(0), \beta_{aug}(1), \beta_{aug} \approx \beta$, then $P_e\left(\pi_d^*; k \in \left(k_{aug}^*, k_{aug}^*(Y_I)\right] \mid X=0, Y_I\right)$ approaches zero. Thus, when the BIPs of the node $v_D$ rely on sufficiently many information-providers of their own, the upper bound on the ratio $\rho_d(v_I)$ approaches one. That is, the error probability of the decision policy $\pi_d*$ approaches that of the sub-optimal decision policy $\hat{\pi}_d^*$.

Therefore, upon becoming aware of an influential node among the sources of its BIPs, the influence-aware RIC anticipates to reduce its error probability by a value that is at most an exponential with the exponent being a multiple of $n$. *However, we can see that from the perspective of the influence-aware RIC: (a) choosing BIPs that are fully-biased against the a-priori likely outcome; (b) each of which, in turn, utilizes sufficiently many sources/information-providers of its own; would take precedence over the RIC being aware of a single influential node.* Thus, we discover that the preference of similarly-and-highly-biased BIPs as the optimal choice is not limited to graph-blind RICs, but is retained for both number-aware and influence-aware RICs. Further, the preference for BIPs with a high number of sources/information-providers of their own extends beyond number-aware RICs, and onto influence-aware RICs.

In this section, we evaluated the effect of a single influential node on the choices perceived as optimal by the number-aware RIC. In the next section, we will numerically evaluate the least number of sources that a number-aware RIC will require from each of its BIPs, and thus, validate the results of Theorem 3.

### D. Numericals: Minimum number of sources used by the BIPs of a number-aware RIC

In the previous section, we obtained the minimum number of sources that a BIP must utilize in order to appear as the optimal choice to the number-aware RIC (cf. Theorem 3). In this section, we will numerically validate the results of Theorem 3 and record our observations. In Figure 8, we only demonstrate the minimum number of information-providers that the $i$-th BIP must use for a number-aware RIC to assume that $\alpha_{aug}(v_i) \approx \alpha$. Later, in the Appendix in Figure A.2, we demonstrate the minimum number of information-providers that the $i$-th BIP must use for a number-aware RIC to assume that $\beta_{aug}(v_i) \approx \beta$. In conjunction with each other, these figures demonstrate Theorem 3 and the minimum number of information-providers that the $i$-th BIP must use for a number-aware RIC to assume that $r_{aug}(v_i) \approx r$, along with $\alpha_{aug}(v_i) \approx \alpha$, $\beta_{aug}(v_i) \approx \beta$.

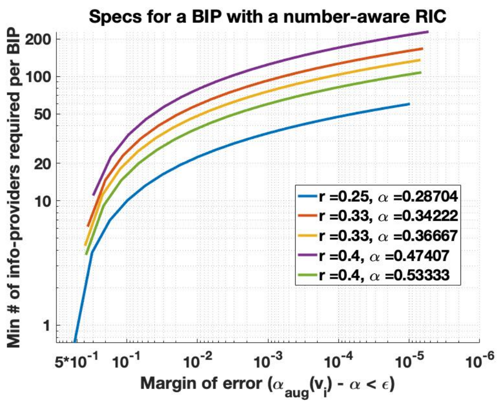
In Figures 8, A.2, we notice that in an information network with a number-aware RIC, the minimum number of information-providers that each BIP must use for $(\alpha_{aug}(v_i), \beta_{aug}(v_i), r_{aug}(v_i)) \approx (\alpha, \beta, r)$ will increase:

1) *When the individual rate of error $r$ increases.* As the individual rate of error $r$ increases across the network, the $i$-th BIP will need to consult more information-providers to identify the ground truth without increasing its probability of making an error.
2) *When the value of $\alpha$ decreases ($\beta$ increases) while $r$ remains the same.* As the information-providers/sources of the $i$-th BIP become more biased, the $i$-th BIP can consult a smaller number of them without increasing its probability of making an error.

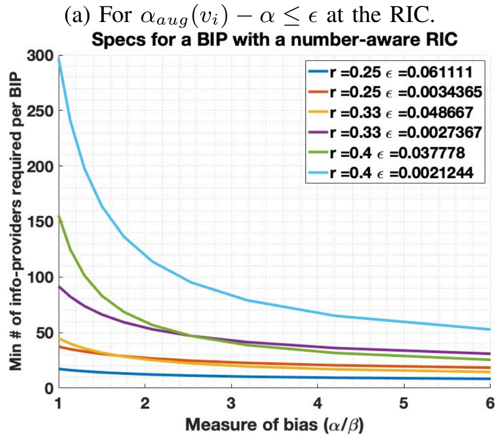
## VI. Conclusion

In this work, we consider information networks with nodes that act as biased-information-providers (BIPs), rational-information-consumers (RICs), or both. Inspired by modern-day social networks, we consider biases and allow the RICs to have limitations. We previously found that a graph-blind RIC would act according to a counter-intuitive, but tractable rule. It would rationally perceive a set of $n$ BIPs that are fully-biased against the a-priori likely outcome to be its optimal choice when compared to any other set of $n$ BIPs with similar individual error rates.

In this work, we extend our analysis to partially-aware RICs. We find that RICs that are aware of the number of sources utilized by each of its BIPs will continue to perceive fully-biased BIPs as its optimal choice for any fixed individual error




(a) For $\alpha_{aug}(v_i) - \alpha \leq \epsilon$ at the RIC.




(b) For $\alpha_{aug}(v_i) - \alpha \leq \epsilon$ at the RIC.

Fig. 8: Minimum number of information-providers that the $i$-th BIP must use for the RIC to assume that $\alpha_{aug}(v_i) \approx \alpha$. With this, when given that $\beta_{aug}(v_i) \approx \beta$, the RIC can assume $r_{aug}(v_i) \approx r$.

rate. With reasonable assumptions, the number-aware RIC can also pick BIPs with sufficiently many sources, making the properties of the individual sources of each BIP unnecessary to its estimation of the ground truth. Finally, we analyze the effect of a single node that influences all $n$ BIPs of a number-aware RIC. We find that the number-aware RIC would anticipate its discovery of a single influential node to reduce its error probability exponentially with increasing $n$. However, we also find that the rate of this exponent would become less significant if the BIPs are highly biased and have sufficiently many sources.

Our analysis further emphasizes the perceived optimality and perceived importance of fully and similarly-biased BIPs to an RIC with human-like limitations. These insights could help adapt information networks to: i) become more RIC-centric, and ii) leverage bias to reduce misinformation.

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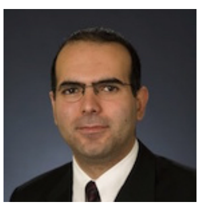

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